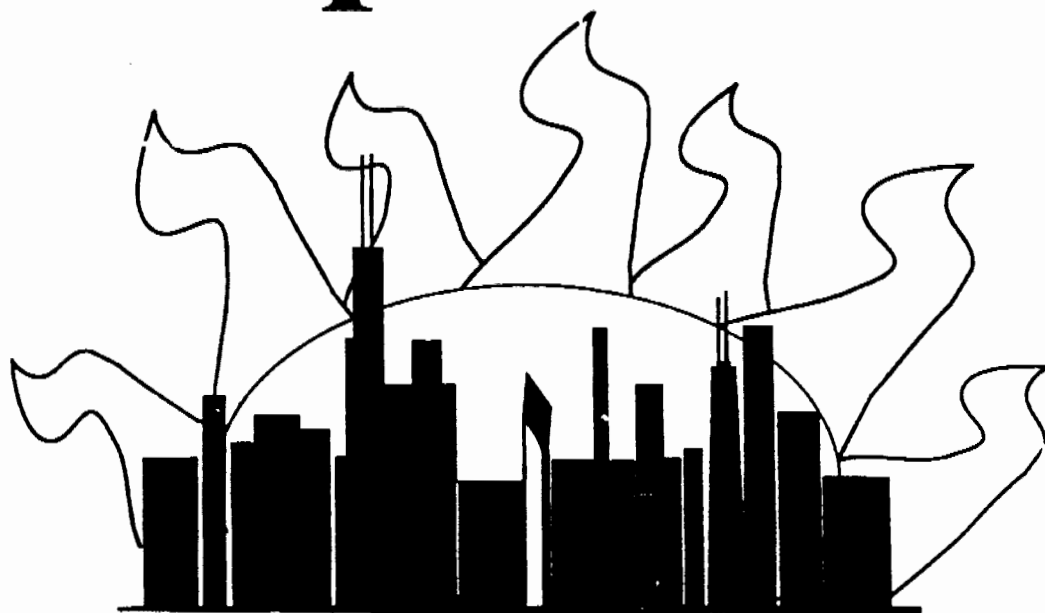


**Alpha Division**

**Topic Test 3**

# **Logs & Exponents**



**Mu Alpha Theta National Convention  
Chicago 1998**

**General Instructions:**

Unless otherwise stated all answers should be written as decimals.

If you are asked to give your answer as a fraction, please give your answer in  $a/b$  form where  $a$  and  $b$  are relatively prime.

**Questions**

1. Find the real value of  $x$  which satisfies  $(\log_x 2x)(\log_{10} x) = 3$ .
2. What is the value of  $\log_5 \frac{(125)(625)}{25}$ ?
3. If  $\log_3 27$  and  $\log_7 243$  are roots of  $2x^2 + bx + k = 0$ , find  $k$ .
4. If  $f(x) = x^{4x-2}$ , find  $(f(x))(f(1-x))$ .
5. You have \$50,000 in an account that pays 10%, compounded annually. You want to withdraw the same amount at the beginning of each year for the next five years, starting immediately, so that after your last withdrawal, there is no money left in the account. How much money (to the nearest dollar) should you withdraw each year?
6. A musical group that plays hard rock has been measured at a sound level of 120 decibels (dB) while a group that plays soft rock music has been measured at a sound level of 95 dB. How many times more intense is the sound of the hard rock group than the sound of the soft rock group? The formula for sound intensity is  $dB = 10 \log \frac{I}{I_0}$  where  $I$  is the sound intensity and  $I_0$  is an arbitrarily chosen standard. Give your answer rounded to the nearest tenth.
7. The sides of a triangle have lengths  $\log_2 3$ ,  $\log_2 7$ , and  $\log_2 x$ . Find the least possible integral value of  $x$ .
8. Find the smaller of the solutions of  $x: 3^{\log_3(3x^2)} - 24 = 0$  Give an exact answer.
9. Find  $x$  where  $x = (\log_a 2)(\log_b a)(\log_4 b)$ . Give your answer as a simplified fraction.

10. If  $3^{8x+1} = 9$ , find  $\log_2 x$ .
11. Find the greatest integer value of  $x$  which satisfies  $4^{x^3 + 5x^2 - 6x} = 1$ .
12. If  $b = \log_3 x$ , find the real value of  $x$  which satisfies  $\log_b (\log_3 x^2) = 2$ .
13. If  $\log_{10} 25 = 1.39794$ , find the number of digits in the complete expansion of  $25^{100}$ .
14. If  $3^{33} + 3^{33} + 3^{33} = 3^x$ , find  $x$ .
15. The expression  $(x^{-\frac{1}{2}} + 3)^7$  is expanded using the Binomial Theorem in order of increasing powers of  $x$ . Find the numeric coefficient of the fourth term.
16. A Christmas tree has  $3^0$  trunks,  $3^3$  branches on each trunk,  $3^6$  branchlets on each branch, and  $3^9$  needles on each branchlet. If  $x$  represents the total number of needles on the tree, find  $\log_3 x$ .
17. If  $4 \sin^2 x + 11 \sin x - 3 = 0$ , then what is the value of  $\log_2 \sin x$ ?
18. Let  $H = 1$ . If  $a$ ,  $b$ , and  $c$  are positive integers, compute the numerical value of
- $$\log\left(\frac{2a}{b}\right) + \log\left(\frac{2b}{c}\right) + \log\left(\frac{5c}{d}\right) + \log\left(\frac{5d}{a}\right) + \log H$$
19. Find, in degrees, the least positive angle  $x$  for which  $\log_2 \cos x = -\frac{1}{2}$ .
20. Find the volume of the cube whose surface area is  $\sqrt[4]{\log_{10} 7.1}$ . Give your answer rounded to four decimal places.