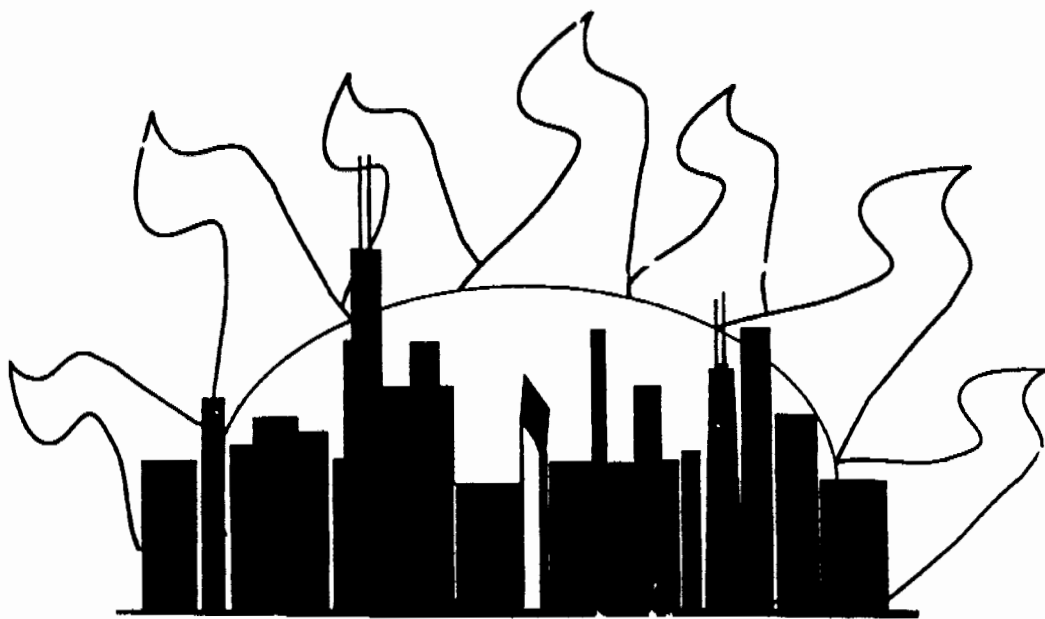


**Alpha Division**

**Topic Test 1**

# **Analytic Geometry**



**Mu Alpha Theta National Convention  
Chicago 1998**

**General Instructions:**

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in  $\frac{a}{b}$  form where  $a$  and  $b$  are relatively prime.

**Questions**

1. Find the distance between the origin and the center of the ellipse with equation  $x^2 + 2y^2 + 2x - 4y - 6 = 0$ .
2. P (4, 2) is on the circle with equation  $(x - 3)^2 + (y - 4)^2 = 5$ . Find the equation of the tangent line to the circle at P.
3. Find the sum of the coordinates of the points where the line  $y = 2x - 2$  and the circle  $x^2 + y^2 = 25$  intersect. Give your answer as a simplified fraction in lowest terms.
4. Find the length of a tangent line segment from (10, 5) to the point of tangency with the circle  $x^2 + y^2 = 25$ .
5. Find the distance from the focus of the parabola whose equation is  $x^2 = 8y - 56$  to the center of the circle whose equation is  $x^2 + y^2 + 6x - 10y + 1 = 0$ .
6. The eccentricity of a certain conic is  $\frac{\sqrt{3}}{5}$ . How many different permutations are there of all the letters in the name of this conic?
7. Find the slope of the line which contains the points of intersection of the circles

$$x^2 + y^2 - 5x + 12y + 2 = 0$$

$$x^2 + y^2 + 7x - 6y - 16 = 0$$

Give your answer as a simplified fraction in lowest terms.

8. A point  $(x, y)$  lies inside the circle  $x^2 + y^2 = 2$  and above the line  $y = 1$ . Find the area of the region that is enclosed by these equations and contains the point. Give answer rounded to four significant digits.
9. In a coordinate plane, a circle which passes through  $(-2, -3)$  and  $(2, 5)$  cannot also pass through  $(1998, y)$ . Find the value of  $y$ .
10. What is the length of the latus rectum of the conic section given by the equation  

$$8x^2 + 9y^2 + 16x + 45y + 10 = 0?$$

11. In a coordinate plane, find the area of the region common to

$$\begin{aligned} x &\geq 0 \\ y &\leq 1 \\ \text{and } x^2 + y^2 &\leq 4y \end{aligned}$$

Give your answer as a decimal rounded to four significant digits.

12. The center of a hyperbola is  $(5, -1)$  and its axes are parallel to the coordinate axes. An asymptote of this hyperbola has equation  $3x + 4y - 11 = 0$ . Find the  $y$ -intercept of the equation of the other asymptote. Give your answer as a simplified fraction in lowest terms.
13. Two perpendicular planes have the equations:  

$$4x - 5y + 2z = 10$$

$$3x + ay + bz = -5$$
 The plane of the second equation also contains the point  $(1, 5, 3)$ . Determine the exact value of  $a + b$ . Give your answer as a simplified fraction in lowest terms.
14. Find the number of degrees, to the nearest minute, in the acute dihedral angle between the planes  $3x - 4y + 5z = 10$  and  $6x + 2y - 3z = 6$ . Give your answer as a fraction:  $\frac{\text{degrees}}{\text{minutes}}$ .
15. A sphere is inscribed in a cube of volume 409.6 cu. cm. The numerical value of the surface area of the cube is what percent of the square of the numerical value of the sphere's surface area? Give your answer rounded to four significant digits.
16. An ellipse, with center  $(3, 0)$  and one focus at  $(0, 0)$  passes through the point  $(-2, 1)$ . Find the length of the minor axis. Give your answer rounded to four significant digits.

17. Find the distance from the point  $(1, 2)$  to the point on the circle  $x^2 + 10x + y^2 + 6y = -30$  that is closest to the point  $(1, 2)$ . Give your answer rounded to four significant digits.
18. A certain parabola passes through the points  $(5, 1)$  and  $(13, -7)$  and has the  $y$ -axis as its directrix. Find the sum of the  $x$  and  $y$  coordinates of the point farthest from the origin at which the vertex of this parabola could be located.
19. Suppose  $a, b, c, d, e, f$  are real numbers. The graph of  $y = \frac{ax^2 + bx + c}{dx^2 + ex + f}$  has asymptotes of  $y = 2$ ,  $x = -3$ , and  $x = 4$ . The graph crosses the  $x$ -axis at  $(-1, 0)$  and  $(3, 0)$ . Find the  $y$ -coordinate of the point where the graph crosses the  $y$ -axis. Give your answer as a simplified fraction in lowest terms.
20. A rifle bullet is shot with an initial velocity of 900 ft/sec. The rifle is at a height of 6 feet and is angled up  $30^\circ$  from the horizontal. The parametric equations which describe the motion are:  $x = 900[\cos(30^\circ)]t$ ,  $y = 6 + 900[\sin(30^\circ)]t - 16t^2$ . Find the distance, in feet, traveled horizontally before the bullet strikes the ground. Give your answer to the nearest foot.