Theta Sequences and Series Test
2015 Mu Alpha Theta National Convention

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Answers:

1) B
2) D
3) B
4) D
5) A
6) E
7) B
8) B
9) C
10) A
11) C
12) C
13) C
14) D
15) D
16) C
17) D
18) A
19) A
20) B
21) A
22) D
23) E
24) A
25) A
26) A
27) B
28) C
29) A
30) D

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Solutions:

1) The problem describes a finite arithmetic series and an infinite geometric series. $\sum_{n=0}^{9}(100000 + 10000n) + \sum_{n=1}^{\infty} 190000/2^n$ = 1640000.
2) Can use the formula for sum of squares from 1 to 9, or simply add up 81 + 64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = 285
3) The 3rd term in the series is 10, so x must equal 24.
4) The last term is ½*(1/3)^49 which is very small, so to the hundredth place the sum can be approximated as the sum of an infinite sequence with first term ½ and a difference of 1/3. The answer is ½/(1-1/3) = 0.75
5) The sequence -4,0,4,8,12,16,20,24,28 contains 28.
6) This can be thought of as 4 series, the 1 character, 2, character, 3 character, and 4 character. The sum of the series are: 1*(9) + 2*(99-9) + 3*(999-99) + 4*(2014-999) = 6949.
7) Separate into two summations – the sum of squares from 1 to 7 and the sum of integers from 1 to 7. 7(7+1)(2*7+1)/6 + (7+1)*7/2 = 168.
8) Can rearrange formula to $a_{n+1} = a_{n-1} - a_n$. Then, a0 = 2, a1 = 3, a2 = -1, a3 = 4, a4 = -5, a5 = 9.
9) 1/(1-1/3) = ½.
10) 2^10 = 1024. 1024*1024 will be > 1000000, so x = 10.
11) Common difference is (31-7)/12 = 2. 7 + 5*2 = 17.
12) 3/(1/2+1/6+1/12) = 4
13) First evaluate the inner sum: 5+7+9 = 21. Then evaluate the outer sum: 24 + 45 + 66 = 135.
14) The summation can be simplified to 6n-4, which is 330-40 = 290.
15) The sum of the first 10 terms is 65 + 145x, so a+b = 210.
16) This summation expands to (81*4+27*7+9*10+3*13+1*16)/243=658/243
17) For the 3 means to be equal, the sequence must be symmetric around zero, so the common difference must equal 2.
18) S = 1/3 + 2/9 + 3/27…. So S/3 = 1/9 + 2/27…. Subtract the two to get an infinite geometric sequence with first term 1/3 and common difference 1/3. S*2/3 = 1/2, making s = ¾.
19) The 2015 term is -3+2015*4 = 8057, so the units digit is 7.
20) 20 + 14/99 = 1994/99.
21) The difference between terms is 13/2, so the last term is the 12th term.
22) There are 2 sequences, 1,2,3,4,7 and 1,2,3,5,6.
23) The inner summation is equal to 1/3, so it reduces to 3^n, which has an infinite summation.
24) 7th pentagonal is 70, 6th hexagonal is 66, 5th heptagonal is 55, and the summation is 65, so the 7th pentagonal is the largest.
25) The roots can be picked off by inspection: -1,-2,-3,-4,-5, so the sum is -15.
26) The change is the same as asking for the difference between the 10th and 12th term, which is the 11th term in the original sequence: 55.
27) 10 choose 3 = 120. There are 8 series with a common difference of 1, 6 with 2, 4 with 3, and 2 with 4. 8+6+4+2 = 20. 20/120 = 1/6.
28) This is 2^11-1 also minus the 2^0 term, so 2^11-1-1=2048-2=2046.

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29) The nested square roots can be rewritten as sqrt(10+x) = x. This is a quadratic, of which only the positive solution makes sense.
30) The sequence is 1,4,11,16,21,64,31,256,41,1024,51, so the sum is 1024+51=1075.