Theta Sequences and Series Test
2015 Mu Alpha Theta National Convention
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1. John has an offer for $100,000 a year in pay and a $10,000 raise each year starting at the end of the first year, effective for the next year. After 10 years of being paid, he retired and receives a pension salary that starts at 50% of his yearly pay and is reduced by 50% each additional year. How much does John get paid in total if he lives forever?

   A) $1,283,000   B) $1,640,000   C) $1,830,000   D) $1,980,000   E) NOTA

2. John uses his retirement money to build a square pyramid consisting of 1x1 blocks. If the base is 9x9, and each layer decreases by 1 in each horizontal dimension (the next level is 8x8), how many blocks does it take to finish the pyramid?

   A) 287   B) 245   C) 305   D) 285   E) NOTA

3. Solve for x given the arithmetic sequence: $6, 8, \sqrt{x + 76}$
   A) 10   B) 24   C) -176   D) 68   E) NOTA

4. Find the sum of the first 50 terms of the geometric series to the nearest hundredth place:
$$\frac{1}{2} + \frac{1}{6} + \frac{1}{18} \cdots$$

   A) 1.49   B) 1.50   C) 0.74   D) 0.75   E) NOTA

5. Which arithmetic sequence includes 28?

   A) -4, 0, 4, …   B) 1, 3, 5, …   C) 2, 6, 10, …   D) 3, 6, 9, …   E) NOTA

6. If you were to write down the integers between 1 and 2014 inclusive, how many characters would you write? (1 is 1 character, 10 is 2 characters, etc)

   A) 7200   B) 1579   C) 6952   D) 6499   E) NOTA

7. Solve for x: $\sum_{n=0}^{7}(n^2 + n) = x$

   A) 140   B) 168   C) 208   D) 96   E) NOTA

8. A new sequence is defined as: $a_n = -a_{n+1} + a_{n-1}$. Find a_5 given $a_0 = 2$ and $a_1 = 3$.

   A) -5   B) 9   C) 4   D) -12   E) NOTA

9. $\sum_{n=1}^{\infty}(1/3)^n$

   A) 0   B) 1/3   C) 1/2   D) 2/3   E) NOTA

10. Find the smallest positive integer x that satisfies the inequality:

$$\prod_{n=1}^{x}(2)^2 > 1000000$$

A) 10   B) 16   C) 20   D) 21   E) NOTA

11. For an arithmetic series with $a_2 = 7$ and $a_{14} = 31$ find a_7

A) 13   B) 15   C) 17   D) 19   E) NOTA

12. Find the harmonic mean of the numbers 2, 6, and 12.

A) 12   B) 6   C) 4   D) 20/3   E) NOTA

13. Evaluate: $\sum_{k=1}^{3}((\sum_{n=1}^{3}(2n+3))*k+3)$

A) 125   B) 63   C) 135   D) 126   E) NOTA

14. Evaluate: $\sum_{n=1}^{10}(2(n+3)+6(n-1)-2(n+2))$

A) 310   B) 270   C) 330   D) 290   E) NOTA

15. Find a+b, where a and b are real, and where a+bx is the sum of the first 10 terms of the arithmetic sequence with first term x+2 and common difference of 3x+1.

A) 75   B) 95   C) 145   D) 210   E) NOTA

16. $\sum_{n=1}^{5}\frac{3n+1}{3^n}$

A) 329/243   B) 175/81   C) 658/243   D) 296/81   E) NOTA

17. What must the positive integral common difference of a 5-term arithmetic sequence be if the first term of the sequence is -4 and the geometric and arithmetic means of the 5 terms in the sequence are the same?

A) 4   B) 1   C) 3   D) 2   E) NOTA

18. $\sum_{n=1}^{\infty}\frac{n}{3^n}$

A) 3/4   B) 1/3   C) 3   D) 3/2   E) NOTA

19. What is the units digit of the 2015th term in the arithmetic sequence 1,5,9,13….

   A) 7   B) 3   C) 9   D) 5   E) NOTA

20. Write $20.\overline{14}$ as a fraction.

   A) 2014/99 B) 1994/99   C) 1996/99   D)19997/909 E) NOTA

21. How many terms does the arithmetic sequence 6.5,13,…,78 have?

   A) 12   B) 11   C) 10   D) 9   E) NOTA

22. How many different strictly increasing sequences of five positive integers whose sum is 17 exist?

   A) 0   B) 4   C) 1   D) 2   E) NOTA

23. Evaluate: $\sum_{n=1}^{\infty}\frac{1}{\left(\sum_{k=1}^{\infty}(1/4^k)\right)^n}$

   A) 3   B) 3/2   C) 1/3   D) 1/2   E) NOTA

24. Which is largest?

   A) The 7th pentagonal number   B) the 6th hexagonal number
   C) the 5th heptagonal number   D) $\sum_{n=1}^{10}(n+1)$ E) NOTA

25. Find the sum of the solutions for x given: $\prod_{n=1}^{5}(x+n)=0$

   A) -15   B) 15   C) -20   D) 20   E) NOTA

26. How much bigger is the 10th term in the Fibonacci sequence if instead of the first 2 terms being $a_1 = 0, a_2 = 1$, they are $a_1 = 1, a_2 = 2$?

   A) 55   B) 34   C) 89   D) 68   E) NOTA

27. 3 numbers are randomly chosen without replacement from between 1 and 10 inclusive. What is the probability they can be arranged to form an arithmetic sequence?

   A) 1/5   B) 1/6   C) 1/7   D) 1/8   E) NOTA

28. Find $\sum_{n=1}^{10} 2^n$

   A) 2047   B) 1023   C) 2046   D) 1022   E) NOTA

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29. Solve for x in : $\sqrt{10 + \sqrt{10 + \sqrt{10 + \cdots}}} = x$

A) $\frac{1+\sqrt{41}}{2}$ B) $\frac{1-\sqrt{41}}{2}$ C) 5/2   D) -5/2   E) NOTA

30. What is the sum of the 10th and 11th terms of the sequence whose odd-numbered terms make up an arithmetic sequence and whose even-numbered terms make up a geometric sequence, the first few terms of which are 1, 4, 11, 16, 21, 64 …?

A) 867   B) 253   C) 322   D) 1075   E) NOTA