### #0 Alpha Ciphering MA⊖ National Convention 2015

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cos 1° · cos 2° · cos 3° · ... · cos 179°

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Solve for x:

$$\left| \left| x - 2 \right| - 5 \right| \le 2$$

Express your answer in interval notation.

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### #2 Alpha Ciphering

#### **MA** National Convention 2015

Solve for the  $2 \times 2$  matrix M:

$$M\begin{bmatrix} 5 & 11 \\ 3 & 3 \end{bmatrix} + 2M = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$$

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### #3 Alpha Ciphering MA⊕ National Convention 2015

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 $12\cos 2x - 5\sin 2x$  can be rewritten as  $A\cos\left(Bx + \arcsin\left(\frac{c}{D}\right)\right)$  for integers A, B, C, D, where A, B > 0 and C, D relatively prime. Find the value of A + B + CD.

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### #8 Alpha Ciphering MA⊕ National Convention 2015

Let  $A_n$  be the *n*th term in the Fibonacci-like sequence 1, 3, 4, 7, 11, 18,..., where  $A_1 = 1$ ,  $A_2 = 3$ , and  $A_k = A_{k-1} + A_{k-2}$  for  $k \ge 3$ .

Let

$$S_n = \sum_{k=1}^n A_k$$

Find

$$S_{2015} - \sum_{k=1}^{2013} S_k$$

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