For each question, "E) NOTA" indicates that none of the above answers is correct.

1. Dr. Morris and the Berkeley Math Club have a pasture of llamas. Everyday, one-fifth of the llamas in the pasture escape. After three days, what fraction of the llamas will be gone?

A) $\frac{1}{125}$         B) $\frac{61}{125}$         C) $\frac{64}{125}$         D) $\frac{124}{125}$         E) NOTA

2. A bathtub can be filled to maximum capacity by a hot-water faucet only in 6 minutes and by a cold-water faucet only in 6 minutes. In addition, it takes the drain 4 minutes to empty the bathtub. With both the faucets running and the drain open, how long will it take to fill the tub? Assume the tub is initially empty. All answers are in minutes.

A) 12         B) 24         C) 25         D) 30         E) NOTA

3. Zach, the master at solving equations, can solve just about any system of equations. Given that Zach is presented the following equations, he knows that $125x + 25y + 5z + w$ is equal to what?

$$x + y + z + w = 2$$
$$8x + 4y + 2z + w = 6$$
$$27x + 9y + 3z + w = 11$$
$$64x + 16y + 4z + w = 20$$

A) 29         B) 32         C) 36         D) 41         E) NOTA

4. The original population of St. Louis, MI was a perfect square. Later, with an increase of 100, the population was one more than a perfect square. Now, with an additional increase of 100, the population is again a perfect square. What is the sum of the digits of the original population of St. Louis?

A) 17         B) 9         C) 7         D) 1         E) NOTA

5. Three mutually tangential, perfectly spherical cannonballs, each with radius 3, rest on a horizontal plane. A fourth cannonball of radius 3 lies on top of them, above the empty space between the three cannonballs. What is the distance from the horizontal plane to the top of the fourth cannonball?

A) $\frac{1}{2}\sqrt{6} + 6$      B) $2\sqrt{6} + 6$      C) $3\sqrt{3} + 6$      D) $6\sqrt{3} + 6$      E) NOTA

6.  At the end of the day Jimmy Johns has 77 dollars in the tip jar.  If Jimmy Johns wants to distribute the money in whole dollar increments to 7 different employees such that no one receives less than $10, in how many different ways can they do this?

A) 512                B) 1716                C) 3432                D) 5040                E) NOTA

7.  Find the minimum value of $\frac{9x^2 \sin^2 x + 4}{x \sin x}$ for $0 < x < \pi$.

A) 12                B) 14                C) 16                D) 18                E) NOTA

8. Nick likes to flip coins.  He flips a coin 5 times and tries to remember the face on each of the flips.  Unfortunately, he only remembers that one flip resulted in heads. Suppose that Nick guesses that the other four flips were heads—what is the probability that he is correct?

A) $\frac{1}{32}$              B) $\frac{1}{31}$              C) $\frac{1}{16}$              D) $\frac{2}{31}$              E) NOTA

9. The width and length of a right rectangular prism each decrease by 50 percent, but the volume remains constant.  By what percent does the height increase?

A) 40%                B) 200%                C) 300%                D) 400%                E) NOTA

10.  Catherine's favorite number is the lucky 7. How many three-digit numbers contains Catherine's favorite number exactly once?

A) 153                B) 154                C) 225                D) 226                E) NOTA

11. The force exerted on a moving particle in the presence of a magnetic field is represented by the following equation: $\vec{F} = q\vec{v} \times \vec{B}$, where $q$ is the magnitude of the charge of the particle, $v$ is the velocity vector, and $B$ is the magnetic field vector.  If a charged particle of magnitude $.5$ Coulombs is going through a magnetic field of vector $\vec{B} = 3i + 4j - 6k$ and a velocity of $\vec{v} = -2i + 6j - 6k$, what is the vector representing the force on the particle?

A) $6i + 15j + 13k$  B) $-6i - 15j + 13k$ C) $12i + 30j + 26k$  D) $-12i - 30j + 26k$   E) NOTA

12.  Suppose I select a number at random on the interval $(-360, 360)$ and that number represents a degree-measure angle. What is the probability that you select a number where the sine of the number is less than $-1/2$?

A) 1/2                B) 5/12                C) 1/3                D) 1/4                E) NOTA

13.  Nate, Nathan, and Theo are all doing a math problem.  Nate and Nathan can each solve the problem in 30 minutes when working alone, while Theo can solve the problem in 10 minutes when working alone.  If all three decided to work together, how many minutes would it take to solve three of the same problem (assuming they do not solve it faster the second or third time, and each works independently)?  All three work on a problem until it is completed, at which point they begin the next problem.

A) 18             B) 20             C) 30             D) 35             E) NOTA

14. 1728 white cubes of volume 1 are arranged into a cube of volume 1728, the surface of which is painted blue. Then Ankit the Destroyer disassembles this cube into its 1728 components, and his friend, Gabe the Painter, paints all the remaining white surface with orange paint. What is the ratio of blue surface area to orange surface area over all 1728 cubes?

A) $1:1$             B) $1:2$             C) $1:6$             D) 1:11             E) NOTA

15. Jake chooses a point at random inside the square with vertices $(0,0), (2,0), (2,2),$ $(0,2)$. What is the probability the point is closer to $(0,0)$ than to $(3,3)$?

A) 1/8             B) 1/4             C) 1/8             D) 7/8             E) NOTA

16. Samantha has a circular sector with central angle 288° with radius 15 and wants to make a cone by rolling the sector so that the two radii coincide.  The volume of the resulting cone is in the form $A\pi$, where $A > 0$.  What is the sum of the digits of $A$?

A) 9             B) 8             C) 7             D) 6             E) NOTA

17. On a certain flight, there are 9 boys, 5 American children, 9 men, 14 Americans, 6 American males, and 7 foreign females. Find the total number of people on the plane. (For this problem, boys are under 18 and men are at least 18).

A) 23             B) 27             C) 33             D) 37             E) NOTA

18. Sebby places four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. He then pours half the coffee from the first cup to the second and, after stirring thoroughly, pours half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

A) 1/4             B) 1/3             C) 3/8             D) 2/5             E) NOTA

19. There exists a sequence of vectors that take the form $a_k = <\frac{1}{k}, \frac{1}{k+1}, \frac{1}{k+2}>$ for integers $k \geq 1$. We then define the real-valued sequence $b_k = a_k \cdot a_{k+1}$ for all integers $k \geq 1$. Find $\sum_{k=1}^{\infty} b_k$.

A) 3/8 B) 3/2 C) 11/6 D) $\infty$ E) NOTA

20. Whenever Nate sees girls, his breathing rate speeds up, and the amount of air in his lungs at $t$ seconds after seeing them is given by $A(t) = 50 - 50\cos\frac{\pi t}{2}$. Suppose Nate is locked up in a room with several girls for an hour. How many full breaths will he have taken in that span of time? Assume one full period of $A(t)$ corresponds to one breath.

A) 7,200 B) 14,400 C) 9,000 D) 4,500 E) NOTA

21. The following chart displays the probability distribution for some variable $B$, which has an infinite number of outcomes:

| $B$ | 1 | 2 | 3 | ... |
|---|---|---|---|---|
| $P(B)$ | $a$ | $a/3$ | $a/9$ | ... |

Assuming that $P(B+1) = \frac{1}{3}P(B)$, find the mean of B.

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) 1 D) 2 E) NOTA

22. Two vertical posts, 2m high, have a 5m wire stretched tight between the tops of the posts. A 6m leash is attached by two rings to the wire at one end and to a dog's collar at the other end. Disregarding the length of the rings and dog, find the area on the ground in square meters on which the dog can roam.

A) $40\sqrt{2} + 32\pi$ B) $20\sqrt{2} + 32\pi$ C) $40\sqrt{2} + 64\pi$ D) $20\sqrt{2} + 32\pi$ E) NOTA

23. Eight distinct points are marked on a circle. How many distinct convex polygons can be drawn using some (or all) of the eight points as vertices?

A) 200 B) 219 C) 250 D) 256 E) NOTA

24. Newton's Law of Cooling says that when the difference in the temperature of an object and its surroundings is measured at equal time intervals, the differences form a geometric sequence. If the temperature of a cup of soup was 32°C at 10:00 a.m. and 24°C at 10:10 a.m., what was its temperature when the soup was taken out of the microwave at 9:40 a.m.? Assume the air temperature remained constant at 20°C .

A) 84° B) 108° C) 116° D) 128° E) NOTA

25. Triangle $ABC$ is defined by the points $A(1, 3), B(4, 7), C(7, 3)$ and encloses a region $R$. What is the volume of the figure generated when $R$ is revolved around the line $x = 7$?

A)  $12\pi$             B) $36\pi$             C) $72\pi$             D) $128\pi$             E) NOTA

26. Regan, a baseball manager, notices that her team is doing so badly that something is definitely wrong with her batting order.  Since all of her players are so similar, she decides to arrange the nine batters in a completely random order.  What is the probability that at most two batters are in a different spot than their original positions after the randomization?

A) $\frac{43}{9!}$             B) $\frac{37}{9!}$             C) $\frac{36}{9!}$             D) $\frac{73}{9!}$             E) NOTA

27. Two cars are at the same location at t = 0 and then begin to move away from each other at an angle of 60°. If one car is moving at 4 m/s and the other at 3 m/s, what is the square of the number of meters between them at t = 2s?

A)  10             B) 13             C) 52             D) 100             E) NOTA

28. The equation of the plane defined by the following three points: $(1, -1, 2)$, $(-2, 3, 4)$, and $(2, -6, 3)$ takes the form $Ax + By + Cz = D$, where $A, B, C,$ and $D$ are relatively prime positive integers.  What is the sum of the digits of $D$?

A) 2             B) 4             C) 6             D) 8             E) NOTA

29.  In the Argand plane, the solutions to the following complex equation form the vertices of a convex n-gon: $x^6 = -8 + 8i$.  Find the area enclosed by this n-gon.

A) $96\sqrt{3}$             B) $192\sqrt{3}$             C) $96\sqrt{6}$             D) $192\sqrt{6}$             E) NOTA

30. A spring is compressed and then released into an oscillatory motion, which can be described by the function $y = 3\sin t + 4\cos t$ where t is time and y gives the vertical position of the spring in meters. What is the measure of the maximum vertical distance that the spring will move?  All choices are in meters.

A) 3             B) 5             C) 7             D) 10             E) NOTA