ANSWERS

1) B
2) D
3) A
4) D
5) C
6) B
7) E (0)
8) D
9) B
10) A
11) A
12) C
13) B
14) A
15) D
16) B
17) D
18) C
19) A
20) C
21) B
22) B
23) D
24) D
25) C
26) A
27) C
28) A
29) A
30) B

SOLUTIONS

(1)

(a) $y = 44x - 24$ (b) $y = 440x - 420$

(c) $y = 440x - 440$ (d) $y = 440x - 460$ (e) NOTA

SOLUTION

$\frac{d}{dx}(20x^{22})\Big]_{x=1} = 440(1)^{21} = 440 \to y - 20 = 440(x-1) \to y = 440x - 420.$ $\boxed{B}$

(2)

(a) 10 (b) $\frac{21}{2}$

(c) 11 (d) $\frac{23}{2}$ (e) NOTA

SOLUTION

$x^{23} = Ax^2 - A + 1 \to x^{23} - 1 = A(x^2 - 1) \to x = 1.$ Further, $23x^{22} = 2Ax$ at $x = 1 \to A = \frac{23}{2}$.
$\boxed{D}$

(3)

(a) $2a^3 - 2a^7 + C$ (b) $2a^7 - 2a^3 + C$

(c) $a^3 - a^7 + C$ (d) $a^7 - a^3 + C$ (e) NOTA

SOLUTION

$dM = -\frac{2}{a^3}da \to \int a^5(7a^4 - 3)\left(-\frac{2}{a^3}\right)da = \int -14a^6 + 6a^2\, da = 2a^3 - 2a^7 + C.$ $\boxed{A}$

(4)

(a) $\frac{\pi}{2}$ (b) π

(c) $\frac{3\pi}{2}$ (d) 2π (e) NOTA

SOLUTION

When $x \geq 2$, $y = \sqrt{1-(x-3)^2}$. When $0 \leq x < 2$, $y = \sqrt{1-(x-1)^2}$. When $-2 \leq x < 0$, $y = \sqrt{1-(x+1)^2}$. When $x < -2$, $y = \sqrt{1-(x+3)^2}$. These are four semicircles of radius one, so the total area is 2π. $\boxed{D}$

(5)

(a) -360 (b) 360

(c) $-\frac{1}{360}$ (d) $\frac{1}{360}$ (e) NOTA

SOLUTION

$\frac{d}{dx}(20e^{2x})\Big]_{x=\ln(3)} = 40e^{2\ln(3)} = 40e^{\ln(9)} = 40(9) = 360$. So the normal slope is $-\frac{1}{360}$. $\boxed{C}$

(6)

(a) $\frac{9^3 10^{13}}{(10^8-9^8)^2}$ (b) $\frac{9^3 10^{12}}{(10^8-9^8)^2}$

(c) $\frac{9^4 10^{13}}{(10^8-9^8)^2}$ (d) $\frac{9^4 10^{12}}{(10^8-9^8)^2}$ (e) NOTA

SOLUTION

$(1)\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right) + (2)\left(\frac{9}{10}\right)^{11}\left(\frac{1}{10}\right) + (3)\left(\frac{9}{10}\right)^{19}\left(\frac{1}{10}\right) + \cdots = \left(\frac{9^{-5}}{10^{-4}}\right)\sum_{n=1}^{\infty} n\left(\frac{9}{10}\right)^{8n} = \left(\frac{10^4}{9^5}\right)\frac{\frac{9^8}{10^8}}{\left(1-\frac{9^8}{10^8}\right)^2} = \frac{\frac{9^3}{10^4}(10^{16})}{(10^8-9^8)^2} = \frac{9^3 10^{12}}{(10^8-9^8)^2}$. $\boxed{B}$

(7)

(a) -2022 (b) -1011

(c) 1011 (d) 2022 (e) NOTA

SOLUTION

$\int_0^\infty \frac{\ln(x)}{2022x^2+x+2022}dx$. Let $x = \frac{1}{u} \to dx = -\frac{du}{u^2}$. $\int_\infty^0 \frac{\ln\left(\frac{1}{u}\right)}{\frac{2022}{u^2}+\frac{1}{u}+2022}\left(-\frac{du}{u^2}\right) = -\int_0^\infty \frac{\ln(u)}{2022+u+2022u^2}du$. If the integral is the negative of itself, it must be zero. $\boxed{E}$

(8)

(a) $\int_0^{2\pi} \sqrt{\theta^2 + 901}\, d\theta$ (b) $\int_0^{2\pi} \sqrt{\theta + 30}\, d\theta$

(c) $\int_0^{20\pi} \sqrt{60\theta + 2}\, d\theta$ (d) $\int_0^{20\pi} \sqrt{\theta^2 + 60\theta + 901}\, d\theta$ (e) NOTA

SOLUTION

$r = 30$ when $\theta = 0$ and $r \to r + 2\pi$ as $\theta \to \theta + 2\pi$. Therefore $r(\theta) = \theta + 30$. Therefore the arc length is $\int_0^{20\pi} \sqrt{r^2 + r'^2} d\theta = \int_0^{20\pi} \sqrt{\theta^2 + 60\theta + 901}\, d\theta$. $\boxed{D}$

(9)

(a) 125 (b) 250

(c) 500 (d) 1000 (e) NOTA

SOLUTION

Let $(x_0, 0)$ be the point the Flash is at. Then $D^2 = (x - x_0)^2 + \left(\sqrt{x}\right)^2 = (x - x_0)^2 + x \to 2DD' = 0 = 2(x - x_0) + 1 \to x = x_0 - \frac{1}{2} \to D = \sqrt{\frac{1}{4} + x_0 - \frac{1}{2}} = \sqrt{x_0 - \frac{1}{4}}$ will be the distance from the curve to the Flash's location. Therefore $\frac{dD}{dt} = \frac{\frac{dx_0}{dt}}{2\sqrt{x_0 - \frac{1}{4}}} = \frac{1000}{2\sqrt{4}} = 250$. $\boxed{B}$

(10)

(a) $\frac{3\ln(20)}{\ln(6) - \ln(5)}$ (b) $\frac{1}{3}\ln\left(\frac{5}{2}\right)$

(c) $\frac{3\ln(20)}{\ln(5) - \ln(2)}$ (d) $\frac{1}{3}\ln\left(\frac{6}{5}\right)$ (e) NOTA

SOLUTION

$\frac{dT}{dt} = k(T - T_0) \to \ln(T - T_0) = kt + C \to T(t) = Ce^{kt} + T_0$. The ambient room temperature $T_0 = 40$ so $T(t) = Ce^{kt} + 40$. Cap starts at -20 so $T(0) = -20 = C + 40 \to C = -60$. Finally after three hours Cap has warmed 10 to -10. That means $-10 = 40 - 60e^{3k} \to k = \frac{1}{3}\ln\left(\frac{5}{6}\right)$. Therefore Cap will reach 37 when $37 = 40 - 60e^{kt} \to kt = \ln\left(\frac{1}{20}\right) \to t = \frac{\ln\left(\frac{1}{20}\right)}{\frac{1}{3}\ln\left(\frac{5}{6}\right)} = \frac{3\ln(20)}{\ln(6) - \ln(5)}$. $\boxed{A}$

(11)

(a) $(e^{2022}-1)\ln(2022)$ (b) 2022

(c) $\ln(2022)$ (d) $e^{2022}-1$ (e) NOTA

SOLUTION

Let $f(x)=\left(1+\frac{2022}{x}\right)^x$. Therefore $f(2022x)=\left(1+\frac{1}{x}\right)^{2022x}$. Now, in general $\frac{1}{a}\frac{d}{db}f(ab)=\frac{d}{d(ab)}f(ab)=\frac{1}{b}\frac{d}{da}f(ab)$. So $\int_0^\infty \frac{f(2022x)-f(x)}{x}dx=\int_0^\infty\int_1^{2022}\frac{1}{x}\frac{d}{dy}f(yx)dy\,dx=\int_0^\infty\int_1^{2022}\frac{1}{y}\frac{d}{dx}f(xy)dy\,dx=\int_1^{2022}\int_0^\infty\frac{1}{y}\frac{d}{dx}f(xy)dx\,dy=\left(\lim_{x\to\infty}f(x)-\lim_{x\to 0}f(x)\right)\int_1^{2022}\frac{dy}{y}=(e^{2022}-1)\ln(2022)$. $\boxed{A}$

(12)

(a) $-\frac{\sqrt[3]{5}}{5}$ (b) $-\frac{\sqrt[3]{25}}{25}$

(c) $-\frac{\sqrt[3]{5}}{25}$ (d) $-\frac{\sqrt[3]{25}}{5}$ (e) NOTA

SOLUTION

By similar triangles we know that $\frac{h}{r}=\frac{200}{50}=4\to h=4r\to\frac{dh}{dt}=4\frac{dr}{dt}\,\&\,V=\frac{1}{3}\pi r^2h=\frac{4}{3}\pi r^3$. Therefore $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$. The initial volume is $V=\frac{1}{3}\pi(50^2)(200)=\frac{500000\pi}{3}$ so half that will be $\frac{250000\pi}{3}=\frac{4}{3}\pi r^3\to r=5\sqrt[3]{5}\to\frac{dV}{dt}=-\pi=20\pi\sqrt[3]{25}\frac{dr}{dt}\to\frac{dr}{dt}=-\frac{1}{20\sqrt[3]{25}}\to\frac{dh}{dt}=-\frac{1}{5\sqrt[3]{25}}=-\frac{\sqrt[3]{5}}{25}$. $\boxed{C}$

(13)

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$ (e) NOTA

SOLUTION

$\lim_{n\to\infty}\sum_{k=1}^n\frac{n}{k^2+n^2}=\lim_{n\to\infty}\sum_{k=1}^n\frac{1}{n}\frac{1}{\left(\frac{k}{n}\right)^2+1}=\int_0^1\frac{dx}{x^2+1}=[\arctan(x)]_0^1=\frac{\pi}{4}$. $\boxed{B}$

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(14)

(a) $\frac{1}{e^2}$ (b) $\frac{1}{e}$

(c) e (d) e^2 (e) NOTA

SOLUTION

The area of R is given by $\int_0^\infty \frac{dx}{(x+1)^2} = \left[-\frac{1}{x+1}\right]_0^\infty = 1$. Therefore we need $\frac{1}{2} = \int_0^\infty b^{-x}dx = \left[\frac{1}{\ln(b)} b^{-x}\right]_0^\infty = -\frac{1}{\ln(b)} \to \ln(b) = -2 \to b = \frac{1}{e^2}$. $\boxed{A}$

(15)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi^2}{2}$ (d) $\frac{\pi^2}{6}$ (e) NOTA

SOLUTION

$\int_0^\infty \frac{x}{e^x-1}dx = \int_0^\infty \frac{xe^{-x}}{1-e^{-x}}dx = \int_0^\infty \sum_{n=1}^\infty xe^{-nx}\,dx = \sum_{n=1}^\infty \int_0^\infty xe^{-nx}dx = \sum_{n=1}^\infty \left[-\frac{1}{n}xe^{-nx} - \frac{1}{n^2}e^{-nx}\right]_0^\infty = \sum_{n=1}^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$ $\boxed{D}$

(16)

(a) $\frac{1}{69}$ (b) $\frac{1}{61}$

(c) 69 (d) 61 (e) NOTA

SOLUTION

$\frac{d}{dx}[f^{-1}(x)]\Big|_{x=22} = \frac{1}{f'(f^{-1}(22))}$. Clearly $f(1) = 22$ so $\frac{1}{f'(f^{-1}(22))} = \frac{1}{f'(1)} = \frac{1}{60(1^4)-9(1^2)+10(1)} = \frac{1}{61}$. $\boxed{B}$

(17)

(a) $3x^2\sin(x^{66}) - 2x\sin(x^{44})$ (b) $\sin(x^{44}) - \sin(x^{66})$

(c) $\sin(x^{66}) - \sin(x^{44})$ (d) $2x\sin(x^{44}) - 3x^2\sin(x^{66})$

(e) NOTA

SOLUTION

$\frac{d}{dx}\int_{x^3}^{x^2} \sin(t^{22})\, dt = 2x \sin(x^{44}) - 3x^2 \sin(x^{66})$. $\boxed{D}$

(18)

(a) 20 (b) 22

(c) 24 (d) 26 (e) NOTA

SOLUTION

$D'(1) + M'(2) = L'(R(1))R'(1) + L'(2)R(2) + L(2)R'(2) = L'(3) * 4 + (-1)(2) + (1)(6) = (5)(4) + 4 = 24$. $\boxed{C}$

(19)

(a) 6 (b) 7

(c) 8 (d) 9 (e) NOTA

SOLUTION

$\int_{-12}^{0} f(x)dx = 6 \rightarrow \int_{0}^{12} f(x)dx = -6$ since $f(x)$ is odd. Now $\int_{-6}^{22} f(x)dx = 22 = \int_{-6}^{6} f(x)dx + \int_{6}^{22} f(x)dx = \int_{6}^{22} f(x)dx$. Finally $\int_{-12}^{6} f(x)dx = 10 = \int_{-12}^{-6} f(x)dx + \int_{-6}^{6} f(x)dx = \int_{-12}^{-6} f(x)dx$. So $\int_{6}^{12} f(x)dx = -10$. Thus $\int_{0}^{22} f(x)dx = \int_{6}^{22} f(x)dx + \int_{0}^{12} f(x)dx - \int_{6}^{12} f(x)dx = 22 - 6 - 10 = 6$. $\boxed{A}$

(20)

(a) 4140 (b) 2025

(c) 2205 (d) 270 (e) NOTA

SOLUTION

Let x be the height of the mouse from the ground. Then, to move a small height dx will result in an amount of work $dW = 4dx + 2xdx \rightarrow W = \int_0^{45}(4 + 2x)dx = [4x + x^2]_0^{45} = 180 + 2025 = 2205$. $\boxed{C}$

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(21)

(a) $\log_2(11)$ (b) $\log_{22}(2)$

(c) $\log_2(22)$ (d) $\log_{11}(2)$ (e) NOTA

SOLUTION

$A = 22e^{kt} \rightarrow 1 = 22e^k \rightarrow k = -\ln(22) \rightarrow \frac{1}{2} = e^{-\ln(22)t_{\frac{1}{2}}} \rightarrow -\ln(2) = -\ln(22)\, t_{\frac{1}{2}} \rightarrow t_{\frac{1}{2}} = \frac{\ln(2)}{\ln(22)} = \log_{22}(2)$. $\boxed{B}$

(22)

(a) 0 (b) e^{23}

(c) e^{46} (d) 23 (e) NOTA

SOLUTION

$\lim_{x\to 23} \frac{e^x - e^{23}}{x-23} = \frac{d}{dx}[e^x]_{x=23} = e^{23}$. $\boxed{B}$

(23)

(a) $\frac{1}{2022!}$ (b) $\frac{1}{2018!}$

(c) $\frac{1}{1011!}$ (d) $\frac{1}{1009!}$ (e) NOTA

SOLUTION

$x^4e^{x^2} = x^4\sum_{n=0}^{\infty}\frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty}\frac{x^{2n+4}}{n!}$. The 2022nd derivative at x=0 will be the coefficient of $x^{2022} \rightarrow 2022 = 2n + 4 \rightarrow n = 1009 \rightarrow$ The answer is $\frac{1}{1009!}$. $\boxed{D}$

(24)

(a) $\frac{33}{49}$ (b) $\frac{33}{94}$

(c) $-\frac{33}{49}$ (d) $-\frac{33}{94}$ (e) NOTA

SOLUTION

$y^4x^3 + x^2 - 14y = 22 \to 4y^3x^3y' + 3y^4x^2 + 2x - 14y' = 0 \to y' = \frac{-3y^4x^2-2x}{4y^3x^3-14} \to y'(3,1) = \frac{-3(9)-2(3)}{4(27)-14} = -\frac{33}{94}$. $\boxed{D}$

(25)

(a) 1 (b) x^{1011}

(c) $\sqrt{2}x^{1011}$ (d) $\frac{1}{2}x^{2022}$ (e) NOTA

SOLUTION

$\frac{d}{d\left(\frac{d}{d\left(\frac{d}{d\left(\frac{d}{d(\because)}[x^{2022}]\right)}[x^{2022}]\right)}[x^{2022}]\right)}[x^{2022}] = \frac{d}{du}[x^{2022}] = 2022x^{2021}\frac{dx}{du} = u \to 2022x^{2021} = u\frac{du}{dx} \to x^{2022} = \frac{1}{2}u^2 \to u = \sqrt{2}x^{1011}$. $\boxed{C}$

(26)

(a) $2 + \ln\left(\frac{3}{2}\right)$ (b) $6 - \ln(2)$

(c) $2 + \ln\left(\frac{2}{3}\right)$ (d) $4 + \ln(3)$ (e) NOTA

SOLUTION

$\int_{3/22}^{8/22} \frac{\sqrt{1+22x}}{x}dx \to u = \sqrt{1+22x} \to x = \frac{u^2-1}{22} \to dx = \frac{u}{11}du \to \int_{3/22}^{8/22} \frac{\sqrt{1+22x}}{x}dx = \int_2^3 \frac{2u^2}{u^2-1}du = \int_2^3 2 + \frac{2}{u^2-1}du = \int_2^3 2 + \frac{1}{u-1} - \frac{1}{u+1}du = [2u + \ln|u-1| - \ln|u+1|]_2^3 = 6 + \ln(2) - \ln(4) - 4 - \ln(1) + \ln(3) = 2 + \ln\left(\frac{3}{2}\right)$. $\boxed{A}$

(27)

(a) $\frac{9}{196\pi}$ (b) $\frac{9}{49\pi}$

(c) $\frac{3}{196\pi}$ (d) $\frac{3}{49\pi}$ (e) NOTA

SOLUTION

$V = \frac{4}{3}\pi r^3 \to \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} \to \frac{3}{4\pi(49)} = \frac{dr}{dt} = \frac{3}{196\pi}$. $\boxed{C}$

(28)

(a) $\frac{160}{3}$ (b) $\frac{80}{3}$

(c) 120 (d) 60 (e) NOTA

SOLUTION

$\int_0^2 20\sqrt{2x}dx = 20\sqrt{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2 = 20\sqrt{2}\left(\frac{2}{3}\right)(2\sqrt{2}) = \frac{160}{3}$. $\boxed{A}$

(29)

(a) 4.70 (b) 4.69

(c) 4.65 (d) 4.60 (e) NOTA

SOLUTION

$\sqrt{22} \approx \sqrt{25} - 3\left(\frac{1}{2\sqrt{25}}\right) = 5 - \frac{3}{10} = 4.70$. $\boxed{A}$

(30)

(a) 0 (b) 1

(c) 2 (d) 3 (e) NOTA

SOLUTION

Divergent since the terms do not limit to zero	Conditionally Convergent via alternating series test.	Absolutely convergent (Taylor series for e^x)
Absolutely convergent via Sterling's approximation & the ratio test.	Absolutely convergent via the root test	Diverges via Raabe's test
Since $\tan(\theta) = \frac{2022}{2023}$, diverges via the p-test	Convergent via the ratio test	Diverges since there is a term dividing by zero
Conditionally convergent via alternating series, and the integral test diverges when negatives are removed.	Absolutely convergent via the p-test.	NA

4-3=1. $\boxed{B}$