Interschool Test

Buy-In Questions

Mu Alpha Theta National Convention

July 10-15, 2022

1 Introduction

The following questions will not award points to your school's overall score on the Interschool, but instead are worth team members. Each question will have a value assigned to it that represents how many team members you can bring in if you answer the question correctly.

For example:

1. (1 member) What is 1+1?

In this example, if you correctly answer 2, then any team member from your school can be brought into the testing room to join you.

Note:

- You can bring in one sponsor, but a sponsor is worth 7 members, so only questions worth 7 or greater can be used to gain a sponsor.
- **You only have one attempt per question, so make sure you check your work and answer correctly.**

Whether you prioritize buy-in questions or point-valued questions is up to you. Good luck!

2 Buy-In Questions

2.1 Geometry

1. (1 Member) If one of the legs of a right triangle has length 60 and the hypotenuse has length 109, what is the length of the other leg?

2. Find the area of the hexagon defined by the points (0,2), (3,2), (4,6), (3,8), (2,9), (-2,3).

3. (5 Members) What is the difference between the maximum and minimum number of regions that can be made from the intersection of 13 lines in a plane?

4. (7 Members) Two circles with distinct radii lie in the first quadrant of the xy-plane. The smaller circle is tangent to the y-axis and the larger circle is tangent to the x-axis. The two circles are externally tangent to each other. If the centers of both circles lie along the line $y = 12 - x$, what is the sum of their radii? 5. (10 Members) Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. What is the probability that the straight-line distance between the points is at least $\frac{1}{2}$?

2.2 Algebra 2

6. (1 Member) What is the equation, in slope-intercept form, of the line that passes through the points (2,6) and (-3,18)?

7. (2 Members) Find the domain of $f(x) = \log(\log_2(\log_3(\ln x)))$.

8. (5 Members) Find the sum of the real roots of $6x^4 - 13x^3 + 12x^2 - 13x + 6$.

9. (7 Members) What is 19!! (where !! represents the double factorial, as in 6!! = 6*4*2)?

10. (10 Members) Given that $M = \begin{bmatrix} -29 & -20 \\ 42 & 29 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ find the sum of the entries of M^{10}.

2.3 Pre-Calculus

11. (1 Member) Evaluate $\tan(\frac{5\pi}{3})$.

12. (2 Members) If $\sin x = -\frac{5}{12}$ and $\frac{\pi}{2} < x < \frac{3\pi}{2}$, what is $\cos(2x)$?

13. (5 Members) Simplify $(\frac{2-4i}{3-i})^3$.

14. (7 Members) What is the direction of the unit vector of the line defined by the intersection of $3x + 2y - z = 6$ and $2x + y - 4z = 4$?

15. (10 Members) If four points are randomly chosen on a circle, what is the probability that the quadrilateral that is defined by those points contains the center of the circle?

2.4 Statistics & Probability

16. (1 Member) Find the mean of the following set $\{1, 4, 5, 6, 8, 10\}$.

17. (2 Members) Statistics that are not overly impacted by outliers are called...

18. (5 Members) Find the standard deviation of the following population $\{1, 3, 5, 9, 23, 25\}$.

19. (7 Members) A fair six-sided die is repeatedly rolled until the third 4 appears. What is the probability this takes exactly 7 rolls?

20. (10 Members) Find the slope for the best fit line of the following points

(0,2), (3,2), (4,6), (3,8), (2,9), (-2,3).

2.5 Calculus

21. (1 Member) Find $\lim_{x\to\infty} \frac{4x^4-16x^2+4x+24}{6x^4-13x^3+12x^2-13x+6}$.

22. (2 Members) Evaluate $\frac{dy}{dx}$ for $x = \cos t$ and $y = t^2 + 1$ at $x = \frac{1}{2}$.

23. (5 Members) Evaluate $\int_0^4 \sqrt{9 + x^2} dx$.

24. (7 Members) Find the sum of the (unsigned) area between $4x^3 - 16x^2 + 4x + 24$ and the x-axis between $x = -3$ to $x = 7$.

25. (10 Members) Evaluate $\int_{\frac{-1}{2}}^{0} \frac{\ln(1+x)}{x} dx$.