Alpha

Complex Numbers

Test #623

Directions:

1. Fill out the top left section of the scantron. Do not abbreviate your school name.

2. In the Student ID Number grid, write your 9-digit ID# and bubble.

3. In the Test Code grid, write the 3-digit test# on this test cover and bubble.

4. Scoring for this test is 5 times the number correct plus the number omitted.

5. TURN OFF ALL CELL PHONES.

6. No calculators may be used on this test.

7. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

8. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

9. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

10. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

11. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

12. Unless a question asks for an approximation or a rounded answer, give the exact answer.

For all questions the answer choice E. means "none of the above". Let $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$, If $z = a + bi$, then $Re(z) = a$, $Im(z) = b$, and $\overline{z} = a - bi$. Unless otherwise noted, assume the argument of a complex number to be the principal argument, which is on $(-\pi, \pi]$. Good luck and have fun!

1. Evaluate $\lim_{z \to (4-3i)} (z^2 - |z| + 1)$

A. $12 - 12i$ B. $3 - 24i$ C. $21 - 24i$ D. $3 - 12i$ E. NOTA

2. Which of the following is equivalent to $\ln(2\sqrt{3} + 6i)$?

A. $\ln(8) + \frac{\pi i}{6}$ B. $\ln(4\sqrt{3}) + \frac{\pi i}{6}$ C. $\ln(8\sqrt{3}) + \frac{\pi i}{3}$ D. $\ln(4\sqrt{3}) + \frac{\pi i}{3}$ E. NOTA

3. Find the area bounded by the locus of points defined by $\frac{1}{2}|z| = |z - 3|$

A. 24π B. 16π C. 4π D. 48π E. NOTA

4. Express in Cartesian form: $\left(\frac{\text{cis } 195°}{\text{cis } 75°}\right)$

A. $-i$ B. $\frac{i\sqrt{3}-1}{2}$ C. $\frac{\sqrt{3}-i}{2}$ D. $\frac{1+i\sqrt{3}}{2}$ E. NOTA

5. Find $Im\left(e^{\frac{7\pi}{12}i}\right)$

A. $\frac{\sqrt{6}+\sqrt{2}}{2}$ B. $\frac{\sqrt{2}-\sqrt{6}}{4}$ C. $\frac{\sqrt{6}+\sqrt{2}}{4}$ D. $\frac{\sqrt{6}-\sqrt{2}}{4}$ E. NOTA

6. Let $f(x)$ be a monic fifth degree polynomial with real coefficients where $f(2) = f(i) = f(2i) = 0$. Find the sum of the coefficients of $f(x)$.

A. -10 B. -6 C. -12 D. 0 E. NOTA

7. Which of the following is equivalent to $\sqrt{-6} \cdot \sqrt{-4}$?

A. $-2\sqrt{6}$ B. $2\sqrt{6}$ C. $4\sqrt{3}i$ D. $2\sqrt{6}i$ E. NOTA

8. How many of the 40th roots of unity are in Quadrant II within the Argand plane?

A. 7 B. 8 C. 9 D. 10 E. NOTA

9. Evaluate $|(6+2i)(3+4i)(4+8i)(1-2i)|$

A. $200\sqrt{2}$ B. $200\sqrt{10}$ C. $400\sqrt{10}$ D. 400 E. NOTA

10. Which of the following the equivalent to $\left(2\sqrt{3}-2i\right)^8$?

A. $2^{15}\left(-1-i\sqrt{3}\right)$ B. $2^{16}\left(-1-i\sqrt{3}\right)$ C. $2^{16}\left(-\sqrt{3}+i\right)$ D. $2^{15}\left(-\sqrt{3}+i\right)$ E. NOTA

11. Determine $\begin{bmatrix} i & 3 \\ -2i & 4 \end{bmatrix}\begin{bmatrix} -3i & 4i & -i \\ 2 & i & 3 \end{bmatrix}$

A. $\begin{bmatrix} 9 & 7i & 8 \\ 14 & 4i-8 & 10 \end{bmatrix}$ B. $\begin{bmatrix} 3 & i & 8 \\ 2 & 4i+8 & 14 \end{bmatrix}$ C. $\begin{bmatrix} 3 & 4+3i & 10 \\ 2 & 8+4i & 14 \end{bmatrix}$ D. $\begin{bmatrix} 9 & 3i-4 & 10 \\ 2 & 8+4i & 10 \end{bmatrix}$ E. NOTA

12. Which of the following is equivalent to the distance between the points on the complex plane representing $2+3i$ and $6-5i$?

A. $2\sqrt{5}$ B. $4\sqrt{5}$ C. $2\sqrt{17}$ D. $4\sqrt{3}$ E. NOTA

13. Which of the following represents the point $(0,1)$ rotated 60° counterclockwise about the point $(-2,-1)$?

A. $\left(\sqrt{3},\sqrt{3}-1\right)$ B. $\left(1-\sqrt{3},1+\sqrt{3}\right)$ C. $\left(-1-\sqrt{3},\sqrt{3}\right)$ D. $\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ E. NOTA

14. Evaluate $\begin{vmatrix} 1+2i & 3 & i \\ 0 & 3 & 1 \\ 0 & 0 & 4i+1 \end{vmatrix}$

A. $18 + 81i$ B. $18i - 21$ C. $6i - 9$ D. $6 + 7i$ E. NOTA

15. When $|z - 6| + |z - 8i| = 26$ is plotted on the Argand plane, it is an ellipse. Find the eccentricity of the ellipse.

A. $\frac{5}{13}$ B. $\frac{3}{5}$ C. $\frac{4}{5}$ D. $\frac{12}{13}$ E. NOTA

16. When $|z - 6| + |z - 8i| = 26$ is plotted on the Argand plane, it is an ellipse. Find the area of the ellipse.

A. 60π B. 65π C. 78π D. 156π E. NOTA

17. Express in complex exponential form: $(\text{cis}\,25°)(2\,\text{cis}\,95°)(\text{cis}\,210°)$

A. $e^{ln2-\frac{\pi}{6}i}$ B. $e^{2+\frac{5\pi}{3}i}$ C. $e^{\ln 2+\frac{5\pi}{6}i}$ D. $e^{2-\frac{\pi}{6}i}$ E. NOTA

18. Which of the following are true?

I. $\log(z) = \ln|z| + i[arg(z)]$

II. $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$

III. $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

A. I and II B. I only C. II and III D. II only E. NOTA

19. Find the number of distinct non-real roots in the equation below

$$f(x) = x^6 - 4x^4 + 3x^2 - 12$$

A. 0 B. 2 C. 4 D. 6 E. NOTA

20. For a complex number z: $Re(z) > Im(z)$, $z\overline{z} = \frac{1}{2}$, and $Im(z^2) = \frac{\sqrt{2}}{3}$. Which of the following is equivalent to $Im(z) - Re(z)$?

A. $\frac{\sqrt{3}-\sqrt{6}}{6}$ B. $\frac{\sqrt{6}-\sqrt{2}}{6}$ C. $\frac{2}{3}$ D. $\frac{\sqrt{6}-2\sqrt{3}}{6}$ E. NOTA

For questions 21-23, let $z_a = 1 + 5i$ and $\overline{z_b} = 2i - 6$

21. What is the $arg(\, z_a\overline{z_b} + \overline{z_a}\overline{z_b})$?

A. $Arctan\left(-\frac{1}{3}\right)$ B. $Arctan\left(\frac{1}{2}\right)$ C. $Arctan\left(-\frac{1}{5}\right)$ D. $Arctan\left(\frac{1}{3}\right)$ E. NOTA

22. Find the magnitude of $\overline{z_a} + z_b$

A. $\sqrt{58}$ B. $\sqrt{74}$ C. 8 D. $\sqrt{34}$ E. NOTA

23. Which of the following is equivalent to $z_a - \overline{\overline{z_a}z_b}$?

A. $17 + 33i$ B. $-3 - 27i$ C. $-15 + 33i$ D. $17 - 23i$ E. NOTA

24. Express in Cartesian form: $e^{\left(3ln2+i\frac{4\pi}{3}\right)}$

A. $-4i\sqrt{3} - 4$ B. $-3\sqrt{3} - 3i$ C. $-3 - 3i\sqrt{3}$ D. $-4 - 4i\sqrt{3}$ E. NOTA

25. Find the imaginary part of *S*, given *S* represents the sum of the series $2 - \frac{4i}{3} - \frac{8}{9} + \frac{16i}{27} + \cdots$

A. $-\frac{1}{3}$ B. $-\frac{6}{5}$ C. $-\frac{12}{13}$ D. $\frac{18}{13}$ E. NOTA

26. Which of the following represents the number of integral solutions for $cis\left(15(x°)\right) = -1$ on the interval $0° < x° < 90°$?

A. 3 B. 4 C. 5 D. 6 E. NOTA

27. When $|z-6|+|z-8i|=10$ is graphed on the Argand plane, which of the following best describes the graph?

A. No graph B. Line C. Segment D. Ellipse E. NOTA

28. Which of the following is equivalent to $\frac{7+5i}{2-i}$?

A. $\frac{9-3i}{5}$ B. $\frac{9+3i}{3}$ C. $\frac{19+3i}{5}$ D. $\frac{9+17i}{5}$ E. NOTA

29. Find the sum of all possible values of $\left(\frac{5}{6i+\frac{5}{6i+\frac{5}{6i+\cdots}}}\right)^2$

A. -46 B. -26 C. 26 D. 46 E. NOTA

30. Simplify i^{2020}

A. -1 B. 1 C. i D. $-i$ E. NOTA