Answers:

1. A	7. B	13. B	19. A	25. D
2. B	8. C	14. A	20. D	26. B
3. C	9. B	15. D	21. D	27. D
4. D	10. B	16. E (378)	22. A	28. D
5. E ($y = \frac{29}{8}$)	11. D	17. A	23. A	29. B
6. D	12. C	18. D	24. B	30. D

Solutions:

1. A A $\boxed{\text{circle}}$ is defined as the locus of points a given distance (the radius) away from a given point (the center).

2. B The shape this produces is a large cone with a smaller, concentric cone removed from it. (Let the origin be point O. The larger cone is triangle AOC revolved around the y-axis, and the smaller cone is triangle AOB revolved around the y-axis.)
The volume can be found by subtracting the volume of the smaller cone from the larger cone:
$$V_l - V_s = \frac{\pi}{3}(3^2)(2) - \frac{\pi}{3}(1^2)(2) = \frac{18\pi}{3} - \frac{2\pi}{3} = \boxed{\frac{16\pi}{3}}$$

3. C The normal vector of the plane is the cross product of the two given vectors which the plane contains:
$$\langle 2,3,-1\rangle \times \langle -4,1,3\rangle = \begin{vmatrix} 2 & 3 & -1 \\ -4 & 1 & 3 \\ \mathbf{i} & \mathbf{j} & \mathbf{k} \end{vmatrix} = \big(9-(-1)\big)\mathbf{i} - (6-4)\mathbf{j} + \big(2-(-12)\big)\mathbf{k}$$
$$= \langle 10,-2,14\rangle$$
These form the coefficients of the plane, so the equation is $10x - 2y + 14z + D = 0$. D can be found by plugging in the given point $(6,4,12)$:
$10(6) - 2(4) + 14(12) + D = 60 - 8 + 168 + D = 220 + D = 0 \rightarrow D = -220$
The equation becomes $10x - 2y + 14z = 220 \rightarrow \boxed{5x - y + 7z = 110}$

4. D Consider the points on a polar graph. A is 6 units from the origin and B is 3 units from the origin, and the radii connecting the origin to each of A and B differ by an angle of π. Thus, A and B lie on exactly opposite sides of the origin, so the distance $\overline{AB}$ is simply the sum of each point's distance from the origin: $6 + 3 = \boxed{9}$.

5. E Rearranging into the form $4p(y-k) = (x-h)^2$, the equation of the parabola becomes $\frac{3}{2}(y-4) = (x+2)^2$. In this form, the distance from the vertex to the directrix is $p = 3/8$. The parabola opens up towards positive y, so the directrix is a horizontal line lying below the vertex, with equation given by the y-coordinate of the vertex minus p: $y = 4 - \frac{3}{8} \rightarrow \boxed{y = \frac{29}{8}}$

6. D <u>Solution 1:</u>
By the rotation matrix,
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix}\begin{bmatrix} 4 \\ -6 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} 4 \\ -6 \end{bmatrix} =$$
$$\begin{bmatrix} 2\sqrt{3}+3 \\ 2-3\sqrt{3} \end{bmatrix} \rightarrow \left(2\sqrt{3}+3\right) + \left(2-3\sqrt{3}\right) = \boxed{5-\sqrt{3}}$$

<u>Solution 2:</u>
Point P is in Quadrant IV and is a distance of $2\sqrt{13}$ from the origin. Let α be the angle $-90° < \alpha < 0°$ formed by the x-axis and the line segment from the origin to P. Then $\cos\alpha = \frac{2}{\sqrt{13}}$ and $\sin\alpha = -\frac{3}{\sqrt{13}}$. A $30°$ counterclockwise rotation to point $P'(a, b)$ will maintain the same distance from the origin, but the angle α will increase by $30°$, so:
$$a = 2\sqrt{13}\cos(\alpha + 30°) = 2\sqrt{13}(\cos\alpha\cos 30° - \sin\alpha\sin 30°) =$$
$$2\sqrt{13}\left(\left(\frac{2}{\sqrt{13}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{-3}{\sqrt{13}}\right)\left(\frac{1}{2}\right)\right) = 2\sqrt{13}\left(\frac{\sqrt{3}}{\sqrt{13}} + \frac{3}{2\sqrt{13}}\right) = 2\sqrt{3}+3$$
and
$$b = 2\sqrt{13}\sin(\alpha + 30°) = 2\sqrt{13}(\sin\alpha\cos 30° + \cos\alpha\sin 30°) =$$
$$2\sqrt{13}\left(\left(\frac{-3}{\sqrt{13}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{2}{\sqrt{13}}\right)\left(\frac{1}{2}\right)\right) = 2\sqrt{13}\left(\frac{-3\sqrt{3}}{2\sqrt{13}} + \frac{1}{\sqrt{13}}\right) = -3\sqrt{3}+2$$
$$\rightarrow \left(2\sqrt{3}+3\right) + \left(-3\sqrt{3}+2\right) = \boxed{5-\sqrt{3}}$$

7. B One definition for the eccentricity of an ellipse is the ratio of the distance between any point P on the ellipse and one focus F to the distance between P and the corresponding directrix l. Take the endpoint of the major axis $(\frac{\sqrt{21}}{5}, -\frac{2}{5})$ as the point P, take the origin to be the focus F, and rewrite the directrix l in the form $Ax + By = C$ to yield $\sqrt{21}x - 2y = 25$.
$$|PF| = \sqrt{(\frac{\sqrt{21}}{5} - 0)^2 + (-\frac{2}{5} - 0)^2} = \sqrt{\frac{21}{25} + \frac{4}{25}} = 1$$

$|Pl| = \frac{|Aa+Bb-C|}{\sqrt{A^2+B^2}} = \frac{\left|\sqrt{21}\left(\frac{\sqrt{21}}{5}\right)-2(-\frac{2}{5})-25\right|}{\sqrt{(\sqrt{21})^2+(-2)^2}} = \frac{20}{5} = 4$

$e = \frac{|PF|}{|Pl|} = \boxed{1/4}$

8. C The distance between the points is $\sqrt{(6-(-1))^2+(5-4)^2+(3-5)^2} = \sqrt{49+1+4} = \sqrt{54} = 3\sqrt{6}$.

There are 3 cases for what dimension of the cube this distance represents, each yielding a different value for the volume V:

1. $3\sqrt{6}$ is the side length $s \rightarrow V = s^3 = (3\sqrt{6})^3 = 162\sqrt{6}$
2. $3\sqrt{6}$ is the diagonal of a face $\rightarrow s = \frac{3\sqrt{6}}{\sqrt{2}} = 3\sqrt{3} \rightarrow V = s^3 = (3\sqrt{3})^3 = 81\sqrt{3}$
3. $3\sqrt{6}$ is a space diagonal of the cube $\rightarrow s^2 + (s\sqrt{2})^2 = 3s^2 = (3\sqrt{6})^2 = 54 \rightarrow s = \sqrt{18} = 3\sqrt{2} \rightarrow V = s^3 = (3\sqrt{2})^3 = 54\sqrt{2}$

Geometric mean $= \sqrt[3]{(162\sqrt{6})(81\sqrt{3})(54\sqrt{2})} = \sqrt[3]{3^{11}2^2\sqrt{36}} = \sqrt[3]{3^{12}2^3} = 3^4 \cdot 2 = \boxed{162}$

9. B Using the substitutions $r^2 = x^2 + y^2$, $\theta = \tan^{-1}(\frac{y}{x})$, and $x = \cos\theta$,

$y = x\tan(2x^2+y^2) \rightarrow \tan^{-1}(\frac{y}{x}) = x^2 + x^2 + y^2 \rightarrow \theta = r^2(\cos\theta)^2 + r^2 \rightarrow r^2 = \frac{\theta}{1+(\cos\theta)^2} \rightarrow \boxed{r = \pm\sqrt{\frac{\theta}{1+(\cos\theta)^2}}}$

(If r is only allowed to be positive, the polar graph will only contain half of the Cartesian graph of $y = x\tan(2x^2+y^2)$, since the graph spirals outwards and never repeats.)

10. B Let the three given vertices of the parallelogram be $B(-8,-2)$, $C(-3,-11)$, and $D(2,-5)$. As a result of the properties of parallelograms, a possible location of the final vertex A can be found by translating point B by the vector given by segment CD (i.e. $2-(-3) = 5$ units right and $-5-(-11) = 6$ units upward). Likewise, this can be done for the remaining two points, resulting in three total possible locations for vertex A. Thus, convex polygon $A_1A_2 \ldots A_n = A_1A_2A_3$ is a triangle. Each possible final vertex A adds a triangle with area equal to that of the original triangle BCD to the total area of $A_1A_2A_3$, so the total area of $A_1A_2A_3$ is $[BCD] + 3[BCD] = 4[BCD]$. (Using this method, there is no need to calculate the coordinates of the possible locations of A, but finding the coordinates is still a viable approach if this one is not discovered.)

$[BCD] = \frac{1}{2}\begin{vmatrix} -8 & -3 & 2 \\ -2 & -11 & -5 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\big(88 + 15 - 4 - (-22 + 40 + 6)\big) = \frac{99-24}{2} = \frac{75}{2}$

$[A_1A_2A_3] = 4[BCD] = 4\left(\frac{75}{2}\right) = \boxed{150}$

11. D The magnitude of the projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$ is equal to $|\vec{\mathbf{u}}|$ multiplied by the cosine of the angle θ between the vectors.

$|\vec{\mathbf{u}}| \cos\theta = 6\cos\theta = |\langle 1, 3, -2\rangle| = \sqrt{1^2 + 3^2 + (-2)^2} = \sqrt{14} \rightarrow \cos\theta = \frac{\sqrt{14}}{6} \rightarrow$

$\sin\theta = \sqrt{1 - (\frac{\sqrt{14}}{6})^2} = \sqrt{1 - \frac{14}{36}} = \boxed{\frac{\sqrt{22}}{6}}$

(Note that $\cos\theta = -\frac{\sqrt{14}}{6}$ in the case that θ is obtuse, but $\sin\theta$ remains positive.)

12. C Rearrange $25(x-8)^2 - 144(y+17)^2 = 3600$ into the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ to obtain the hyperbola $\frac{(x-8)^2}{12^2} - \frac{(y+17)^2}{5^2} = 1$. The focal radius c of a hyperbola has the property that $c^2 = a^2 + b^2$.

$c^2 = 12^2 + 5^2 \rightarrow c = \boxed{13}$

13. B

I) 3 distinct points do not define a unique plane in the case where all are colinear.

II) 2 distinct lines may be skew, in which case no plane can contain both lines.

III) If one vector is a scalar multiple of the other, their cross product will produce the vector $\langle 0, 0, 0\rangle$, which does not define a unique plane even when given a point.

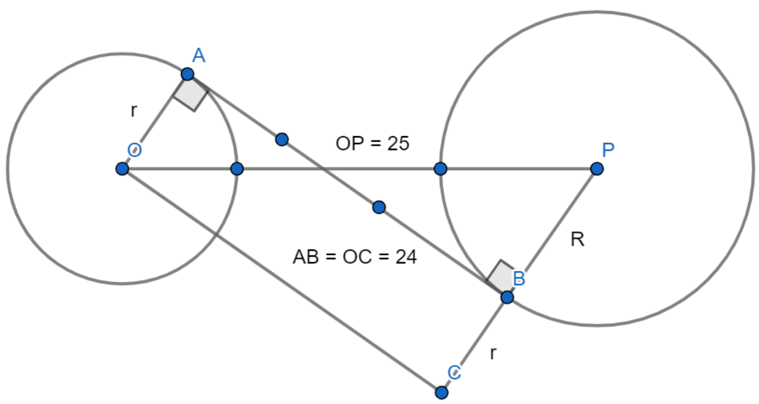
IV) Given a line l and a point P not on l, a plane containing both can be defined by P and 2 vectors that are not scalar multiples of each other. One vector is in the direction of l, and the other vector is in the direction of a second line m intersecting both P and l. Regardless of which line m is chosen, the same nonzero normal vector to the plane will be produced, which along with point P (or any point on l or m) is sufficient to define a unique plane.

Thus, $\boxed{\text{Only IV}}$ always defines a unique plane.

14. A It is best to approach this problem by ignoring the coordinate plane and instead focusing purely on geometry.

By the distance formula, the distance between the circles' centers $|OP|$ is $\sqrt{(9-(-6))^2 + (-17-3)^2} = \sqrt{15^2 + 20^2} = 25$. Arbitrarily, let the lengths of

the radii of the circles be r and R. The interior tangent of length 24 meets the radii of each circle at right angles at points A and B, as per the diagram below. The interior tangent $\overline{AB}$ may be translated to the segment $\overline{OC}$ of the same length, forming rectangle $ABCO$. Since all interior angles of a rectangle are 90°, $\angle OCP$ is a right angle. Thus, $\triangle OCP$ is a right triangle, with legs of length 24 and $r+R$, and a hypotenuse of length 25. By the Pythagorean theorem, $24^2+(r+R)^2=25^2 \rightarrow (r+R)^2=(25+24)(25-24)=49 \rightarrow r+R=7$. Since all we need to find is the shortest distance between a point on circle O and a point on circle P, this length is trivially the distance $|OP|$ minus the lengths of the radii of each circle.
$|OP|-r-R=|OP|-(r+R)=25-7=\boxed{18}$



15. D Let $z=a+bi$. Then $z+\bar{z}=2a$, $z-\bar{z}=2bi$, and $|z|=\sqrt{a^2+b^2}$. Plugging these values into the equation $4(z+\bar{z})^2-(z-\bar{z}-20i)^2=144$, we obtain the equation $\frac{a^2}{9}+\frac{(b-10)^2}{36}=1$. This is the graph of an ellipse in the complex plane, with center $(0,10)$, semi-major axis 6, and semi-minor axis 3.
We are looking to minimize the value of $|z|^2$, or the square of a point's distance from the origin in the complex plane. This is the same as simply minimizing $|z|$. The ellipse is centered directly above the origin, so graphically, the closest point on the ellipse to the origin is the endpoint of the axis below the ellipse's center, the point $(0,4)$. (Picture a circle $a^2+b^2=r^2$ expanding out from the origin until it intersects the graph of the ellipse; the first intersection will be when $r=4$.)
$|z|^2=a^2+b^2=0^2+4^2=\boxed{16}$

16. E $|3x-2|+|7y+41|=63 \rightarrow 3\left|x-\frac{2}{3}\right|+7\left|y+\frac{41}{7}\right|=63$
This produces the graph of a rhombus. Shifting the center of the graph does not affect the area, so this is equivalent to the graph of $3|x|+7|y|=63$. The vertices

of the rhombus are where $|x|$ or $|y|$ is maximized, so they occur when the other variable in the equation has its minimum absolute value, which is 0.

$3|x| + 7|0| = 63 \rightarrow 3|x| = 63 \rightarrow x = \pm 21$

$3|0| + 7|y| = 63 \rightarrow 7|y| = 63 \rightarrow y = \pm 9$

Then the vertices of the rhombus are $(21, 0)$, $(0, 9)$, $(-21, 0)$, and $(0, -9)$. Its diagonals have lengths 42 and 18 and intersect at right angles, so the total area enclosed is $\frac{1}{2}(42)(18) = \boxed{378}$.

17. A $r = 2 + 3\cos\theta \rightarrow r^2 = 2r + 3r\cos\theta \rightarrow 3x = r^2 - 2r \rightarrow x = \frac{1}{3}r^2 - \frac{2}{3}r$

Since x is now written as a quadratic in terms of r with a positive leading coefficient, the value of x will be minimized at the vertex of the quadratic.

$$-\frac{b}{2a} = -\frac{-\frac{2}{3}}{2\left(\frac{1}{3}\right)} = 1 \rightarrow x_{min} = \frac{1}{3}(1)^2 - \frac{2}{3}(1) = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$$

$$y = \pm\sqrt{r^2 - x^2} = \pm\sqrt{(1)^2 - \left(\frac{1}{3}\right)^2} = \pm\sqrt{\frac{8}{9}} = \pm\frac{2\sqrt{2}}{3} \rightarrow |y| = \boxed{\frac{2\sqrt{2}}{3}}$$

18. D $x + 3y^2 = 5 \rightarrow -\frac{1}{3}(x - 5) = y^2$

This is a parabola with vertex $(5, 0)$ that opens up to the left, towards negative x. Plug in $x = -1$ to find where the line intersects the parabola:

$y^2 = -\frac{1}{3}(-1 - 5) = 2 \rightarrow y = \pm\sqrt{2}$

The base B of the parabolic sector is the segment of the line $x = -1$ between its intersection points with the parabola, so the length of the base is $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$. The height h is the length from the base to the vertex $(5, 0)$, a distance of 6.

$A = \frac{2}{3}Bh = \frac{2}{3}\left(2\sqrt{2}\right)(6) = \boxed{8\sqrt{2}}$

19. A Conic F is in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. The conic discriminant $B^2 - 4AC = (-10)^2 - 4(4)(6) = 100 - 96 = 4$ is greater than 0, so Conic F is a hyperbola, if it is not degenerate. To check for degeneracy, calculate the determinant $\begin{vmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{vmatrix}$, which equals 0 for degenerate conics.

$$\begin{vmatrix} 4 & -5 & 0 \\ -5 & 6 & 7 \\ 0 & 7 & 2 \end{vmatrix} = 48 - 196 - 50 = -198 \neq 0,$$

so conic F is a non-degenerate $\boxed{\text{hyperbola}}$.

20. D The acute angle θ that the coordinate axes must be rotated counterclockwise by to eliminate the rotational xy term of a conic in the form $Ax^2 + Bxy + Cy^2 + Dx +$

$Ey + F = 0$ is given by the formula $\cot 2\theta = \frac{A-C}{B}$. (This is difficult to derive, but it possible to do so in case the formula is not known.)

$\cot 2\theta = \frac{4-6}{-10} = \frac{1}{5} \rightarrow \tan 2\theta = 5 \rightarrow \frac{2\tan\theta}{1-(\tan\theta)^2} = 5 \rightarrow 2\tan\theta = 5 - 5(\tan\theta)^2$

$\rightarrow 5(\tan\theta)^2 + 2\tan\theta - 5 = 0 \rightarrow \tan\theta = \frac{-2\pm\sqrt{2^2-4(5)(-5)}}{2(5)} = \frac{-2\pm\sqrt{4+100}}{10} = \frac{-2\pm2\sqrt{26}}{10} = \frac{-1\pm\sqrt{26}}{5}$

θ is in the first quadrant, so $\tan\theta > 0 \rightarrow \tan\theta = \boxed{\frac{\sqrt{26}-1}{5}}$

21. D A) Conic F is a hyperbola, so its eccentricity is greater than 1.

B) $36x^2 + 31y^2 = 1116 \rightarrow \frac{x^2}{31} + \frac{y^2}{36} = 1$

This is an ellipse with $a = 6$ and $b = \sqrt{31} \rightarrow c = \sqrt{a^2 - b^2} = \sqrt{36-31} = \sqrt{5} \rightarrow e = \frac{c}{a} = \frac{\sqrt{5}}{6}$

C) $r = \frac{84}{11-11\sin\theta} \rightarrow r = \frac{\frac{84}{11}}{1-\sin\theta}$

This is in the form $r = \frac{ep}{1-e\sin\theta}$, so $e = 1$ and this is a parabola.

D) $r = \frac{20}{22+7\cos\theta} \rightarrow r = \frac{\frac{10}{11}}{1+\left(\frac{7}{22}\right)\cos\theta}$

This is in the form $r = \frac{ep}{1+e\cos\theta}$, so this is an ellipse with $e = \frac{7}{22}$.

This leaves only the ellipses in B) and D) as contenders for the least eccentricity.

We find that $\frac{\sqrt{5}}{6} > \frac{\sqrt{4}}{6} = \frac{2}{6} = \frac{7}{21} > \frac{7}{22}$, so $\boxed{\text{D}}$ has the least eccentricity.

22. A The vector $\vec{\mathbf{u}}$ from $(-3, 5)$ to $(7, -1)$ is $\langle(7-(-3)), (-1-5)\rangle = \langle 10, -6\rangle$.

$\hat{\mathbf{u}} = \frac{\vec{\mathbf{u}}}{|\vec{\mathbf{u}}|} = \frac{1}{\sqrt{10^2+(-6)^2}}\langle 10, -6\rangle = \frac{1}{2\sqrt{34}}\langle 10, -6\rangle = \langle\frac{5\sqrt{34}}{34}, -\frac{3\sqrt{34}}{34}\rangle$

$\hat{\mathbf{u}} \cdot \langle 1, 1\rangle = \frac{5\sqrt{34}}{34} - \frac{3\sqrt{34}}{34} = \boxed{\frac{\sqrt{34}}{17}}$

23. A Let the measure of $\angle CAD$ be 3θ. Then $\tan 3\theta = \frac{143}{26} = \frac{11}{2}$ and $\tan\theta = \frac{|DF|}{|AD|} = \frac{|DF|}{26}$.

Expand $\tan 3\theta$ to find $\tan\theta$ and thus $|DF|$.

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\tan 3\theta = \tan(2\theta+\theta) = \frac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}$

$\tan 2\theta = \tan(\theta+\theta) = \frac{2\tan\theta}{1-(\tan\theta)^2}$

$$\tan 3\theta = \frac{\left(\frac{2\tan\theta}{1-(\tan\theta)^2}+\tan\theta\right)}{\left(1-\frac{2\tan\theta}{1-(\tan\theta)^2}(\tan\theta)\right)} = \frac{\left(\frac{2\tan\theta+\tan\theta-(\tan\theta)^3}{1-(\tan\theta)^2}\right)}{\left(\frac{1-(\tan\theta)^2-2(\tan\theta)^2}{1-(\tan\theta)^2}\right)} = \frac{3\tan\theta-(\tan\theta)^3}{1-3(\tan\theta)^2} = \frac{11}{2}$$

$$\frac{3\tan\theta-(\tan\theta)^3}{1-3(\tan\theta)^2} - \frac{11}{2} = 0 \rightarrow \frac{2\left(3\tan\theta-(\tan\theta)^3\right)-11\left(1-3(\tan\theta)^2\right)}{1-3(\tan\theta)^2} = 0 \rightarrow$$

$$\frac{2(\tan\theta)^3-33(\tan\theta)^2-6\tan\theta+11}{1-3(\tan\theta)^2} = 0$$

The numerator is a polynomial in terms of $\tan\theta$. $\tan\theta = \frac{1}{2}$ is a rational root, so $|DF| = 26\tan\theta = 13$.

(There are other real roots, namely $8 \pm 5\sqrt{3}$, but these produce values of θ which lie outside the acceptable range $0 < \theta < \frac{90°}{3} = 30°$.)

Then $|FC| = |DC| - |DF| = 143 - 13 = 130$. By the Pythagorean theorem, $|BF|^2 = |FC|^2 + |BC|^2 = 130^2 + 26^2 \rightarrow |BF| = \sqrt{130^2+26^2} = 13\sqrt{10^2+2^2} = 13\sqrt{104} = \boxed{26\sqrt{26}}$

24. B Eliminate the parameter t by substituting x for $\sqrt{t}$ in the equation of $y(t)$:

$$y(t) = \left(\sqrt{t}\right)^2 - 1 + 16\sin\left(\frac{3}{2}\sqrt{t}\right) \rightarrow y(x) = x^2 - 1 + 16\sin\left(\frac{3}{2}x\right)$$

Note that $t > 0$ and $x(t) = \sqrt{t}$, so the domain of $y(x)$ is restricted to $x > 0$.

We are looking for the number of times that $y(x) = 0$ on its domain.

$y = x^2 - 1 + 16\sin\left(\frac{3}{2}x\right) = 0 \rightarrow x^2 - 1 = -16\sin\left(\frac{3}{2}x\right)$, so this is equivalent to the number of times $f(x) = x^2 - 1$ and $g(x) = -16\sin\left(\frac{3}{2}x\right)$ intersect for $x > 0$.

Analyze the graphs of $f(x)$ and $g(x)$ in the intervals $\frac{\pi}{2}k \le \frac{3}{2}x < \frac{\pi}{2}(k+1)$, $k \in \mathbb{Z}$.

($\sin\left(\frac{3}{2}x\right)$ and $x^2 - 1$ are both monotonic in each of these intervals, so using intervals of $\pi/2$ is ideal for finding every intersection.)

Computing the values of $f(x)$ and $g(x)$ for the endpoints of each interval, we have:

$\frac{3}{2}x$	x	$f(x) = x^2 - 1$	$g(x) = -16\sin\left(\frac{3}{2}x\right)$
0	0	-1	0
$\pi/2$	$\pi/3 \approx 1$	≈ 0	-16
π	$2\pi/3 \approx 2$	≈ 3	0
$3\pi/2$	$\pi \approx 3$	≈ 8	16
2π	$4\pi/3 \approx 4$	≈ 15	0
$5\pi/2$	$5\pi/3 \approx 5$	≈ 24	-16

The following can then be determined (with the added help of an accurate picture):

- In $(0, \frac{\pi}{2})$, $g(x)$ changes from $> f(x)$ to $< f(x)$, meaning there is at least one intersection. $g(x)$ does not change direction on the interval, so there is exactly one intersection.
- In $(\frac{\pi}{2}, \pi)$, $g(x) < f(x)$ on the entire interval, so there are no intersections.
- In $(\pi, \frac{3\pi}{2})$, $g(x)$ changes from $< f(x)$ to $> f(x)$, meaning there is at least one intersection. $g(x)$ does not change direction on the interval, so there is exactly one intersection.
- In $(\frac{3\pi}{2}, 2\pi)$, $g(x)$ changes from $> f(x)$ to $< f(x)$, meaning there is at least one intersection. $g(x)$ does not change direction on the interval, so there is exactly one intersection.
- In $(2\pi, \frac{5\pi}{2})$, $g(x) > f(x)$ on the entire interval, so there are no intersections.

The maximum value of $g(x) = -16\sin\left(\frac{3}{2}x\right)$ is 16, so since $f(x) = x^2 - 1$ is monotonically increasing for $x > 0$, there are no further intersections past where $f(x) = 16$.
$x^2 - 1 = 16 \rightarrow x^2 = 17 \rightarrow x = \sqrt{17} < 5 < 5\pi/3$, so all intersections have been found. Thus, there are a total of 3 intersections of $f(x)$ and $g(x)$ for $x > 0$, meaning that $y(x) = 0$ exactly $\boxed{3}$ times on its domain.

25. D Let triangle ABC be the base of the tetrahedron. Since the final point D is a distance of 2 away from the plane containing the base, the tetrahedron's height is 2. Then the volume $V = \frac{1}{3}B \cdot h = \frac{1}{3}[ABC](2)$.

$$[ABC] = \frac{1}{2}\begin{vmatrix} 3 & 4 & -2 \\ 1 & 1 & 1 \\ 3 & 6 & 6 \end{vmatrix} = \frac{1}{2}|18 + 12 - 12 - (-6 + 18 + 24)| = \frac{1}{2}|-18| = 9$$

$V = \frac{1}{3}(9)(2) = \boxed{6}$

26. B $x^2 + y^2 - 12x - 22y = -121 \rightarrow (x-6)^2 + (y-11)^2 = -121 + 36 + 121 = 36$. This is a circle centered at $(6, 11)$ with radius 6.
The distance from the center $(6, 11)$ to the line $4x + 3y = 42$ is $\frac{|Aa+Bb-C|}{\sqrt{A^2+B^2}} = \frac{|4(6)+3(11)-42|}{\sqrt{3^2+4^2}} = \frac{|24+33-42|}{\sqrt{25}} = \frac{15}{5} = 3 < 6$, so the line intersects the circle at 2 points to form a chord. Let the center of the circle be point O, let the chord formed by the line's intersection with the circle be $\overline{AB}$, and let point C be the midpoint of chord $\overline{AB}$. To find the area of the larger section, we will split the section into $\triangle OAB$ and the circular sector enclosed by major arc AB.

Since a radius through the midpoint of a chord is a perpendicular bisector of the chord, segment $\overline{OC}$ is a perpendicular bisector of $\overline{AB}$. Then $\angle OCA = \angle OCB = \frac{\pi}{2}$.
Since $\angle OCA$ is a right angle, $\overline{OC}$ is also the shortest distance from O to line $\overleftrightarrow{AB}$, which was found to be 3. In addition, $\overline{OA}$ and $\overline{OB}$ are radii of the circle, so $|OA| = |OB| = 6$. $\cos\angle AOC = \cos\angle BOC = \frac{|OC|}{|OA|} = \frac{|OC|}{|OB|} = \frac{3}{6} = \frac{1}{2} \rightarrow \angle AOC = \angle BOC = \pi/3$ $\rightarrow \angle AOB = 2\pi/3$.
Then $[OAB] = \frac{1}{2}|OA||OB|\sin\angle AOB = \frac{1}{2}(6)(6)\sin\frac{2\pi}{3} = 18\left(\frac{\sqrt{3}}{2}\right) = 9\sqrt{3}$.
In addition, the circular sector defined by major arc AB has a central angle of $2\pi - \angle AOB = 2\pi - \frac{2\pi}{3} = 4\pi/3$. Thus, the sector is $\frac{4\pi/3}{2\pi} = 2/3$ of the area of the entire circle. $\frac{2}{3}(\pi(6)^2) = \frac{2}{3}(36\pi) = 24\pi$.
The total area of the larger section of the circle is then $\boxed{24\pi + 9\sqrt{3}}$.

27. D Every point (x, y, z) on line l can be expressed parametrically as $(1 + 2t, 4 + 4t, -3 - t)$. By the distance formula, the distance between $(4, -2, 3)$ and some point on line l is $\sqrt{(4-(1+2t))^2 + (-2-(4+4t))^2 + (3-(-3-t))^2} = \sqrt{(3-2t)^2 + (-6-4t)^2 + (6+t)^2} = \sqrt{21t^2 + 48t + 81}$.
The value of t for which $\sqrt{21t^2 + 48t + 81}$ is minimized is the same as the value of t for which $21t^2 + 48t + 81$ is minimized. $21t^2 + 48t + 81$ is a quadratic with a positive leading coefficient, so it will have its minimum value at its vertex. The value of t at the vertex is $-\frac{b}{2a} = -\frac{48}{2(21)} = -\frac{48}{42} = -8/7$, so $\boxed{-8/7}$ is the value of t at the minimum distance between line l and $(4, -2, 3)$.

28. D The slope of any line l can be calculated as the tangent of the angle (on the right side of l) that l makes with any horizontal line. For this problem, use the horizontal line $y = -2$ for reference. Let the acute angle between $\overline{AB}$ and $y = -2$ be θ. Then the angle between $\overline{BC}$ and $y = -2$ on the right side is $\theta + \pi/3$, since $\angle ABC$ is the interior angle of an equilateral triangle.
Slope of $\overline{BC} = \tan\left(\theta + \frac{\pi}{3}\right) = \frac{\tan\theta + \tan\frac{\pi}{3}}{1 - \tan\theta\tan\frac{\pi}{3}} = \frac{\tan\theta + \sqrt{3}}{1 - \sqrt{3}\tan\theta}$
$\tan\theta =$ slope of $\overline{AB} = \frac{2-(-2)}{2-(-1)} = \frac{4}{3}$
Slope of $\overline{BC} = \frac{\frac{4}{3}+\sqrt{3}}{1-\frac{4}{3}\sqrt{3}} = \frac{4+3\sqrt{3}}{3-4\sqrt{3}} = \frac{(4+3\sqrt{3})(3+4\sqrt{3})}{(3-4\sqrt{3})(3+4\sqrt{3})} = \frac{12+36+16\sqrt{3}+9\sqrt{3}}{9-48} = -\frac{48+25\sqrt{3}}{39} = \frac{-48-25\sqrt{3}}{39} \rightarrow -48 - 25 + 39 = \boxed{-34}$

29. B

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} = \begin{vmatrix} 3 & -13 & -8 \\ -2 & -5 & 0 \\ \mathbf{i} & \mathbf{j} & \mathbf{k} \end{vmatrix} = (0-40)\mathbf{i} - (0-16)\mathbf{j} + (-15-26)\mathbf{k}$$

$$= \langle -40, 16, -41 \rangle$$

$$\langle -40, 16, -41 \rangle \cdot \vec{\mathbf{c}} = \langle -40, 16, -41 \rangle \cdot \langle 1, 1, 1 \rangle = -40 + 16 - 41 = \boxed{-65}$$

30. D The graph of $r = 2022\theta$ is a two-sided Archimedean spiral, which spirals outward counterclockwise for $\theta > 0$ and outward clockwise for $\theta < 0$. $r \sin\theta = y$, so to find intersections we must find where the y-coordinate of the spiral exceeds or falls below 2022. An accurate drawing will then help to explain the following:
When $\theta = 0$, the spiral is at the origin, so its y-coordinate is 0. At both $\theta = \frac{\pi}{2}$ and $\theta = -\frac{\pi}{2}$, the y-coordinate of the spiral is $2022(1)\left(\frac{\pi}{2}\right) > 2022$, so there is an intersection in each of $(-\frac{\pi}{2}, 0)$ and $(0, \frac{\pi}{2})$. At both $\theta = \pi$ and $\theta = -\pi$, the y-coordinate of the spiral drops back down below 2022 to 0, so there are additional intersections in each of $(-\pi, -\frac{\pi}{2})$ and $(\frac{\pi}{2}, \pi)$. Thus, there are a total of $\boxed{4}$ intersections in the interval $-\pi < \theta < \pi$.