Theta

Conics and Analytic Geometry

Test #611

Directions:

1. Fill out the top left section of the scantron. Do not abbreviate your school name.

2. In the Student ID Number grid, write your 9-digit ID# and bubble.

3. In the Test Code grid, write the 3-digit test# on this test cover and bubble.

4. Scoring for this test is 5 times the number correct plus the number omitted.

5. TURN OFF ALL CELL PHONES.

6. No calculators may be used on this test.

7. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

8. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

9. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

10. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

11. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

12. Unless a question asks for an approximation or a rounded answer, give the exact answer.

1. The midpoint between (6, 17) and the intersection of $3x-4y=8$ and $-2x+9y=1$ is (a, b). Find the value of $a+b$.

A. 9 B. 14 C. 3.5 D. 12 E. NOTA

2. Find the equation of the circle centered at (0, 3) that passes through the foci of $9x^2+16y^2=144$.

A. $x^2+y^2-6y-7=0$ B. $x^2+y^2-6y+7=0$
C. $x^2+y^2-6y-5=0$ D. $x^2+y^2-6y+5=0$ E. NOTA

3. One diagonal of a square lies on $3x-4y+8=0$ and one of its vertices is (–1, 1). Find the area of the square in square units.

A. $\frac{1}{50}$ B. $\frac{1}{25}$ C. $\frac{3}{50}$ D. $\frac{2}{25}$ E. NOTA

4. The points $(a, 4)$ and $(-2, b)$ lie on the line passing through (2, –1) and (5, –3). Which of the following lines contains (a, b)?

A. $6(x+y)=25$ B. $2x+6y=-1$
C. $2x+3y=6$ D. $6(x+y)=-25$ E. NOTA

5. Isosceles triangle *RAT* has base coordinates $A(1, 3)$ and $T(-2, 7)$. Which of the following points could be vertex *R*?

A. $(1, 6)$ B. $\left(-\frac{1}{2}, 5\right)$ C. $\left(\frac{5}{6}, 6\right)$ D. $(-1, 4)$ E. NOTA

6. Which of the following sets of points forms an equilateral triangle?

A. $(1, 0)$, (4, 0), (7, –1) B. (0, 0), $\left(\frac{3}{2}, \frac{4}{3}\right)$, $\left(\frac{4}{3}, \frac{3}{2}\right)$
C. $\left(\frac{2}{3}, 0\right)$, $\left(0, \frac{2}{3}\right)$, (1, 1) D. $\left(\frac{1}{6}, 2\right)$, $\left(\frac{2}{3}, -3\right)$, $(4, 1)$ E. NOTA

7. Points W and F on the line joining $A(-2, 5)$ and $B(3, 1)$ such that $AW = WF = FB$. Find the distance from (0, 0) to the midpoint of $\overline{WF}$.

A. 3 B. $\frac{\sqrt{37}}{2}$ C. 4 D. 3.5 E. NOTA

8. One vertex of an equilateral triangle is (2, 2) and its centroid is $\left(-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$. Find the length of one side of the triangle.

A. $4\sqrt{2}$ B. $4\sqrt{3}$ C. $3\sqrt{2}$ D. $5\sqrt{2}$ E. NOTA

9. Find the area of the triangle described in problem 8.

A. $\frac{32\sqrt{3}}{3}$ B. $8\sqrt{3}$ C. $\frac{9\sqrt{3}}{2}$ D. $16\sqrt{3}$ E. NOTA

10. Find the area of a rhombus having three consecutive vertices located at (2, –3), (6, 5), and (–2, 1). All choices are in square units.

A. 24 B. 36 C. 18 D. 48 E. NOTA

11. A triangle has (4, –3) and (–2, 5) as two of its vertices and its orthocenter at (1, 2). Find the coordinates of the third vertex.

A. (30, 24) B. (33, 26) C. (26, 33) D. (24, 36) E. NOTA

12. Triangle BUG is an acute triangle with medians $\overline{BD}$, $\overline{UE}$, and $\overline{GF}$. Points E and F are located at (3, 4) and (1,2), respectively, and centroid H is at (3, 2). If point D is located at (m, n), find the value of $m + n$.

A. 3 B. 5 C. 11 D. –3 E. NOTA

13. Using the information from problem 12, find the length of the altitude from point *B*.

A. $4\sqrt{2}$ B. $3\sqrt{2}$ C. $6\sqrt{2}$ D. $2\sqrt{3}$ E. NOTA

14. A mirror runs along the y-axis. A laser pointer is at (2, 3) and pointed at a point *K* on the mirror. If the reflection of the laser passes through the point (5, 10). Find the coordinates of point *K*.

A. (0, 3) B. (0, 2) C. (0, 5) D. (0, 4.5) E. NOTA

15. For positive real numbers *A* and *M*, which of the following polygons is formed by consecutively connecting the points (*M*, 0), (*A*, 0), (0, *M*), and (0, *A*)?

A. parallelogram B. rhombus
C. cyclic quadrilateral D. rectangle E. NOTA

16. Which of the following describes the graph formed by $x^2y^2 - 9y^2 - 6x^2y + 54y = 0$?

A. A pair of lines and a circle
B. A pair of lines and a parabola
C. A pair of lines and a hyperbola
D. A set of four lines
E. NOTA

17. When an aircraft travels faster than the speed of sound, a cone-like wave is created. If the Aircraft does not change direction or altitude, which shape (or part of which shape) is created when the cone intersects flat ground?

A. line B. ellipse C. parabola D. hyperbola E. NOTA

18. A reflecting telescope has a mirror in the form of a paraboloid. The diameter of the mirror is 9 inches and the depth of the mirror is 2 inches. How far from the center of the mirror will the incoming light collect?

A. $\frac{81}{32}$ B. 8 C. $\frac{81}{8}$ D. 9 E. NOTA

19. A point moves so that the sum of its distances from two distinct points is always 10. Which of the following *could* be the eccentricity of the path of movement?

A. 0 B. 1 C. $\frac{2}{3}$ D. $\frac{3}{2}$ E. NOTA

20. A boat is traveling due east on a course that is 200 miles from—and parallel to—a straight shoreline. The boat sends an SOS signal from point (*x*, *y*) that is received at two lighthouses, *P* and *Q*, situated along the shoreline and 100 miles apart. It was determined that the boat is 80 miles closer to *Q* than to *P*. If lighthouse *P* is located at $(-50, 0)$ and lighthouse *Q* is located at $(50, 0)$, what is the value of *x*?

A. $\frac{40\sqrt{391}}{3}$ B. $\frac{20\sqrt{409}}{3}$ C. $\frac{40\sqrt{409}}{3}$ D. $\frac{20\sqrt{391}}{3}$ E. NOTA

21. Find the length of the tangent from (7, 8) to $x^2 + y^2 = 9$.

A. $2\sqrt{26}$ B. 10 C. $4\sqrt{6}$ D. $6\sqrt{3}$ E. NOTA

22. Find the area enclosed by the graphs of $y = \sqrt{-x^2 - 4x}$ and $y = -\sqrt{64 - 16(x+2)^2}$. All answers are in square units.

A. 8π B. 9.5π C. 10π D. 12π E. NOTA

23. The graphs of $x^2 + y^2 = 5$ and $y^2 = 4x$ intersect at (x_1, y_1) and (x_2, y_2). Find the product $x_1x_2y_1y_2$.

A. 2 B. $2\sqrt{2}$ C. 3 D. 4 E. NOTA

24. If the point (*a*, *b*) is the closest point on the curve $y = \sqrt{x}$ to the point (1, 0), find *ab*.

A. $\frac{\sqrt{2}}{4}$ B. $\frac{\sqrt{2}}{3}$ C. $\frac{\sqrt{2}}{2}$ D. $\sqrt{2}$ E. NOTA

25. The graph of $4x^2 - 3xy + 9y^2 + 17x - 12y + 19 = 0$ is fully contained in which quadrant(s)?

A. II B. IV C. II, III D. II, III, IV E. NOTA

26. Given that $2x^2 - xy + y^2 - 7y + 10 = 0$ generates a non-degenerate conic, what is the graph?

A. hyperbola B. circle C. parabola D. ellipse E. NOTA

27. The range of $y = -\dfrac{4}{x^2 - x - 2}$ can be written in the form $(-\infty, A) \cup \left[\dfrac{B}{C}, \infty\right)$, where B and C are relatively prime integers. Find the value of $A + B - C$.

A. 7 B. 8 C. 12 D. 3 E. NOTA

28. A locus of points moves so that the product of the slopes of the lines connecting them with (–2, 1) and (3, 2) is 4. Find the distance between the vertices of this graph.

A. $3\sqrt{11}$ B. $\dfrac{3\sqrt{11}}{2}$ C. $\dfrac{4\sqrt{11}}{33}$ D. $\dfrac{8\sqrt{11}}{33}$ E. NOTA

29. Find the sum of the coordinates of the point $\dfrac{5}{8}$ of the distance from (7, 1) to (–5, –9).

A. $-\dfrac{21}{4}$ B. $-\dfrac{23}{4}$ C. $-\dfrac{21}{2}$ D. $-\dfrac{23}{2}$ E. NOTA

30. Find the oblique asymptote of $y = \dfrac{x^3 - 3x^2 + 2x}{4x^2 - 8x - 12}$.

A. $y = \dfrac{1}{4}x$ B. $y = \dfrac{1}{4}x - \dfrac{1}{4}$ C. $y = -\dfrac{1}{4}x$ D. $y = \dfrac{1}{4}x + \dfrac{3}{4}$ E. NOTA