Theta

Individual

Test #511

Directions:

1. Fill out the top left section of the scantron. Do not abbreviate your school name.

2. In the Student ID Number grid, write your 9-digit ID# and bubble.

3. In the Test Code grid, write the 3-digit test# on this test cover and bubble.

4. Scoring for this test is 5 times the number correct plus the number omitted.

5. TURN OFF ALL CELL PHONES.

6. No calculators may be used on this test.

7. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

8. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

9. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

10. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

11. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

12. Unless a question asks for an approximation or a rounded answer, give the exact answer.

1. A varies directly with B^3 and inversely with C^2 and D. If $A = 2$ when $B = 2$, $C = 4$, and $D = 10$, what is A when $B = 3$, $C = 3$, and $D = 6$?

   A. 10   B. 20   C. 30   D. 40   E. NOTA

2. When $x^5 + x^3 + x + 1$ is divided by $x^2 + 2$, the result can be expressed as

   $Ax^4 + Bx^3 + Cx^2 + Dx + E + \dfrac{Fx + G}{x^2 + 2}$, where all letters are integers. Find $ACE + BDFG$.

   A. -3 B. 0   C. 1   D. 3   E. NOTA

3. For the system $\begin{cases} 2x - y + 5z = 8 \\ 3x + 9y - 2z = 4 \\ -7x + y + z = 5 \end{cases}$ being solved by Cramer's rule, find the tens digit of D_y.

   A. 0   B. 3   C. 4   D. 5   E. NOTA

4. In rectangle $MRLU$ point F lies on side $\overline{LU}$ such that the area of triangle LRF minus the area of triangle MUF equals the area of triangle MFR minus the area of LRF. If the area of triangle MUF equals 8, what is the area of triangle MRF?

   A. $40/3$ B. 16   C. 20   D. 24   E. NOTA

5. Find the area of the triangle enclosed by the horizontal asymptote of: $y = \dfrac{3}{x^2(x+3)}$. The slant

   asymptote of: $y = \dfrac{x^3}{9 - x^2}$ and the vertical asymptote of: $y = \dfrac{5}{(x-3)^2}$

   A. 4.5   B. 6   C. 9   D. 18   E. NOTA

6. A circle with center at the origin passes through $(-4,3)$. The equation of the tangent line at this point can be expressed in $Ax + By = C$ form, where A, B, and C are relatively prime integers and $A > 0$. Compute $A + B + C$.
   A. -24 B. -22 C. 0 D. 8 E. NOTA

7. A pyramid with height 15 is separated into two pieces by a plane parallel to the base and 6 units above it. What is the positive difference in the volumes of these two pieces if the volume of the original pyramid is 250 cubic units?
   A. 50   B. 142   C. 160   D. 196   E. NOTA

8. Find the distance from the directrix to the focus for the parabola $y^2 - 4y + 8x - 28 = 0$.
   A. 0.5   B. 3   C. 4   D. 7   E. NOTA

9. The sequence $-3, -2, 3, 12, 25, \ldots$ can be represented by the rule $a_n = An^2 + Bn + C$, where $a_1 = -3$. Find the value of $A - B - C$.
   A. -6 B. 2   C. 3   D. 7   E. NOTA

10. You are given 2 three-digit positive integers L and U, not necessarily distinct. One is the number formed by reversing the digits of the other. How many distinct values are possible for $|L - U|$?
    A. 5   B. 8   C. 9   D. 10   E. NOTA

11. In triangle ZLU, angles Z and L measure 60 and 45 degrees, respectively. The angle bisector of Z intersects $\overline{LU}$ at W with $ZW = 24$, and the altitude drawn from U intersects $\overline{ZL}$ at F. If $UF + FL + ZU + ZF$ can be represented as $a + b\sqrt{3}$ for integers a and b. Compute $a - b$.
    A. -12 B. 0   C. 12   D. 24   E. NOTA

12. Find the shortest distance between the point (6,0) and the curve $\sqrt{x-2}$.

    A. 1.75   B. $\frac{\sqrt{13}}{2}$ C. $\frac{\sqrt{15}}{2}$ D. 2   E. NOTA

13. Solve the following inequality: $\left|2x^2+5x-5\right|<2$

    A. $\left(-\frac{7}{2},1\right)$ B. $\left(-\frac{7}{2},-3\right)\cup\left(\frac{1}{2},1\right)$

    C. $\left(-3,\frac{1}{2}\right)$ D. $(-\infty,-3)\cup\left(\frac{1}{2},\infty\right)$ E. NOTA

14. How many negative real roots does $3x^5+2x^4-7x^2+4x+3=0$ have?

    A. 0   B. 1   C. 2   D. 3   E. NOTA

15. Find the sum of the integers from -10 to 10 inclusive that are in the range of
    $f(x)=1-2\sqrt{8+2x-x^2}$?

    A. -14 B. -9 C. 3   D. 7   E. NOTA

16. Given: $|x|<5$. Find $\sqrt{(x-6)^2}+\sqrt{(x+9)^2}$.

    A. $2x+3$ B. $-2x+3$ C. 3   D. 15   E. NOTA

17. If L is 40% greater than U, and U is 40% less than 600, find $L-U$.

    A. 144   B. 240   C. 256   D. 504   E. NOTA

18. Which of the following equations have the same graph?

    I. $y=x-4$ II. $y=\frac{x^2-16}{x+4}$ III. $(x+4)y=x^2-16$

    A. I and III   B. II and III   C. I, II, and III   D. None   E. NOTA

19. Let the incircle of triangle ZLU be tangent to sides $\overline{LU}$, $\overline{ZU}$, and $\overline{ZL}$ at points W, J, F, respectively. Given that $\angle Z = 32°$, find the degree measure of angle JWF.

   A. 32   B. 58   C. 64   D. 74   E. NOTA

20. Find the real number k such that the roots of $y^3 - 6y^2 + 21y - k = 0$ (not necessarily all real) can be arranged to form an arithmetic sequence. What is the sum of the digits of k?

   A. 4   B. 6   C. 8   D. 12   E. NOTA

21. Given: $\log_y 9x = 3\log_x y = -3$, Find $\frac{x}{y}$.

   A. $\frac{1}{3}$ B. 1   C. 8   D. 9   E. NOTA

22. How many real solutions exist for the equation: $\sqrt{2x+9} + \sqrt{x+1} = \sqrt{x+4}$

   A. 0   B. 1   C. 2   D. 3   E. NOTA

23. Find all ordered pair solutions for the following system of equations: $L + U = 6$ and $U^{L^2 - 7L + 12} = 1$. What is the sum of the values of L?

   A. 7   B. 12   C. 14   D. 19   E. NOTA

24. J-Wigs wants to split his 10 Mu students into a group of 4 and a group of 6. If he wishes to keep two of them, Jason and Lu, in separate groups, how many ways can he split them into groups?

   A. 112   B. 154   C. 168   D. 182   E. NOTA

25. Find the ratio of the shorter leg to the longer leg in a right triangle where an altitude drawn to the hypotenuse divides the hypotenuse in the ratio 1:2.
    A. $1:2$ B. $1:3$ C. $2\sqrt{3}:3$ D. $\sqrt{2}:2$ E. NOTA

26. In the binomial expansion of $(J-W)^n$ the 3rd term and the 5th term have the same coefficient. What is the coefficient of the 4th term?
    A. -35 B. -20 C. 20 D. 35 E. NOTA

27. In triangle PRQ, point S lies on side PR and point T lies on side RQ such that triangle RST is similar to triangle RQP. If $RT = 12$, $TQ = 3$, and RP is 8 more than RS, find RS.
    A. 6   B. 8   C. 10   D. 18   E. NOTA

28. Compute the number of positive divisors of the number $11^5 \cdot 20^4$ that are perfect cubes.
    A. 11   B. 12   C. 15   D. 16   E. NOTA

29. How many pairs of non-negative integer numbers (x, y) satisfy $101x + 5y = 2020$?
    A. 4   B. 5   C. 6   D. 10   E. NOTA

30. Find the sum of all rational solution to the equation $4^{2x+1} - 3 \cdot 4^{x+1} = -5$.
    A. -0.5 B. 0   C. 2   D. 2.5   E. NOTA