Mu

Integration

Test #432

Directions:

1. Fill out the top section of the Round 2 Google Form answer sheet and select **Mu-Integration** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select “E) NOTA” and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like “which of the following could be” or “what is one solution of”, an answer choice providing one of the possibilities is considered to be correct. Do not select “E) NOTA” in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select “E) NOTA” in that instance.

10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

If you believe one of the answer choices presented is not the correct answer please select "E" (None of the Above). Good luck and have fun 😊!

1. Compute:

$$\int_{2\cdot 0\cdot 2\cdot 2}^{2+0+2+2} (2\cdot x^3 + 0\cdot x^2 + 2\cdot x + 2)dx$$

A. 676 B. 686 C. 696 D. 706 E. NOTA

2. Compute:

$$\int_{0}^{2022\pi} \sin(20x)\cos(22x)\,dx$$

A. -10/21 B. -5/21 C. 5/21 D. 10/21 E. NOTA

3. Let a and b be real numbers such that $b \geq a$. If:

$$I(a,b) = \int_a^b (-15 - x^2 + 8x)dx$$

then let M denote the maximum value of $I(a,b)$. Evaluate $\left\lfloor \frac{1}{M} \right\rfloor$ where $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

A. 0 B. 1 C. 2 D. 3 E. NOTA

4. Compute:

$$\int_{2021}^{2022+e} \frac{dx}{x-2022}$$

A. 0 B. 1 C. e D. ln(2022) E. NOTA

5. Compute:

$$\int_0^{2022} \sqrt{x}\sqrt{2022-x}\,dx$$

A. $1011^2\pi$ B. $\frac{1011^2\pi}{2}$ C. 1011π D. $\frac{1011\pi}{2}$ E. NOTA

6. Let:

$$I = \int_1^2 \frac{dx}{x^5 + x}$$

Then I can be written in the form $\frac{A\ln\left(\frac{B}{C}\right)}{D}$ where A, B, C, D are positive integers, A and D are relatively prime and B and C are relatively prime. Then compute $A + B + C + D$.

A. 29 B. 30 C. 53 D. 54 E. NOTA

7. Let:

$$I = \int_0^1 \frac{x}{\sqrt{x^2 + 1}} dx.$$

Which of the following is the value of I, rounded to the nearest tenth?

A. 0.2 B. 0.4 C. 0.8 D. 1.1 E. NOTA

8. Let:

$$I = \int_{-1}^{0} (x^4 + 4x^3 + 6x^2 + 4x + 2) dx$$

Given that I can be written in the form $\frac{m}{n}$ where m and n are relatively prime positive integers, then compute $m + n$.

A. 8 B. 9 C. 10 D. 11 E. NOTA

9. Compute:

$$\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{4 - \cos^2(x)} dx$$

A. $\frac{\pi\sqrt{3}}{18}$ B. $\frac{\pi\sqrt{3}}{9}$ C. $\frac{\pi}{8}$ D. $\frac{\pi}{4}$ E. NOTA

10. Let:

$$I = \int_0^1 \left(\sqrt{x} + 2\sqrt[3]{x} + \sqrt[6]{x}\right) dx$$

Given that I can be written in the form $\frac{m}{n}$ where m and n are relatively prime positive integers, then compute m + n.

A. 85 B. 106 C. 137 D. 169 E. NOTA

For Questions 11- 14:

Denote R to be the region bound above by $f(x) = x^2 + 4x + 5$ and bound below by the x axis on the interval $[-1,3]$.

11. Let V_1 denote the volume of the resultant solid (ignoring units) when R is revolved about the y axis. Given that $V_1 = \frac{m\pi}{n}$, where m and n are relatively prime positive integers, then compute the remainder when $m + n$ is divided by 1000.

A. 235 B. 317 C. 319 D. 467 E. NOTA

12. Let A denote the area of R. Given that $A = \frac{p}{q}$ where p and q are relatively prime positive integers, then compute $p + q$.

A. 130 B. 133 C. 136 D. 139 E. NOTA

13. Compute the value of c that satisfies the mean value theorem for integrals for $f'(x)$ on $[-1,3]$.

A. -1 B. 0 C. 1 D. 2 E. NOTA

14. Let V_2 denote the volume of the resultant solid (ignoring units) when R is revolved about the x axis. Given that $V_2 = \frac{m\pi}{n}$, where m and n are relatively prime positive integers, then compute the remainder when $m + n$ is divided by 1000.

A. 677 B. 687 C. 697 D. 707 E. NOTA

15. Given that: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, compute:

$$\int_{-\infty}^{\infty} \frac{2022^{-x^2+x}}{1+2022^x} dx$$

A. $\frac{\sqrt{\pi}}{\sqrt{\ln{(2022)}}}$ B. $\frac{\sqrt{\pi}}{\sqrt{2\ln{(2022)}}}$ C. $\frac{\sqrt{\pi}}{2\sqrt{\ln{(2022)}}}$ D. $\frac{\sqrt{\pi}}{2\sqrt{2\ln{(2022)}}}$ E. NOTA

16. Let:

$$I = \int_{-3}^{3} |x^3 - x^2 - 2x|.$$

Given that I can be written in the form $\frac{p}{q}$, where p and q are relatively prime positive integers then compute $p + q$.

A. 69 B. 79 C. 89 D. 99 E. NOTA

17. Compute:

$$\int_{0}^{\infty} \frac{x^4 - x^2}{1 + x^8} dx$$

A. $-\frac{\pi}{8}$ B. $-\frac{\pi}{4}$ C. $\frac{\pi}{8}$ D. $\frac{\pi}{4}$ E. NOTA

18. Denote:

$$A = \int_{0}^{\frac{\pi}{2}} \sin^3(x)\, dx$$

Given that A can be written in the form $\frac{m}{n}$ where m and n are relatively prime positive integers then compute m + n.

A. 5 B. 8 C. 13 D. 21 E. NOTA

19. Let $f(x) = x^5 + x^3 + x$.
Denote:
$$A = \int_0^1 \left(\left(uf(u)\right)' - f(u)\right) du$$
Given that A can be written in the form $\frac{p}{q}$ where p and q are relatively prime positive integers, then compute $p + q$.

A. 31 B. 37 C. 52 D. 73 E. NOTA

20. Denote:
$$I = \int_1^2 \left(|x|^3 (x^2)^{\frac{3}{2}}\right) dx$$
What integer is closest to the value of I?

A. 16 B. 17 C. 18 D. 19 E. NOTA

21. Denote $\left(f_1(x), f_2(x), \dots f_n(x)\right)$ to be the distinct solutions to the following equation , given that that the equation holds for all $x \in \mathbb{R}$.
$$x + \int_0^x f(t)dt = -\frac{f(x)}{2022}\ ,$$
compute:
$$\sum_{i=1}^{n} \lim_{x\to\infty} f_i(x)$$

A. -1/1011 B. 0 C. 2/1011 D. 4044 E. NOTA

22. Let:
$$I = \int_0^1 \frac{(-3x^2 - 4x + 11)}{(x^2 + 2x + 5)^2} dx$$
Given that I can be written in the form $\frac{m}{n}$ where m and n are relatively prime positive integers, then compute $m + n$.

A. 39 B. 49 C. 59 D. 69 E. NOTA

For Questions 23- 25:

It may be helpful to know that $\int_0^\infty \frac{\sin(x)}{x} dx = \frac{\pi}{2}$.

23. Evaluate:

$$\int_0^\infty \frac{\sin(4x)}{x} dx$$

A. $\frac{\pi}{32}$ B. $\frac{\pi}{8}$ C. $\frac{\pi}{2}$ D. 2π E. NOTA

24. Evaluate:

$$\int_0^\infty \sin^2(x)\, dx$$

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. π D. 2π E. NOTA

25. Evaluate:

$$\int_0^\infty \frac{\sin^3(x)}{x} dx$$

A. $\frac{\pi}{8}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{2}$ D. π E. NOTA

26. Evaluate:

$$\int_0^\infty e^{-x}\, dx$$

A. -1 B. 0 C. 1 D. ∞ E. NOTA

27. Evaluate:

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+(\tan x)^{2022}}$$

A. $\frac{\pi}{8088}$ B. $\frac{\pi}{4044}$ C. $\frac{\pi}{4}$ D. $\frac{\pi}{2}$ E. NOTA

For Question 28 – 29:

Denote $F(a,b,n) = \int_a^b \frac{dx}{1+x^n}$.

28. Given that $F(0,1,-1)$ can be written in the form:

$$A + B \cdot \ln(C),$$

where A, B, C are all integers, compute $A + B + C$.

A. 1 B. 2 C. 3 D. 4 E. NOTA

29. Compute $\lim\limits_{b\to\infty} F(0,b,2)$.

A. $\frac{\pi}{12}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{2}$ D. π E. NOTA

30. In what interval does:

$$\iint_R \left(x^{\ln(y)} - y^{\ln(x)}\right)dx$$

lie, where R is the triangle whose vertices are (3,4), (5, 1), and (8,3) ?

A. (0, 2] B. (2, 4] C. (4, 6] D. (6, 10] E. NOTA