#1 Alpha Ciphering
MAΘ National Convention 2021

Given the equation $\frac{x(x-1)-(k+1)}{(x-1)(k-1)}=\frac{x}{k}$, for what value of k are the solutions for x equal?

#2 Alpha Ciphering
MAΘ National Convention 2021

The roots of $64x^3 - 144x^2 + 92x - 15 = 0$ form an arithmetic sequence. What is the largest root?

#3 Alpha Ciphering
MAΘ National Convention 2021

Mr. Lu bought N Chinese romance novels for d dollars (d is a positive integer). He sold two of them to the Snowman at half their cost. The rest he sold at a profit of $8 per novel. If the overall profit was $72, then the least possible value of N is?

#4 Alpha Ciphering
MAΘ National Convention 2021

In triangle ZLU, the median from vertex Z is perpendicular to the median from vertex L. If the lengths of sides ZU and LU are 6 and 7 respectively, what is the length of side ZL?

#5 Alpha Ciphering
MAΘ National Convention 2021

How many 3-digit positive integers have three different digits in increasing order or in decreasing order?

#6 Alpha Ciphering
MAΘ National Convention 2021

Point X is 9 units from the center of a circle of diameter 30. How many different chords of the circle contain X and have integer lengths?

#7 Alpha Ciphering
MAΘ National Convention 2021

Compute $\sum_{k=1}^{2021}\left[\sin^k\left(\frac{k\pi}{2}\right)+\cos^k\left(\frac{k\pi}{2}\right)\right]$.

#8 Alpha Ciphering
MAΘ National Convention 2021

Given that this expression is true for all values of x where both sides are defined:

$$\tan\frac{1}{5}x - \tan x = \frac{\sin\frac{k}{2}x}{\left(\cos\frac{1}{5}x\right)(\cos x)}.$$

Solve for k.

#9 Alpha Ciphering
MAΘ National Convention 2021

Given: $2^{2023} = 2 + \sum_{n=0}^{2021} \log\left(x^{2^n}\right)$. What is x?

#10 Alpha Ciphering
MAΘ National Convention 2021

What is the area of the triangle formed by the end points of the Latus Rectum and the intersection point of the axis of symmetry and directrix for the parabola $y = \frac{-3}{2}x^2 + 3x - 4$.

#11 Alpha Ciphering
MAΘ National Convention 2021

A triangle has side lengths X, Y and Z such that, $Y + Z = 2X$ and $YZ = X^2$. Find the cosecant of the angle opposite the side of length X.

#12 Alpha Ciphering
MAΘ National Convention 2021

Given: z = a + bi, there is a real ordered pair (a,b) that is a solution to $|z+3| = 1 - iz$. What is $b - a$?