Theta Ciphering solutions Nationals 2021

0. 4
1. 584
2. 20
3. 3
4. 7
5. 12
6. $\frac{9}{4}$
7. $\frac{1296}{625}$
8. $\frac{-1}{2}-\frac{\sqrt{3}}{2}i$
9. $\frac{3}{14}$
10. $\frac{6}{7}$
11. 72
12. $\frac{8}{27}$

#0- $\frac{-4}{n}=\frac{3}{n-1}\rightarrow -4n+4=3n\rightarrow 7n=4\rightarrow n=\frac{4}{7}$ 7n=4

$-\left(13+14-15+2\sqrt{13\bullet 14}\right)\left(13+14-15-2\sqrt{13\bullet 14}\right)$

1. $-\left(12+2\sqrt{13\bullet 14}\right)\left(12-2\sqrt{13\bullet 14}\right)$

$4(13)(14)-144=8(91-18)=8\bullet 73=584$

2. ${}_{10}C_3=120$ 8 with a common difference of 1. 6 with a common difference of 2. 4 with a common difference of 3 and 2 with a common difference of 2. 8+6+4+2=20

$\frac{a\left(r^6-1\right)}{r-1}=91$

3. $\frac{a\left(r^2-1\right)}{r-1}=7$

$\frac{r^6-1}{r^2-1}=13\rightarrow \frac{\left(r^2-1\right)\left(r^4+r^2+1\right)}{r^2-1}=13\rightarrow \left(r^4+r^2+1\right)=13\rightarrow \left(r^2+4\right)\left(r^2-3\right)=0\rightarrow r^2=3$

$n,\frac{2n(n-1)}{2},\frac{n(n-1)(n-2)}{6}\rightarrow n+\frac{n(n-1)(n-2)}{6}=\frac{2n(n-1)}{2}$

4. $1+\frac{n^2-3n+2}{6}=n-1\rightarrow 6+n^2-3n+2=6n-6\rightarrow n^2-9n+14=0$

$(n-2)(n-7)=0\rightarrow n=7$

5. Draw a picture and set up similar triangles. Cal side of square x. You get ratio of triangles legs 1/6. So, area is 12:1

6. Draw a picture and bisect the 54-degree angle. This creates a parallelogram and breaks up side XY in the ratio 4 to 3. Then use angle bisector theorem. $\frac{3}{4}=\frac{x}{3}\rightarrow x=\frac{9}{4}$

7. $k^{\frac{-1}{4}}=x\rightarrow 5=17x-6x^2\rightarrow 6x^2-17x+5=0$

$k^{\frac{-1}{4}}=\frac{5}{6}\rightarrow k=\frac{1296}{625}$

8. $\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)^3=1$ so recycles every 3 times. Divide 3 into exponent and check the remainder, which is 2. $\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)^2=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

9. $\begin{vmatrix}\log_{128}\frac{1}{5} & \log_{1024}\frac{1}{49}\\ \log_{49}32 & \log_{125}64\end{vmatrix}=\frac{\log\frac{1}{5}}{\log 128}\bullet\frac{\log 64}{\log 125}-\frac{\log\frac{1}{49}}{\log 1024}\bullet\frac{\log 32}{\log 49}$

$\frac{-1}{3}\bullet\frac{6}{7}+\frac{1}{2}=\frac{-12+21}{42}=\frac{3}{14}$

10. Must use 3,5,7, and 9. A little trial and error and you get $\frac{6}{7}$

11.Subtractthe two equations to get $4y^2=36\rightarrow y=\pm 3$ plug into either equation to get:

$x^2-6x-55=0\rightarrow(x-11)(x+5)=0\rightarrow x=11,-5$

$x^2-6x-7=0\rightarrow(x-7)(x+1)=0\rightarrow x=7,-1$

The points are (3,11), (3,-5), (-3,7), and (-3-1)

Draw the picture and you see this is a Trapezoid with height 6 and bases of 16 and 8.

$\frac{1}{2}(6)(16+8)=72$

12.

C	A	T
1	1	1
1	-1/3	-1/3
0	1/3	0
2/3	1	2/3

2/3	1	2/3
2/3	-1/3	-2/9
0	1/3	0
4/9	1	4/9

You can see it is geometric and we do 1 more time we get: $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$