Attend the 35<sup>th</sup> Annual Convention in Honolulu, Hawaii



#### News from the President:

Welcome to another new and exciting year for Mu Alpha Theta. There are many wonderful things going on this year. Please read this newsletter completely to find out what activities are available to member schools.

The Mu Alpha Theta Executive Board welcomes newly elected Governing Council Members. Mattie White from Louisiana was reelected as Region II Governor. Tom Tosch from Washington joins us as Region I Governor. Mary Emma Bunch from Tennessee is our new President-Elect.

The 34<sup>th</sup> annual Mu Alpha Theta National Convention was held in Huntsville, Alabama July 27 – August 1. It was a wonderful convention with many exciting activities and competitions. The hosts were Mary Rhein from Lakota West HS in Ohio, Cindy Rodgers and Beth Andrada from Grissom HS in Huntsville, AL, and Allen MacKenzie from Virginia Tech. One new competition offered was the Mathematics Olympiad. Students competed in two specific topic tests for their division, the division individual test, and ciphering events. Twenty-five students with the highest cumulative scores on these 4 events competed in the Mathematics Olympiad. The Olympiad included 5 open response questions with a 2.5 hour time limit. The winner of this prestigious event was Ryan Williams from Miami Springs HS in Florida. Thanks again to our hosts for an outstanding convention.

At the national convention, several committee meetings were held to get input from sponsors who were present. Sessions were held on the National Convention, the Mu Alpha Theta Newsletter, the Future of Mu Alpha Theta, Membership, and Scholarships. Many ideas were discussed. If you are interested in giving input to any of these committees contact Kay Weiss, Executive Director of Mu Alpha Theta, at <u>matheta@ou.edu</u>.

For the second year, Mu Alpha Theta will be sponsoring a free mail in competition for member schools. This competition has been named the Log 1 Contest. Member schools can register and find more information for the Log 1 competition at <u>www.mualphatheta.org</u> by hitting the link at the bottom of the home page. If you have problems registering, contact the National Office at <u>matheta@ou.edu</u>.

Another free contest offered this year is the Rocket City Math League sponsored by Grissom High School in Huntsville, Alabama. There are two divisions: Algebra II/Geometry and Algebra I/Pre-Algebra. Go to <u>http://www.rocketcitymath.org/</u> to register.

Mu Alpha Theta has become associated with the International Science and Engineering Fair (ISEF). Certificates were awarded to competitors at Regional Fairs and a monetary award was given at the ISEF competition. Mu Alpha Theta is very excited to be involved in rewarding students doing high level mathematics projects.

Next year's national convention will be held at the Ala Moana Hotel in Honolulu, Hawaii and will be hosted by Calvin Fukahara and Kimlynne Slagel from the Kamehameha Schools. They have spent several years planning for this convention so it is sure to be a great one. You can download an initial convention packet of information at <u>www.mualphatheta.org</u> under National Convention.

As Mu Alpha Theta grows, the Governing Council is always looking for ways to serve all members. If you have any ideas or suggestions, please be sure to contact any Council member or Kay Weiss at the National Office.

Again, welcome to a new and exciting year for Mu Alpha Theta. Should you have any questions or concerns, you can contact me at <u>susiehiller@bellsouth.net</u>.

Susan Hiller, MA0 President Vero Beach High School.

#### Sponsors:

Please copy this newsletter to give to your members! Or, at least have a copy available for your students to read.

A copy will also be available online at our website.

# Included in this issue:

Students with some calculus background may be interested in Dr. Goodey's article about *Finding Roots* using Newton's Method.

The article leads to an interesting math website to visit.

### **National Office News from Kay:**

In an effort to provide chapters with as many options as possible, we will now be able to take credit cards for purchases. If you wish to use a credit card for a purchase of certificates or merchandise, call the office at 405-325-4489. We will be adding the option to use your credit cards for online purchases, as well.

We have spent the summer moving data to a new database and have worked on our online order system. You can now order certificates online more easily. Just type in names or cut and paste them from a Word document or a spreadsheet. Our program will put the names into a table for you. You can edit names and then submit. If you don't have time to enter all names at one sitting, you can now come back and enter more names at a later time by remembering your order number. We will still take names sent in or submitted by email or fax, but our online system is the most efficient way to get your certificates quickly.

Our Merchandise Order area will soon have a "Shopping Cart" added. We should have it back up online soon. A few price increases were necessary for the first time in years due to increases in costs. Medallions will now be selling for \$5 instead of \$4 each. Our button has gone to \$7 from \$6 and is now the same price as the pin. Shoulder patches will be \$2.50 each, the small seals will be 6 cents each and banners will be \$12. We will no longer be charging an extra shipping cost. The \$1.50 or \$2.50 was too confusing and our price increases should allow us to recoup our mailing costs. New prices are effective immediately and have been posted online.

If we do not have your email address or you do not have a password to enter the Online System, email Kay Weiss at <u>matheta@ou.edu</u>. We will be trying to send emails monthly with updated information of interest to you during the year, so please get your email address to us so we can include you in our mailings. Sponsors can update email addresses online. Press the Update Information button on the left after logging in. Also, make sure you are signed up for our listserve, go to

#### mualphatheta-subscribe@topica.com

Check out all the new features at our website. Under Chapter Resources, you will now find an updated sample constitution, an updated Initiation Ceremony, the Certificate Order form to download, pictures of our merchandise and spirit items, activities for your chapter meetings, and copies of past newsletters. We hope to have old issues of the Mathematical Log available, soon.

We have added a number of years of old exams and answers from past National Conventions and the State competitions in Florida under National Convention, Past Tests.

Watch the bottom of the homepage for updates on contests and information of interest.

The National Office is open Monday through Fridays from 9:30 am – 6:00 pm, Central time. We close during national holidays, as well as during:

Thanksgiving: Nov. 24 – Nov 29 Winter Break: Dec. 22 - Jan. 5 Spring Break: Mar. 13 – Mar. 15 Summer Break: July 20 – 30 National Convention: Aug. 1 – 6

My assistant Susmita Chakraborty will be available to answer your questions when I am out of the office attending the AMATYC convention in mid-November and the NCTM convention in early April. If ever we are not in the office when you call, please leave a message or email, and we will respond as soon as we are able.

#### **National Officers:**

President:	Susan Hiller Vero Beach HS, Vero Beach, FL
Presdient-Elect	susiehiller@bellsouth.net Mary Emma Bunch
	Knoxville, TN mmbear@comcast.net
Secretary-Treasurer:	Paul Goodey
	Norman, OK
Governor Region I:	Tom Tosch
	Mount Rainer High School, Des Moines, WA
Governor Region II:	tomtosch@wamath.net Mattie White
	Benjamin Franklin HS,
	mattiewmathII@aol.com
Governor Region III:	Susan Doker Lincoln High School
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Governor Region: IV:	Mary Rhein
	West Chester, OH
MAA Representative:	Julie Clark
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National Student Officers:	
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	cliu05@vahoo.com
Secretary/Treasurer:	Wilson Po
,,	Bellevue High School
	Bellevue, WA
	iampo1987@yahoo.com
Parliamentarian:	Yufei Pan
	Doordon Wigh Coheal
	Bearden High School

Each year, the National Office presents a number of monetary awards and prizes to outstanding members and chapters. These awards are listed online at our website at <u>www.mualphatheta.org</u> under Awards. Each Award is described and forms are available to submit nominations.

This year the Governing Council has established a new chapter award, called the Diane Rubin Award, to be presented annually. The award was named in honor of Diane Rubin who worked tirelessly at the National Office of Mu Alpha Theta for twenty years, before her retirement in December, 2003. The winning chapter will receive a prize of \$500 and a distinguished plaque. The award will be presented for the first time at the 2005 National Convention, but a representative of the chapter need not be present to win. However, to encourage the chapter to attend, two free registrations and up to \$500 towards travel expenses to the convention will be provided.

The award will be presented to a chapter of Mu Alpha Theta which has shown a history of outstanding service to its community for at least two years. To be eligible, the chapter must submit no more than a two page, double spaced description of their community service project(s) and two supporting letters from someone on the receiving end of the service project(s). Benefit from such a community project should be to a group outside of the chapter's school or its students. Benefit from such a community project should be to a group outside of the chapter's school or its students. Forms are provided online at <u>www.mualphatheta.org</u> under Awards. They must be submitted to the National Office, postmarked no later than February 1. The Governing Council will select the winning chapter who will be notified no later than March 1.

The Governing Council also awards the Kalin Award to a student at the National Convention that shows an unusual capability in mathematics and/or service to his/her chapter of Mu Alpha Theta. One winner and up to three finalists are chosen. The winner receives \$1000 and each finalist receives \$300. Applications are due at the National Office by June 15

The Andree Award is presented to a student planning on becoming a mathematics teacher. The deadline for nominations for this award has been moved up to February 1 beginning in 2005. The student receives a \$1000 cash prize and a free registration to the National Convention. Please remember deserving students that might qualify for these awards.

Students or other sponsors can nominate a deserving sponsor for the Huneke and/or Sister Scholastica awards. These are presented each year to outstanding sponsors attending the National Convention. Each award carries a \$500 cash prize. All awardees above receive a plaque, as well. Nomination forms are due by June 15. For more details and forms, please check the Awards menu at our website.

#### 2005 National Convention

The 35<sup>th</sup> Annual Mu Alpha Theta National Convention hosted by Calvin Fukuhara and Kimlynne Slagel from the Kamehameha Schools will be held August 1<sup>st</sup>-6<sup>th</sup>, 2005 in Honolulu, Hawaii. Housing will be at Ala Moana Hotel in Honolulu, Hawaii. **The registration fee will be \$570 per person, if registered by April 1.** A \$50 non-refundable deposit per person is due with the Early Bird Registration Form postmarked by that date. The additional \$520 per person must be sent with a postmark no later than May 15. **The registration fee will be \$595 for all attendees who did not register by the April 1 deadline.** Full payment for all registrations must be postmarked no later than May 15. The registration fee includes convention activities, housing, T-shirt, and most meals. Single rooms for sponsors are available at an additional cost of \$200.

All students participating in the National Convention must be registered with the National Office of Mu Alpha Theta as Full or Associate members of the organization. Associate members can be entered by sponsors online at Mu Alpha Theta's website, <u>www.mualphatheta.org.</u> Log in to the online system and press the Add Members button.

Check out the schedule for the 2005 National Convention online and download the Preliminary Information Packet from the National Convention page.

# SPONSORS

#### **DISTRINGUISHED TEACHERS AMONG US**

Each year the American Mathematics Competitions analyzes the results of the AMC12. They award the Edyth May Sliffe Distinguished Award for High School Mathematics Teaching based on the results. Out of twenty-seven teachers awarded this recognition in 2004, ten come from schools with a chapter of Mu Alpha Theta, six are present sponsors, one is the present Governor of Region III, and one is the past Governor of Region I. We congratulate sponsors: Barbara Currier of Greenhill School in Addison, TX; Sue Doker of Lincoln High School in Tallahassee, FL; Sam Koski of Miami Springs Senior High School in Miami Springs, FL; Josie Mallerv of Klein High School in Klein, TX; Renee Fish of Palm Harbor University High School, Palm Harbor, FL; and Tom Norris of Thomas Jefferson High School in Auburn, WA.

#### SURVEY INFORMATION REQUESTED:

The National Office is trying to find out how many of you were Mu Alpha Theta members when you were a student. Also, we would like to know how many are currently members of NCTM. A quick email can be sent to <u>matheta@ou.edu</u>.

Thanks for your response!

We are trying to gather activities for monthly meetings. Please share some of your best with us! If you have interesting fund raising activities, please share those with us also. Email suggestions to matheta@ou.edu. We've been working on our website at <u>www.mualphatheta.org</u>! See Chapter Resources for new features and information about our Educational Foundation. There are copies of old exams under National Convention, Past Tests.

See information about our Online System and its features. An online Roster to find other Active Chapters in your city, state, or region is now online.

#### MAPLE SOFTWARE STILL AVAILABLE FREE TO SPONSORS AND AT REDUCED PRICES FOR STUDENTS!

You can still order your free Maple computer algebra system, CAS, online at our website. After the homepage going to at www.mualphatheta.org, press the button in the lower left corner marked "Order Certificates and Merchandise Online", enter your school ID number and password, press the View/Update Information button and get your password next to your name. Back up to the homepage and press the Order Maple Products button." Maple will send a resource CD with high school-specific applications and tutorials, a copy of "Integrating Maple into the Math Curriculum", a free home use license number of Maple 9, and more. Remind your students that they can buy the program at greatly reduced prices!

Do you have an **outstanding member**? Nominate them for a Mu Alpha Theta Award. Download forms online at <u>http://www.mualphatheta.org</u> under the Andree or Kalin Awards. Be sure to get them in before the spring deadlines! See page 3 for details.

#### **Convention Hosts and Test Coordinators Needed**

Think about hosting a National Convention! Hosts are needed for 2008 and beyond. Mu Alpha Theta will help you organize and pays a stipend. At each convention, we also need someone to supervise test writing and proofing. A stipend also comes with this position. For information about these important jobs, email Susan Hiller at Susie.hiller@bellsouth.net.

#### Mu Alpha Theta wants to recognize the following retiring sponsors with our thanks for a job well done:

Illinois Mathematics and Science Academy, Aurora, IL: Ron Vavrinek retired 1/04

Pike High School, Indianapolis, IN: Eddie Lenderman, 6/04

North Baltimore High School, North Baltimore, OH: Barbara Chrisman, 6/04 (1991-2004)Central High School: Betty Petrow, 6/03

Marion High School, Marion, AR: Susan J. Creekmore 6/04

Strongsville High School, Strongville, OH: Wade Zwingler 6/04

Easley High School: Patricia Y. Chang 6/04

Marion High School, Marion, IN: Nancy Balle 6/04

The Academy of Saint Joseph, Brentwood, NY. Jane Cahill 6/04

If you know of other retiring sponsors, please contact the National Office.

#### CERTIFICATE OF APPRECIATION NOW AVAILABLE FROM THE NATIONAL OFFICE

Mu Alpha Theta will now offer a Certificate of Appreciation that can be awarded to sponsors, speakers or anyone you wish to recognize for their Outstanding Contributions to Mathematics Education. These will be available for \$3 and have a space for the person's name, the date and the Chapter Sponsor's signature of the chapter presenting the certificate. The certificate is similar to the one awarded to our student members but reads:

"Mu Alpha Theta Certificate of Appreciation from the National High School and Two-Year College Mathematics Honor Society. The National Office of Mu Alpha Theta Recognizes

For Outstanding Contributions to Mathematics Education."

# CONTESTS FOR MU ALPHA THETA MEMBERS

#### **NATIONAL LOG 1 CONTEST**

Last year's free Write-In Contest run by National Assessment and Testing drew over 2000 contestants and proved to be so popular that Mu Alpha Theta is again running a three test contest that is absolutely free to active chapters. (Log 1 cost – get it?)

The 2004-2005 Mu Alpha Theta Contest will consist of three rounds: an "Individual Test", a round of three "Topic Tests", and a "Ciphering Test". The first test will be a 15-problem, 30-minute, individual test of general mathematics knowledge. Problems, to be solved without a calculator, will range from easy to difficult, to provide confidence and challenges to all students. The second round will consist of three Topic Tests covering the topics of Probability, Geometry, or Equations and Inequalities. Students select one of these topic tests to take. The third round of the contest will be a Ciphering Test consisting of ten three-problem rounds of general open-answer problems to be solved without a calculator. Each Ciphering round will last three minutes. This contest is an excellent opportunity for your school to participate in your national organization and experience the breadth of topics being explored by schools across the country.

Go online to http://www.mualphatheta.org/Contests/MAO\_Contests\_Write\_In.htm and register your students now! There is a link to the Online Registration site at the bottom of the page. You can just go straight to the log in page to register students by pressing the "Order Certificates and Merchandise Online" button. Sponsor's can select students who wish to compete for prizes. Full and Associate members may sign up to compete. Sponsors will need the chapter ID and password to enter the Registration area. Associate members may be entered by pressing the "Add Members" button in the Online System. If your chapter has no members registered with the National Office, please contact Kay Weiss at <u>matheta@ou.edu</u> or (405) 325-4489. Sponsors will need to select the division of each of their members: Mu, Alpha, or Theta, depending on the math classes in which they are enrolled. Sponsors may update or add participants and student's division until March 1, 2005. Students need not take all three tests but will have a chance at scoring more points, if they do.

The three rounds of this contest will be administered approximately two months apart on November 18th, January 13, and on March 3<sup>rd</sup>. Testing materials, including detailed solutions, will be sent by postal or electronic mail, but schools whose registrations are received within two weeks of a test date will receive that round's testing materials via e-mail only. Should your school have difficulty with a particular testing date, please contact Tom Clymer at <u>clymer@natassessment.com</u> On a test date, you may administer the test once at any time that is convenient. Please ask your students not to share questions or answers for a week following the test date. After a week from the test date, you may make as much use of the test at your school as you like, including using it to supplement your regular curriculum. Each school must return the scored answer forms of all participating students postmarked by midnight of the Monday following each test. Sponsors will be able to enter scores online for participants. Standings will be posted for top scoring schools, students and perfect scorers.

Your school may post and return results for as few or as many of the rounds as you like. You are welcome to register for the contest simply to receive the free materials.

Plaques and certificates will be awarded to the top individuals in three divisions: Mu, Alpha, and Theta, and in each of the four Mu Alpha Theta regions. Top schools in each region will receive awards based on the sum of their two highest student scores in each of the three divisions. Furthermore, Mu Alpha Theta will award a free mousepad to all new and reactivating schools that compete. All participating schools will be entered in a random drawing for 20 blue Mu Alpha Theta t-shirts and 50 TI graphlinks. Other prizes may be added at a later time.

#### **Rocket City Math League**

This is a free competition for talented and motivated math students in Pre-Algebra, Algebra I, Geometry, and Algebra II. There will be three rounds during the year in December, January, and March and awards for the top students at each level will be sent out in April. You may enter as many students as you choose. The top five scores in each division will determine your team's score. This contest is run by the math club at Grissom High School in Huntsville, AL. See information about the contest online at <u>www.rocketcitymath.org</u>

#### **ARML** Power Contest

For the first time this year, the American Regions Math League Power Contest will be offered at a reduced cost of \$30 per school for up to 50 Mu Alpha Theta Chapters. This year's contests will run the weeks of November 4 - 13, 2004 and February 10 - 19, 2005. Each year the contest consists of two problem sets. The mathematics level of the contest problems has been geared so that students in an honors class, math club, or on a math team can have a unique problem solving and mathematical writing experience. There is no limit to the size of the team that can work on the problem sets, but the time for solving each set is limited to 45 minutes.

Coaches will receive the contest materials at least one week prior to the above dates and may schedule the contest anytime during the designated week and a half. After completing the contest, the student solutions are then mailed back and are graded using a forty point rubric. Trophies are awarded to the top three scoring teams at next year's ARML competition. Up to 50 Mu Alpha Theta chapters are eligible to participate for the reduced price of \$30. Those schools wishing to participate should email your request to Tom Kilkelly at tpkilkel@ties.k12.mn.us. He will confirm your registration before you send in money with the form found on our website under Contests, ARML. A sample Power Contest exam with answers is online, as well.

#### National Invitational

Lawton Chiles, Lincoln, and Rickards High Schools are inviting you to the first ever Mu Alpha Theta National Invitational. The event will take place on Friday & Saturday, November 12 & 13<sup>th</sup> at Lawton Chiles High School, Tallahassee, FL. Competitions will include Ciphering, Algebra I, Geometry, Algebra II, Precalculus, Calculus, Statistics, Sweepstakes and an Interschool Test. For further information, contact Darryl Hill at <u>darrylvhill@comcast.net</u> or Sue Doker at DokerS@mail.leon.k12.fl.us

**Students, Honor your sponsor:** By nominating her/him for a Mu Alpha Theta sponsor award. Download forms at <u>http://www.mualphatheta.org</u> under Huneke or Sister Scolastica Awards. Be sure to get them in before the deadlines listed!

**Or:** Make a donation to the Mu Alpha Theta Educational Foundation in honor of your sponsor. Donations support awards, grants, prizes and scholarships for Mu Alpha Theta students and help to defray the cost to participants at the yearly National Convention.

Students can also donate in honor of a graduating friend or a favorite math teacher. If Mu Alpha Theta has been a special part of your life, consider making a contribution to help other interested members participate in Mu Alpha Theta competitions or continue their education in Mathematics. See page 8 for further information about donating to the Foundation.

#### Maple Software available at Reduced Prices for Students!

Thanks to our Partnership with Maplesoft, you can still order your Student Edition of the Maple computer algebra system, CAS, online at our website at greatly reduced prices. After going to the homepage at <u>www.mualphatheta.org</u>, press the button in the lower left corner marked "Order Maple Products". At the bottom of the page is a link to the Maplesoft order page. You will need to get your student ID number from your sponsor to order. This offer is for Mu Alpha Theta members only.

#### **2004 Awards and Scholarships**

During 2004, the Mu Alpha Theta Educational Foundation awarded numerous prizes, grants and scholarships. This year's Andree Award, given to a graduating senior or former member who has completed no more than two years of college work and is interested in becoming а presented mathematics teacher, was to Christopher Terndrup of Vestavia Hills High School in Birmingham, AL. Christopher received a check for \$1000, a plague and a free registration to the Huntsville Convention.

Nickolas M. VanMeter from Archbishop Rummel High School in Metairie, LA was our 2004 Kalin Award winner. Finalists for the Kalin Award were Shaanan Shetty from Rickards High School in Tallahassee, FL and Jing-Jing Mao from Lakota West High School in West Chester, OH. Both students received a check for \$300 and a plaque.

The Huneke Distinguished Sponsor Award was presented to Carol Selph of Vero Beach High School in Vero Beach, FL. The Student Delegates presented Sister Scholastica Award to the Most Committed Sponsor to Mary Rhein from Lakota West High School in West Chester, OH.

Ten schools received Convention Grants to attend the Huntsville Convention this year. From Region I: Todd Beamer High School from Federal Way, WA and Highline High School from Burien, WA. From Region II: Stephen F. Austin High School. From Region III: Middleton High School from Tampa, FI, Cypress Bay High School from Westen, FL, Alabama School of Fine Arts from Birmingham, AL, and Felix Varela High School from Miami, FL. From Region IV:

Bearden High School from Knoxville, TN, The Peddie School from Hightstown, NJ, and St. Joseph High School from Huntington, WV. Each school received two free registrations to the Huntsville Convention and up to \$500 in reimbursed transportation costs.

The Grissom High School Math Club was awarded a \$2000 grant to run the Rocket City Math League again this year. You can read about this free competition for Mu Alpha Theta members online at: <u>www.rocketcitymath.org</u>.

Three Vector Scholarships were presented this year. **Alice Pang** used her \$3000 scholarship to attend Stanford University in California. Alice won first place in our Math On the Web competition in 2003. She was a student at Baton Rouge High School. **Mila Singh** and The first Mu Alpha Theta Math Olympiad contest at the Huntsville Convention required full written solutions to five math problems that were mostly proofs. Winning the top prize of \$1000 was **Ryan Williams of Miami Springs Sr. High School** with a perfect score on the exam. **Muzhou Wang from Lakota West High School in West Chester, OH** came in second winning the \$500 prize, **Jie Tang from Bellevue High School in Washington** came in third winning \$300 and **Nimish Ramanlal from Seminole High School in Sanford, FL** won the \$100 prize.

Another first this year was the Mu Alpha Theta Award given for the most challenging, thorough, and creative investigation of a problem involving modern mathematics at each of 600 Regional and International Science Fairs. Winners from these fairs met in Portland, Oregon in May, 2004 to compete in the Intel International Science and Engineering Fair. A \$1000 prize was presented to Andrew Lewis Matteson of Randall High School in Amarillo. Andrew's Project was TX. entitled. "Generalizations of Schur's Problem and the Search for S(5). You can see the winners of this year's Mu Alpha Theta Awards, see their project topics and read Andrew's abstract online at

www.mualphatheta.org/Science Fair/science fair winners.htm

#### The Mu Alpha Theta Educational Foundation

In 1996, Mu Alpha Theta established the Mu Alpha Theta Educational Foundation to support the educational activities of Mu Alpha Theta. Over the years, this 501 (c) (3) non profit organization has accepted both monetary and product donations to reduce the cost of the National Convention and to award as prizes, scholarships, and grants to deserving mathematics students, Mu Alpha Theta sponsors, and chapters.

At their 2004 summer meeting, the Governing Council of Mu Alpha Theta decided to launch a campaign to seek public contributions to the Educational Foundation. Donations are tax deductible and will be accepted in any amount. For a \$10 donation, we will send a card to the person you are honoring with your donation. Donations can also be made in memory of someone. In this case, please tell us where to send the card. For a \$50 to \$99 donation, we will send a Mu Alpha Theta pin. You may also request a card to be sent. For a \$100 donation, we will send a Mu Alpha Theta tote bag and a card. Contributions may be sent by check. Please include your name and address, to whom the donation response card and gift should go and their address, whether the donation is in their honor or their memory, and if you wish to waive receipt of a pin or tote bag. You may use a credit card to make your donation by calling the office at 405-325-4489.

## Geometric Aspects of Newton's Method for Finding Roots

One of the most familiar iterative techniques for finding roots of equations is that developed by Newton. If f(x) = 0 is the equation we wish to solve we make a guess at a solution,  $x_n$  say, and then apply Newton's method to find a (hopefully) better guess. If we denote this subsequent guess by  $x_{n+1}$  then we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Geometrically speaking, we draw the tangent to the graph of f at the point  $(x_n, f(x_n))$  and denote by  $x_{n+1}$  the point where this tangent intersects the x-axis. The idea is that, if  $x_n$ is close to a root, then  $x_{n+1}$  will be closer. This will then give us an iterative method of finding roots of equations to a given degree of accuracy.

It is interesting, though, to take an arbitrary point  $x_1$  and apply Newton's method to it. If there is any justice in the world, we will find a sequence of points approximating one of the roots of f(x) = 0. Actually there is no justice in the world, but we will frequently get good approximations. The game is this: given  $x_1$ , predict which root Newton's method will lead you to.

Let's look at the (boring) case where f is a quadratic with two distinct real roots  $r_1$ ,  $r_2$ . If we denote by m the value where f achieves its maximum or minimum, then m will be the midpoint of  $r_1$  and  $r_2$ . You can use calculus to prove this, but it's quicker if you don't know any calculus. We'll assume  $r_1 < m < r_2$ . The answer to our question is the following:

if  $x_1$  is chosen to be less than m then Newton's method will lead us to the root  $r_1$  and if  $x_1 > m$  we are led to  $r_2$ . If  $x_1 = m$  then Newton's method doesn't work – I told you there's no justice – however this is the only case of failure.

The situation becomes much more interesting if we turn our attention to cubic equations (with three distinct roots), for example  $f(x) = x^3 - x$ . We put  $a = 1/\sqrt{3} \approx 0.577$ and note that f has local extrema at  $\pm a$  (Using Calculus, how could you prove this?). We draw the tangent from the point (a, 0) to the graph and get the number  $b \approx 0.466$  (Can you verify this is the correct value of b? A graphing calculator may by helpful.)



In a similar fashion, we obtain the number  $c \approx 0.450$  by drawing the tangent from (0, b)

to the graph. (Try to verify the value of c.)



If  $x_1 < -a$  then Newton's method takes us to the root  $r_1 = -1$ . Similarly, if  $x_1 > a$ , we are led to  $r_3 = 1$ . Rather more intriguing, if  $-a < x_1 < -b$ , we are led to  $r_3 = 1$ . If you've understood where the previous calculation came from, you will see that for  $-b < x_1 < -c$  we are led to  $r_1 = -1$ . There are also values, for example  $x_1 = -0.446$  that lead us to the root  $r_2 = 0$ . In this context, it is interesting to find a value of x such that the tangent to the graph at  $(x, x^3 - x)$  intersects the x-axis again at the point -x. If nothing else is clear, you should be convinced that it is a dangerous game to predict, for a given  $x_1$ , where you will end up.

To appreciate the real beauty (eye of the beholder and all that) in this you have to turn to complex numbers. These are numbers that don't exist but are essential to life as we know it. It is an interesting fact that Newton's method works for polynomials in a complex variable, for example  $z^3 - z$  which

has real roots or  $z^3 - 1$  which has real and complex roots.

And now for something completely different, or ...? Imagine someone has asked you to paint the the whole plane using just three colours. But there's a catch, no point is allowed to be on a boundary of just two regions with different colours. To put it another way, for every point of the plane you draw a disc centred at that point. No matter what is the radius of the disc there must be either just one or all three colours in the disc (that is, never just two). Is it possible to do this? Well, yes, but it's not easy. What you do is apply Newton's process to the polynomial  $z^3 - 1$ . This polynomial has three roots, call them  $z_1, z_2, z_3$ . If a point of the plane is led by Newton's process to  $z_1$ , colour it red, if it's led to  $z_2$ , colour it blue and if it's led to  $z_3$  colour it green. You will then have the desired colouring.

To get an idea what is going on with  $x^3 - x$ try the above colouring technique with  $z^3 - z$ , you should see some quite amazing pictures.

If you find this interesting, talk to your doctor – it might not be too late to get help. Otherwise go to

#### http://www.aleph0.clarku.edu/

#### ~djoyce/newton

where you will learn much more than you never wanted to know about this.

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