Question 0

A: $2x + 5 = 11$, so $x = 3$. $\boxed{A = 3}$

B: $\cos\frac{\pi}{A} = \cos\frac{\pi}{3} = \frac{1}{2}$. $\boxed{B = \frac{1}{2}}$

C: $f'(x) = 8x + 3$. $f'\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right) + 3 = 7$. $\boxed{C = 7}$

Question 1

A: $(\log_2 16)(\log_{49} 7) + (\log_5 1331)(\log_{11} 625) = 4\left(\frac{1}{2}\right) + \left(\frac{\log 1331}{\log 5}\right)\left(\frac{\log 625}{\log 11}\right)$
$= 2 + \frac{3\log 11}{\log 5}\left(\frac{4\log 5}{\log 11}\right) = 2 + 3(4) = 14$. $\boxed{A = 14}$

B: $y = x^2 - 14x - 72 = (x - 18)(x + 4)$. Zeros are 18 and -4, so the positive difference of the zeros is $18 - (-4) = 22$. $\boxed{B = 22}$

C: $x(t) = \frac{1}{12}t^3 + \frac{1}{4}t^2 + \ln t$. $v(t) = x'(t) = \frac{1}{4}t^2 + \frac{1}{2}t + \frac{1}{t}$.
$a(t) = v'(t) = \frac{1}{2}t + \frac{1}{2} - 1/t^2$.
$a(22) = \frac{1}{2}(22) + \frac{1}{2} - \frac{1}{22^2} = 11 + \frac{1}{2} - \frac{1}{484} = 11 + \frac{242}{484} - \frac{1}{484} = 11\frac{241}{484}$.
$\boxed{C = 11\frac{241}{484} \ or \ \frac{5565}{484}}$

Question 2

A: $y^2 - 20x - 4y + 304 = 0 \rightarrow \; y^2 - 4y + 4 = 20x - 304 + 4.$
So, $(y-2)^2 = 20(x-15)$. The vertex is at $(15,2)$. The parabola opens to the right, and the distance from the vertex to the focus is $\frac{20}{4} = 5$, so the focus is at $(15+5\,,2) = (20,2)$. $2p + 5q = 2(20) + 5(2) = 40 + 10 = 50.$ $\boxed{A = 50}$

B: The center is $(1,50)$ and the major axis is $2a = 50$, so $a = 25$. The area of the ellipse is $ab\pi = 25b\pi = 100\pi$, so $b = 4$. The coordinates of the endpoints of the minor axis are $(1, 50 \pm 4)$, so $r = 50 - 4 = 46$ and $s = 50 + 4 = 54$. $\boxed{B = 54}$

C: The side of the square cross-section is $x + k$, so the area of the cross-section is $(x+k)^2 = x^2 + 2kx + k^2$. The volume of the solid is found by
$\int_0^k (x^2 + 2kx + k^2)dx = \left[\frac{x^3}{3} + kx^2 + k^2x\right]_0^k = \frac{k^3}{3} + k^3 + k^3 = 54.$
So, $\frac{7}{3}k^3 = 54$, and $k^3 = \frac{162}{7}$. $\boxed{C = \frac{162}{7}}$

Question 3

A: $12.3_6 = 1(6^1) + 2(6^0) + 3(6^{-1}) = 6 + 2 + \frac{1}{2} = \frac{17}{2}$. $\boxed{A = \frac{17}{2} \;\; or \;\; 8.5}$

B: If $X = \begin{bmatrix} 1 & 2 \\ 3 & k \end{bmatrix}$, then $X^{-1} = \frac{1}{k-6}\begin{bmatrix} k & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{k}{k-6} & -\frac{2}{k-6} \\ -\frac{3}{k-6} & \frac{1}{k-6} \end{bmatrix}$ and $X^T = \begin{bmatrix} 1 & 3 \\ 2 & k \end{bmatrix}$. So, the row-2 column-2 entry of $X^{-1}X^T$ is $-\frac{3}{k-6}(3) + \frac{1}{k-6}(k) = \frac{17}{2}$.
So, $\frac{k-9}{k-6} = \frac{17}{2} \rightarrow 2k - 18 = 17k - 102 \rightarrow 84 = 15k \rightarrow \;\; k = \frac{84}{15} = \frac{28}{5}$. $\boxed{B = \frac{28}{5}}$

C: Integrate to get $y = \frac{x^3}{3} - \frac{x^5}{5} + K$. Using the initial condition, $\frac{28}{5} = \frac{8}{3} - \frac{32}{5} + K$.
So, $K = \frac{28}{5} - \frac{8}{3} + \frac{32}{5} = \frac{84-40+96}{15} = \frac{140}{15} = \frac{28}{3}$. When $x = 1$,
$y = \frac{1}{3} - \frac{1}{5} + \frac{28}{3} = \frac{5-3+1}{15} = \frac{142}{15}$. $\boxed{C = \frac{142}{15}}$

Question 4

A: $\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)g'(x)$, so at $x = 3$ this is
$f'\left(g(3)\right)g'(3) = f'(2)g'(3) = 8(6) = 48$.
$\frac{d^2}{dx^2}\left(x^2\, g(x)\right) = \frac{d}{dx}\left(x^2\, g'(x) + 2x\, g(x)\right) = x^2 g''(x) + 2xg'(x) + 2xg'(x) + 2g(x)$.
At $x = 3$ this is $9(4) + 2(3)(6) + 2(3)(6) + 2(2) = 36 + 36 + 36 + 4 = 112$.
The sum of these values is $112 + 48 = 160$. $\boxed{A = 160}$

B: $|\lfloor 160 \rfloor| = 160 = 32(5) = 2^5(5)$. The total number of positive integer factors is $(5 + 1)(1 + 1) = 6(2) = 12$. $\boxed{B = 12}$

C: $1 + i\sqrt{3} = 2cis\left(\frac{\pi}{3}\right)$, so this is $2^{12}cis\left(\frac{12\pi}{3}\right) = 2^{12} = 4096$. $\lfloor |4096| \rfloor = 4096$.
The sum of the digits is $4 + 0 + 9 + 6 = 19$. $\boxed{C = 19}$

Question 5

A: $f(1) = 1 + 1 = 2$. $f'(x) = 2x + \frac{1}{3}x^{-\frac{2}{3}}$, so $f'(1) = 2 + \frac{1}{3} = \frac{7}{3}$. Equation of the tangent line is $y - 2 = \frac{7}{3}(x - 1) \rightarrow y = \frac{7}{3}x - \frac{1}{3}$. If $66 = x^2 + \sqrt[3]{x}$, then by inspection $x = 8$. On the tangent line, when $x = 8$, $y = \frac{7}{3}(8) - \frac{1}{3} = \frac{55}{3}$. $\boxed{A = \frac{55}{3}}$

B: The polygon has $\left\lfloor \frac{55}{3} \right\rfloor = 18$ sides, so there are 18 interior angles with measures of k, $k + 1, k + 2, \dots, k + 17$. So, $k + k + 1 + k + 2 + \cdots + k + 17 = 360$. This is $18k + \frac{18}{2}(0 + 17) = 360$, so $18k + 153 = 360$. $k = \frac{207}{18} = \frac{23}{2}$. $\boxed{B = \frac{23}{2}\ or\ 11.5}$

C: The x –intercepts will occur at multiples of $\frac{2\pi}{23}$, which for the interval given is at $x = 0, \frac{2\pi}{23}, \frac{4\pi}{23}, \dots, \frac{44}{23}, 2\pi$. There are 24 x –intercepts. $\boxed{C = 24}$

Question 6

A: This area is $\int_0^{1/2}\left(2-x-\frac{1}{\sqrt{1-x^2}}\right)dx=\left[2x-\frac{1}{2}x^2-\arcsin x\right]_0^{1/2}=1-\frac{1}{8}-\frac{\pi}{6}$ $=\frac{7}{8}-\frac{1}{6}\pi.$ $P+Q=\frac{7}{8}-\frac{1}{6}=\frac{21-4}{24}=\frac{17}{24}.$ $\boxed{A=\frac{17}{24}}$

B: This can be written as $\frac{x}{1+A}=A$, so $\frac{x}{1+\frac{17}{24}}=\frac{17}{24}.$ $\frac{24x}{41}=\frac{17}{24}$, so $x=\frac{697}{576}.$ The sum of the numerator and denominator is $697+576=1273.$ $\boxed{B=1273}$

C: $k=1+2+7+3=13.$ $\sin\left(\frac{13\pi}{3}\right)+\sin\left(\frac{1273}{3}\right)=\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\sqrt{3}.$ $\boxed{C=\sqrt{3}}$

Question 7

A: Separate into $\lim\limits_{x\to\infty}\frac{3x}{x}+\lim\limits_{x\to\infty}\frac{2}{x}+\lim\limits_{x\to\infty}\frac{2\sin x}{x}=3+0+0=3.$ $\boxed{A=3}$

B: Approximating $\int_1^3(x\ln x)dx$ using a right Riemann sum with two equal subintervals gives $1(2\ln 2+3\ln 3)=\ln 4+\ln 27=\ln 108.$ $\boxed{B=108}$

C: $s=1+0+8=9.$ $108=\frac{9}{1-r}$, so $108-108r=9$, so $r=\frac{99}{108}=\frac{11}{12}.$ $\boxed{C=\frac{11}{12}}$

Question 8

A: $\mathbf{v}\cdot\mathbf{w} = 2(7) + (-5)(-6) + 4(-9) = 14 + 30 - 36 = 8.$

$$\mathbf{v}\times\mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -5 & 4 \\ 7 & -6 & -9 \end{vmatrix}$$

$= \mathbf{i}\big((-5)(-9) - (-6)(4)\big) - \mathbf{j}\big(2(-9) - 7(4)\big) + \mathbf{k}\big(2(-6) - 7(-5)\big)$

$= 69\mathbf{i} + 46\mathbf{j} + 23\mathbf{k}$. Final answer is $8 + 69 - 46 - 23 = 8$. $\boxed{A = 8}$

B: $f(x) = 8\sqrt{x}$. Average rate of change on $[0, 8]$ is $\frac{8\sqrt{8}-0}{8-0} = \sqrt{8}$.

$f'(x) = \frac{8}{2\sqrt{x}} = \frac{4}{\sqrt{x}}$. So, $\frac{4}{\sqrt{c}} = \sqrt{8} \rightarrow\ 4 = \sqrt{8c} \rightarrow 16 = 8c \rightarrow\ c = 2$. $\boxed{B = 2}$

C: Looking for the first 3 terms of $(2x-1)^6$, which are

$\binom{6}{0}(2x)^6 + \binom{6}{1}(2x)^5(-1) + \binom{6}{2}(2x)^4(-1)^2 = 64x^6 - 6(32)x^5 + 15(16)x^4$.

This is $64x^6 - 192x^5 + 240x^4$. Sum of these coefficients is $64 - 192 + 240 = 112$.

$\boxed{C = 112}$

Question 9

A: The constant term of the expansion is $\binom{6}{2}(x^2)^2\left(\frac{2}{x}\right)^4 = 15(2^4) = 2^4(3)(5)$. The number of positive integer factors of this term is $(4+1)(1+1)(1+1) = 5(2)(2) = 20$. $\boxed{A = 20}$

B: The particle changes direction at $t = \frac{20}{10} = 2$ and $t = \frac{20}{5} = 4$. $v(t) = t^2 - 6t + 8$.

$\int_0^2 v(t)dt = \left[\frac{t^3}{3} - 3t^2 + 8t\right]_0^2 = \frac{8}{3} - 12 + 16 = \frac{20}{3}$, so the particle has travelled $\frac{20}{3}$ units from $t = 0$ to $t = 2$.

$\int_2^4 v(t)dt = \left[\frac{t^3}{3} - 3t^2 + 8t\right]_2^4 = \frac{64}{3} - 48 + 32 - \frac{20}{3} = -\frac{4}{3}$, so the particle has travelled another $\frac{4}{3}$ units from $t = 2$ to $t = 4$. This is a total of $\frac{20}{3} + \frac{4}{3} = \frac{24}{3} = 8$ units travelled in the first 4 seconds.

$\int_4^5 v(t)dt = \left[\frac{t^3}{3} - 3t^2 + 8t\right]_4^5 = \frac{125}{3} - 75 + 40 - \left(\frac{64}{3} - 48 + 32\right) = \frac{4}{3}$, so the particle travelled another $\frac{4}{3}$ units from $t = 4$ until $t = 5$. The total distance travelled is $8 + \frac{4}{3} = \frac{28}{3}$. $\boxed{B = \frac{28}{3}}$

C: Slope between $\left(1, \frac{28}{3}\right)$ and $\left(\frac{28}{3}, k\right)$ is $\frac{\left(k - \frac{28}{3}\right)}{\frac{28}{3} - 1} = \frac{3k - 28}{25}$. So, $\frac{3k-28}{25} = \frac{28}{3}$, making

$9k - 84 = 700 \rightarrow\ 9k = 784 \rightarrow k = \frac{784}{9}$. $\boxed{C = \frac{784}{9}}$