MENTAL MATH SOLUTIONS

ANSWERS

1. $18\frac{1}{16} = 18.0625 = \frac{289}{16}$ 2.673 $3.\frac{1}{13}$ 4.60 5.56 6. = 7. B5 8. $\frac{57}{64}$ 9. $\frac{1}{7}$ 10.143 $11.\frac{4}{3}(\sqrt{13}+2)$ 12.01 13. $\sqrt[3]{2019}$ 14.28 15.5.5 16.49 $17.\frac{\sqrt{3}}{3}$ 18.4 19.8 20. 13.5 21. $\frac{6057}{10}$ 22.80 23.1023 $24.\frac{3}{2}$ 25. –9 26. 1, -2 27.12321 28. -72 29.3600 $30.\frac{17}{42}$ 31.7 32.0 33. $27\frac{1}{25} = 27.04 = \frac{676}{25}$ 34.81 35.2020 36.2018 $37. \frac{2021!}{2019!}$ 38. 0. 0198 39.11 40.714

- 1. Notice $4.25 = 4 + \frac{1}{4}$, when squared this is $16 + 2 + \frac{1}{16} = 18\frac{1}{16}$.
- 2. Notice 3 divides into 2019. Since there are only four factors 673 must be prime.
- 3. The first draw is irrelevant if unknown since all cards are equally likely, so the second draw remains at $\frac{1}{12}$ probability for an ace.

4. $24 = 2^3 * 3$. Sum of factors is 60.

- 5. This is just $\binom{n+2}{3}$ and n = 6, so $\binom{8}{3} = 56$.
- 6. Take the log of each side and they are equal.
- 7. We group the top 4 binary digits and the bottom 4 binary digits to obtain $B5_{16}$
- 8. Ew arithmetic. $\frac{19}{12} * \frac{9}{16} = \frac{57}{64}$.
- 9. That is $\frac{1}{4}$ of one square and there is total area $\frac{7}{4}$. (since one overlap). So probability is $\frac{1}{7}$.
- 10. The sum of the first *n* Fibonacci is $F_{n+2} 1$. So 144 1 = 143.
- 11. Rationalize and see $\frac{12(\sqrt{13}+2)}{9} = \frac{4}{3}(\sqrt{13}+2).$
- 12. Take mod100 for last two digits. Clearly $101^{101} \equiv 1 \mod 100$. And thus the last two digits are 01.
- 13. There's only one real zero. $\sqrt[3]{2019}$
- 14. This is 28². So 28.
- 15. If he rerolls, the expected value is 4.5, so he'd only reroll 1-4, which is half of all possibilities. So overall expected value is $2.25 + \frac{5+6+7+8}{4} = 2.25 + 3.25 = 5.5$
- 16. This is a prime squared. We have 4, 9, 25, and then 49.
- 17. Side length is $\frac{2}{\sqrt{3}}$. So $\frac{1}{2} * 1 * \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.
- 18. 27 is 6 above the old mean and since the mean dropped by 2 when 27 was removed, this means that there is now 3 numbers. So there was 4 numbers.
- 19. 2 per side so 8.
- 20. Know powers of 2. 13.5

 $21.\ 30 * \frac{2019}{100} = \frac{6057}{10}$

- 22. This is 2222_3 , which is 1 less than the 4th power of 3, which is 80.
- 23. Almost 2¹⁰, but there's no 0 included, so it is 1023.

24. This is $2 - \frac{1}{2} = \frac{3}{2}$.

- 25. Should have years memorized from past. 63 * 32 = 2016 and $45^2 = 2025$. So -9.
- 26. The inverse of (a, b) is (b, a), so this only works if b = a and thus f(x) = x, so setting this equal, we have x = 1, -2.
- 27. Stack 'em, it's 12321.
- 28. This is 50 * 50 9 * 9 50 * 50 + 3 * 3 = -72
- 29. Take 12^2 out and we have $3^2 + 4^2 = 25$, and 25 * 144 = 25 * 4 * 36 = 3600
- 30. Bleh arithmetic, $\frac{17}{42}$
- 31. $x^2 8x + 7 = 0$, but make sure both are in the domain, which only 7 is.
- 32. This is clearly $(x^2 1)(x 1)$, which has distinct zeroes 1, -1 so sum is 0.
- 33. 5.2² is another chance for you to realize the gimmick for question 1. Squaring $5 + \frac{1}{5} = 25 + 2 + \frac{1}{25} = 27\frac{1}{25} = 27.04$.
- 34. This was supposed to hint again for previous questions. Raise to the 4th power and get 81.
- 35. 2020, something's matching.
- 36. Each game results in one loser, which is out of the tournament. So we need 2018 people out of the tournament so 2018 games are needed.
- 37. Since $\frac{2021!}{2019!} = \frac{2021}{2019} * \frac{2020}{2018} * \frac{2019!}{2017!}$ the former is clearly larger.
- 38. Multiply numerator and denominator by 99 to obtain $\frac{198}{9999}$, which is 0. $\overline{0198}$.
- 39. 1235 so sum of digits is 11.
- 40. Sum of squares of first *n* Fibonacci numbers is F_nF_{n+1} , which is F_8F_9 . F_9 was given earlier as 34, and $F_8 = 21$. So this is 21 * 34 = 714.