Answers:

1. $\left(-2, 2\sqrt{3}\right)$
2. 13
3. 2
4. -2 and 3
5. 7
6. 489,888
7. i (must be in rectangular form)
8. -5
9. $y = 2x^2 + 6$
10. -15
11. $\langle -20, -4, 13 \rangle$ (must be in this form)
12. $\frac{2}{3}$
13. $\frac{\sqrt{230}}{5}$
14. $-\sqrt{3} - i$ (must be in rectangular form)
15. -2
16. $(-5, 0)$ (must be in this form)
17. -2 and -1
18. 3
19. 4389
20. $(-\infty, 2] \cup (7, \infty)$ (must be in interval notation)
21. $\frac{9}{17}$ (must be in fraction form)
22. $\frac{7\pi}{2}$
23. -126
24. 42
25. 16

Solutions:

1. The Cartesian ordered pair would be $\left(4\cos\frac{2\pi}{3},4\sin\frac{2\pi}{3}\right)=\left(-2,2\sqrt{3}\right)$.

2. Let S_n be the sum of the *n*th powers of the roots of the equation. Using the Newton's sums method, $S_1+3\cdot 1=0\Rightarrow S_1=-3$, so $S_2+3S_1-2\cdot 2=0\Rightarrow S_2=13$.

3. All of the properties apply to real numbers. Matrices behave like real numbers with respect to addition, and matrices have the associative property of multiplication, but since order matters for matrices with respect to multiplication, matrices do not have the commutative property of multiplication, which is number 2.

4. $(f\circ g)(x)$ will not be defined if 1) *x* cannot be plugged in to *g*, or 2) $g(x)$ cannot be plugged in to *f*. For 1), $x\neq -2$; for 2), $g(x)\neq 2\Rightarrow \frac{3x+1}{x+2}\neq 2\Rightarrow x\neq 3$. Therefore, the domain of $(f\circ g)(x)$ is all real numbers except -2 and 3.

5. Let *S* be the sum of the series. $S=\frac{4}{2}+\frac{7}{4}+\frac{10}{8}+\frac{13}{16}+\frac{16}{32}+\ldots$, and if we divide both sides of this equation by 2 and subtract the result from this equation, we get

$$\frac{S}{2}=\frac{4}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\frac{3}{32}+\ldots=2+\frac{\frac{3}{4}}{1-\frac{1}{2}}=2+\frac{3}{2}=\frac{7}{2}\Rightarrow S=7.$$

6. The 7th term in this expansion is $\binom{9}{6}(2x)^3(3)^6=84\cdot 8\cdot 729x^3=489{,}888x^3$, so the coefficient is 489,888.

7. $\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^{2019}=\left(cis\frac{\pi}{6}\right)^{2019}=cis\frac{2019\pi}{6}=cis\frac{673\pi}{2}=cis\frac{\pi}{2}=i$

8. $f(x)=x^3+9x^2+27x+19=(x+3)^3-8$, so this function is one-to-one. Therefore, $\left(f^{-1}(-16)+3\right)^3-8=-16\Rightarrow\left(f^{-1}(-16)+3\right)^3=-8\Rightarrow f^{-1}(-16)+3=-2\Rightarrow f^{-1}(-16)=-5$.

9. The non-vertical asymptote will be the parabolic asymptote. Since $\frac{2x^4+x-3}{x^2-3}$ $=2x^2+6+\frac{x+15}{x^2-3}$, the non-vertical asymptote is $y=2x^2+6$.

10. Based on the previous problem, since the remainder is not the zero function but it can be made equal to 0 by plugging in $x=-15$, this is where the function intersects its non-vertical asymptote.

11. $\vec{u}\times\vec{v}=\begin{vmatrix} i & j & k \\ 3 & -2 & 4 \\ -1 & 5 & 0 \end{vmatrix}=-4j+15k-2k-20i=\langle -20,-4,13\rangle$

12. Based on the structure of conic section polar equations, $r=\frac{2}{3+2\sin\theta}=\frac{2/3}{1+2/3\sin\theta}$, and when written in this form, the coefficient of $\sin\theta$ is the eccentricity, which is $\frac{2}{3}$.

13. Since the mean of the numbers in the data set is 0, the standard deviation is $\sqrt{\frac{(-4)^2+(-2)^2+0^2+1^2+5^2}{5}}=\sqrt{\frac{46}{5}}=\frac{\sqrt{230}}{5}$.

14. $x^4=-8+8\sqrt{3}i=16cis\left(120^\circ+360^\circ k\right)\Rightarrow x=2cis\left(30^\circ+90^\circ k\right)$, so the solution in the third quadrant is $x=2cis210^\circ=-\sqrt{3}-i$.

15. If we consider the graph of the rational function $y=\frac{1-4x}{2x}$, since the degrees of the numerator and denominator are both 1, the horizontal asymptote of the function is $y=\frac{-4}{2}=-2$. Therefore, as $x\to\infty$, the graph of the function continues to approach this asymptote, making the value of the limit -2.

16. $\frac{x^3+6x^2+3x-10}{5x^2+5x}=\frac{1}{5}x+1+\frac{-2x-10}{5x^2+5x}$, so the graph will intersect its non-vertical asymptote (the quotient above) when the remainder equals 0, which occurs when $x=-5$. Plugging this into either the function or the asymptote yields $y=0$, so the point of intersection I $(-5,0)$.

17. Descartes' Rule of Signs can be used to show this polynomial has exactly one real zero, one that is negative. Since $g(-2)=-47<0$ and $g(-1)=3>0$, by the Intermediate Value Theorem, this zero must fall between -2 and -1.

18. $\log_{(x+1)}(3x+7)=2 \Rightarrow 3x+7=(x+1)^2 \Rightarrow 0=x^2-x-6=(x-3)(x+2) \Rightarrow x=3$ or $x=-2$. However, $x=-2$ is not a solution because it makes the base of the logarithm negative. Therefore, the only solution is $x=3$, meaning the sum of the solutions is 3.

19. We need to find how many terms are in this series. Using the formula for the terms of an arithmetic sequence, $161=-7+3(n-1) \Rightarrow n=57$. Therefore, the sum of this series is $S_{57}=\frac{57}{2}(-7+161)=4389$.

20. For this function to have real-valued outputs, $\frac{3x-6}{x-7} \geq 0$, which occurs when both the numerator and denominator have the same sign or when the numerator is 0 and the denominator is non-zero. Using sign analysis, this occurs when $x \leq 2$ or $x>7$, so the domain is $(-\infty,2] \cup (7,\infty)$.

21. The number of ways to draw two cards of the same color is $\binom{2}{1}\binom{26}{2}=650$. The number of ways to draw two cards of the same rank is $\binom{13}{1}\binom{4}{2}=78$. The number of ways to draw two cards that are both of the same color and the same rank is $\binom{13}{1}\binom{2}{1}\binom{2}{2}=26$. Since there are $\binom{52}{2}=1326$ different possible drawings, the probability is $\frac{650+78-26}{1326}=\frac{702}{1326}=\frac{9}{17}$.

22. $0=2\sin^2\theta-\sin\theta-1=(2\sin\theta+1)(\sin\theta-1) \Rightarrow \sin\theta=-\frac{1}{2}$ or $\sin\theta=1 \Rightarrow \theta=\frac{\pi}{2}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$ in the restriction, so the sum is $\frac{\pi}{2}+\frac{7\pi}{6}+\frac{11\pi}{6}=\frac{7\pi}{2}$.

23. $\langle 2,-4,9\rangle \cdot \langle 3,15,-8\rangle = 2\cdot 3+(-4)\cdot 15+9\cdot(-8)=6-60-72=-126$

24. The line can be written in the form $y=-\frac{1}{A}x+\frac{B}{A}$, so $-\frac{1}{A}=\frac{-32-(-13)}{-22-16}=\frac{1}{2}\Rightarrow A=-2$.

Therefore, $B=x+Ay=16+(-2)(-13)=42$.

25. $\frac{3}{8}=6r^4\Rightarrow r^4=\frac{1}{16}\Rightarrow r=\pm\frac{1}{2}$, so the sum of all possible sums is $\frac{6}{1-\frac{1}{2}}+\frac{6}{1-\left(-\frac{1}{2}\right)}$

$=12+4=16$.