2016 – 2017 Log1 Contest Round 2

Theta Logs and Exponents

Name: ___________________

Units do not have to be included.

	4 points each	
1	Write in logarithmic form: $2^{-3} = \frac{1}{8}$	
2	Evaluate: $\log_5 40 - \log_5 8$	
3	Simplify the expression $\log_6 216 + \frac{(\log 42 - \log 6)}{\log 49}$ Leave your answer as an improper fraction.	
4	Solve for x: $\log_{37}(\log_2(\log_{12} x)) = 0$	
5	Solve for z: $z = \log_{\log_{\log \cdots 27} 27} 27$	

	5 points each	
6	Find the last digit in the sum of $2^{2016} + 7^{2017}$	
7	Evaluate: $\log_9(8) \cdot \log_{10}(9) \cdot \log_{11}(10) \cdots \log_{4096}(4095)$	
8	Express $\log(1228)$ in terms of **a and b** given that $\log(307) = a$ and $\log(625) = b$. Any term that does not contain **a or b** must be expressed as an integer or fraction.	
9	Solve for x: $-15 = -8\ln(3x) + 7$	
10	Solve for x and express your answer in root form. $x^{x^{x^{\cdots}}} = 5$	

6 points each		
11	Solve for x and express your answer in root form. $\left(36\sqrt{3}\right)^{8} = 108^{4}\left(\sqrt{6}\right)^{\left(\sqrt{x}\right)^{3}}$	
12	A population of ants follows an increasing, exponential growth model, multiplying by a factor of $15\frac{5}{8}$ every 3 days. If the initial population was 10, what is the population after 4 days? Note that there are no such things as "fractions of an ant" when talking about population growth.	
13	Solve for x in terms of **a**. $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^{2}$	
14	Solve for x: $\frac{49 * 7^{-2x+6}}{343^{4x}} = 49^{x-4}$	
15	Evaluate: $\log_{81}(\sqrt[3]{3} \cdot \sqrt[9]{3} \cdot \sqrt[27]{3} \cdots)$	

2016 – 2017 Log1 Contest Round 2

Alpha Logs and Exponents

Name: ___________________

Units do not have to be included.

	4 points each	
1	Write in logarithmic form: $2^{-3} = \frac{1}{8}$	
2	Evaluate: $\log_5 40 - \log_5 8$	
3	Simplify the expression $\log_6 216 + \frac{(\log 42 - \log 6)}{\log 49}$ Leave your answer as an improper fraction.	
4	Which of the following is smaller, 3^{20} or 6^{12}?	
5	Solve for z: $z = \log_{\log_{\log_{\cdots 27}27}}27$	

	5 points each	
6	Find the last digit in the sum of $2^{2016} + 7^{2017}$	
7	Evaluate: $\log_9(8) \cdot \log_{10}(9) \cdot \log_{11}(10) \cdots \log_{4096}(4095)$	
8	Express $\log(1228)$ in terms of $\mathbf{a}$ **and** $\mathbf{b}$ given that $\log(307) = \text{a}$ and $\log(625) = \text{b}$. Any term that does not contain $\mathbf{a}$ **or** $\mathbf{b}$ must be expressed as an integer or fraction.	
9	Solve for x: $\log(x) - 1 = -\log(x - 9)$	
10	Solve for x and express your answer in root form. $x^{x^{x^{x^{\cdots}}}} = 5$	

6 points each		
11	Solve for x and express your answer in root form. $\left(36\sqrt{3}\right)^{8} = 108^{4}\left(\sqrt{6}\right)^{\left(\sqrt{x}\right)^{3}}$	
12	A population of ants follows an increasing, exponential growth model, multiplying by a factor of $15\frac{5}{8}$ every 3 days. If the initial population was 10, what is the population after 4 days? Note that there are no such things as "fractions of an ant" when talking about population growth.	
13	Solve for x in terms of **a**. $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^{2}$	
14	Evaluate: $\dfrac{\log_3 \sqrt{243\sqrt{81\sqrt[3]{3}}}}{\log_2 \sqrt[4]{64} + \log_e e^{-10}}$	
15	Evaluate: $\log_{81}(\sqrt[3]{3} \cdot \sqrt[9]{3} \cdot \sqrt[27]{3} \cdots)$	

2016 – 2017 Log1 Contest Round 2

Mu Logs and Exponents

Name: ___________________

Units do not have to be included.

	4 points each	
1	Write in logarithmic form: $2^{-3} = \frac{1}{8}$	
2	Evaluate: $\log_5 40 - \log_5 8$	
3	Simplify the expression $\log_6 216 + \frac{(\log 42 - \log 6)}{\log 49}$ Leave your answer as an improper fraction.	
4	Which of the following is smaller, 3^{20} or 6^{12}?	
5	Find the y-value of the point on the curve $y = \int \ln(x)dx$ when $x = 3$, given that when $x = 1$, $y = 8$	

	5 points each	
6	Find the last digit in the sum of $2^{2016} + 7^{2017}$	
7	Evaluate: $\log_9(8) \cdot \log_{10}(9) \cdot \log_{11}(10) \cdots \log_{4096}(4095)$	
8	Express $\log(1228)$ in terms of $\mathbf{a}$ **and** $\mathbf{b}$ given that $\log(307) = a$ and $\log(625) = b$. Any term that does not contain $\mathbf{a}$ **or** $\mathbf{b}$ must be expressed as an integer or fraction.	
9	Solve for x: $\log(x) - 1 = -\log(x - 9)$	
10	Evaluate: $\log_3(\int_0^4 3^x \ln(3)dx + 1)$	

6 points each		
11	Solve for x and express your answer in root form. $\left(36\sqrt{3}\right)^{8} = 108^{4}\left(\sqrt{6}\right)^{\left(\sqrt{x}\right)^{3}}$	
12	A population of ants follows an increasing, exponential growth model, multiplying by a factor of $15\frac{5}{8}$ every 3 days. If the initial population was 10, what is the population after 4 days? Note that there are no such things as “fractions of an ant” when talking about population growth.	
13	Solve for x in terms of **a**. $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2$	
14	Evaluate: $\dfrac{\log_3\sqrt{243\sqrt{81\sqrt[3]{3}}}}{\log_2\sqrt[4]{64} + \log_e e^{-10}}$	
15	Given that $\mathrm{f(x)} = \ln \mathrm{e}^{2\ln \mathrm{e}^{\left(\frac{3}{x}\right)\ln \mathrm{e}^{\sqrt{x}}}}$ evaluate the derivative of $\mathrm{f(x)}$ at $\mathrm{x} = 2$. That is, find $\mathrm{f}'(2)$	

2016 – 2017 Log1 Contest Round 2

Theta Logs and Exponents – Answer Key

Name: ___________________

Units do not have to be included.

	4 points each	
1	Write in logarithmic form: $2^{-3} = \frac{1}{8}$	$\log_2\left(\frac{1}{8}\right) = -3$
2	Evaluate: $\log_5 40 - \log_5 8$	1
3	Simplify the expression $\log_6 216 + \frac{(\log 42 - \log 6)}{\log 49}$ Leave your answer as an improper fraction.	$\frac{7}{2}$
4	Solve for x: $\log_{37}(\log_2(\log_{12} x)) = 0$	144
5	Solve for z: $z = \log_{\log_{\log_{\cdots 27} 27}} 27$	3

	5 points each	
6	Find the last digit in the sum of $2^{2016} + 7^{2017}$	3
7	Evaluate: $\log_9(8) \cdot \log_{10}(9) \cdot \log_{11}(10) \cdots \log_{4096}(4095)$	$\frac{1}{4}$
8	Express $\log(1228)$ in terms of **a and b** given that $\log(307) = a$ and $\log(625) = b$. Any term that does not contain **a or b** must be expressed as an integer or fraction.	$a - \frac{b}{2} + 2$
9	Solve for x: $-15 = -8\ln(3x) + 7$	$\frac{1}{3}e^{\frac{11}{4}}$
10	Solve for x and express your answer in root form. $x^{x^{x^{x^{\cdots}}}} = 5$	$\sqrt[5]{5}$

6 points each		
11	Solve for x and express your answer in root form. $\left(36\sqrt{3}\right)^8 = 108^4\left(\sqrt{6}\right)^{\left(\sqrt{x}\right)^3}$	$4\sqrt[3]{4}$
12	A population of ants follows an increasing, exponential growth model, multiplying by a factor of $15\frac{5}{8}$ every 3 days. If the initial population was 10, what is the population after 4 days? Note that there are no such things as "fractions of an ant" when talking about population growth.	390
13	Solve for x in terms of **a**. $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2$	$\mathrm{x} = \mathrm{a}$
14	Solve for x: $\frac{49 * 7^{-2x+6}}{343^{4x}} = 49^{x-4}$	1
15	Evaluate: $\log_{81}(\sqrt[3]{3} \cdot \sqrt[9]{3} \cdot \sqrt[27]{3} \cdots)$	$\frac{1}{8}$

2016 – 2017 Log1 Contest Round 2

Alpha Logs and Exponents – Answer Key

Name: ___________________

Units do not have to be included.

	4 points each	
1	Write in logarithmic form: $2^{-3} = \frac{1}{8}$	$\log_2\left(\frac{1}{8}\right) = -3$
2	Evaluate: $\log_5 40$- $\log_5 8$	1
3	Simplify the expression $\log_6 216 + \frac{(\log 42 - \log 6)}{\log 49}$ Leave your answer as an improper fraction.	$\frac{7}{2}$
4	Which of the following is smaller, 3^{20}or 6^{12}?	6^{12}
5	Solve for z: $z = \log_{\log_{\log_{\cdots 27} 27}} 27$	3

	5 points each	
6	Find the last digit in the sum of $2^{2016} + 7^{2017}$	3
7	Evaluate: $\log_9(8) \cdot \log_{10}(9) \cdot \log_{11}(10) \cdots \log_{4096}(4095)$	$\frac{1}{4}$
8	Express $\log(1228)$ in terms of **a and b** given that $\log\,(307) = \mathrm{a}$ and $\log\,(625) = \mathrm{b}$. Any term that does not contain **a or b** must be expressed as an integer or fraction.	$\mathrm{a} - \frac{\mathrm{b}}{2} + 2$
9	Solve for x: $\log(\mathrm{x}) - 1 = -\log(\mathrm{x} - 9)$	10
10	Solve for x and express your answer in root form. $\mathrm{x}^{\mathrm{x}^{\mathrm{x}^{\mathrm{x}^{\cdots}}}} = 5$	$\sqrt[5]{5}$

6 points each		
11	Solve for x and express your answer in root form. $\left(36\sqrt{3}\right)^{8} = 108^{4}\left(\sqrt{6}\right)^{\left(\sqrt{x}\right)^{3}}$	$4\sqrt[3]{4}$
12	A population of ants follows an increasing, exponential growth model, multiplying by a factor of $15\frac{5}{8}$ every 3 days. If the initial population was 10, what is the population after 4 days? Note that there are no such things as “fractions of an ant” when talking about population growth.	390
13	Solve for x in terms of **a**. $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2$	$\mathrm{x} = \mathrm{a}$
14	Evaluate: $\dfrac{\log_3 \sqrt{243\sqrt{81\sqrt[3]{3}}}}{\log_2 \sqrt[4]{64} + \log_e e^{-10}}$	$-\dfrac{43}{102}$
15	Evaluate: $\log_{81}(\sqrt[3]{3} \cdot \sqrt[9]{3} \cdot \sqrt[27]{3} \cdots)$	$\dfrac{1}{8}$

2016 – 2017 Log1 Contest Round 2

Mu Logs and Exponents – Answer Key

Name: ___________________

Units do not have to be included.

	4 points each	
1	Write in logarithmic form: $2^{-3} = \frac{1}{8}$	$\log_2\left(\frac{1}{8}\right) = -3$
2	Evaluate: $\log_5 40 - \log_5 8$	1
3	Simplify the expression $\log_6 216 + \frac{(\log 42 - \log 6)}{\log 49}$ Leave your answer as an improper fraction.	$\frac{7}{2}$
4	Which of the following is smaller, 3^{20} or 6^{12}?	6^{12}
5	Find the y-value of the point on the curve $y = \int \ln(x)dx$ when $x = 3$, given that when $x = 1$, $y = 8$	$3\ln(3) + 6$

	5 points each	
6	Find the last digit in the sum of $2^{2016} + 7^{2017}$	3
7	Evaluate: $\log_9(8) \cdot \log_{10}(9) \cdot \log_{11}(10) \cdots \log_{4096}(4095)$	$\frac{1}{4}$
8	Express $\log(1228)$ in terms of **a and b** given that $\log(307) = a$ and $\log(625) = b$. Any term that does not contain **a or b** must be expressed as an integer or fraction.	$a - \frac{b}{2} + 2$
9	Solve for x: $\log(x) - 1 = -\log(x - 9)$	10
10	Evaluate: $\log_3(\int_0^4 3^x \ln(3)dx + 1)$	4

6 points each		
11	Solve for x and express your answer in root form. $\left(36\sqrt{3}\right)^8 = 108^4\left(\sqrt{6}\right)^{\left(\sqrt{x}\right)^3}$	$4\sqrt[3]{4}$
12	A population of ants follows an increasing, exponential growth model, multiplying by a factor of $15\frac{5}{8}$ every 3 days. If the initial population was 10, what is the population after 4 days? Note that there are no such things as "fractions of an ant" when talking about population growth.	390
13	Solve for x in terms of **a**. $2\log_b x = 2\log_b(1-a) + 2\log_b(1+a) - \log_b\left(\frac{1}{a} - a\right)^2$	$\mathrm{x} = \mathrm{a}$
14	Evaluate: $\frac{\log_3\sqrt{243\sqrt{81\sqrt[3]{3}}}}{\log_2\sqrt[4]{64} + \log_e e^{-10}}$	$-\frac{43}{102}$
15	Given that $\mathrm{f(x)} = \ln \mathrm{e}^{2\ln \mathrm{e}^{\left(\frac{3}{x}\right)\ln \mathrm{e}^{\sqrt{x}}}}$ evaluate the derivative of $\mathrm{f(x)}$ at $\mathrm{x} = 2$. That is, find $\mathrm{f}'(2)$	$-\frac{3}{4}\sqrt{2}$

2016 – 2017 Log1 Contest Round 2
Logs and Exponents Solutions

Mu	Al	Th	Solution
1	1	1	If $a = b^c$, then $\log_b a = c$. Therefore, the logarithmic form of $2^{-3} = \frac{1}{8}$ is $\log_2\left(\frac{1}{8}\right) = -3$
2	2	2	Since the quotient of two logs is the difference of the logs: $\log_5 40 - \log_5 8 = \log_5 \frac{40}{8} = \log_5 5 = 1$
3	3	3	$3 + \frac{\log(7)}{2\log(7)} = \frac{7}{2}$
4	4		$3^{12}3^8$ compared to $3^{12}2^{12}$ Factor out 3^{12} $(3^2)^4$ compared to $(2^3)^4$ $9 > 8$ Therefore, $3^{20} > 6^{12}$
		4	Since the log expression is equal to 0, it must be true that $\log_2(\log_{12} x) = 1$. If this is to be true as well, then $\log_{12} x = 2$. In exponential form, this is $x = 12^2 = 144$
5			$\int \ln(x)dx = x(\ln(x) - 1) + C$ $8 = 1(\ln(1) - 1) + C, \ln(1) = 0$ $y = x(\ln(x) - 1) + 9$ $y = 3\ln(3) + 6$ when $x = 3$
	5	5	$z = \log_z 27$ $z = 3$

6	6	6	For the base-2 number, the last digit follows the sequence 2,4,8,6. Thus every power of 4 for a base of 2 ends in 6. The next one is a 2. 2016 is a multiple of 4 so 2^{2016} ends in a 6. For the base-7 number, the last digit follows the sequence 7,9,3,1. This parallels the base-2 number sequence so 7^{2017} ends in a 7. Since 6 + 7 = 13, the last digit in the sum is 3.
7	7	7	$\log_9 8 = \frac{\log 8}{\log 9}$ $\log_{10} 9 = \frac{\log 9}{\log 10}$ $\log_{4096} 4095 = \frac{\log 4095}{\log 4096}$ With the above change of bases, carrying out the multiplication sequence stated in the problem reduces the expression to $\frac{\log 8}{\log 4096} = \frac{\log 8}{\log 8^4} = \frac{\log 8}{4\log 8} = \frac{1}{4}$
8	8	8	$\log(1228) = \log(307 * 4)$ $\log(1228) = \log(307) + \log(4)$ $\log(1228) = \mathrm{a} + \log\left(\frac{2500}{625}\right)$ $\log(1228) = \mathrm{a} + \log(2500) - \log(625)$ $\log(1228) = \mathrm{a} + \log(25) + \log(100) - \mathrm{b}$ $\log(1228) = \mathrm{a} + \log(625)^{\frac{1}{2}} + 2 - \mathrm{b}$ $\log(1228) = \mathrm{a} + \frac{1}{2}\log(625) - \mathrm{b} + 2$ $\log(1228) = \mathrm{a} + \frac{1}{2}\mathrm{b} - \mathrm{b} + 2$ $\log(1228) = \mathrm{a} - \frac{\mathrm{b}}{2} + 2$
9	9		$\log \mathrm{x} + \log(\mathrm{x} - 9) = 1$ $\log(\mathrm{x}(\mathrm{x} - 9)) = 1$ $\mathrm{x}(\mathrm{x} - 9) = 10^1 = 10$ $\mathrm{x}^2 - 9\mathrm{x} - 10 = 0$ $(\mathrm{x} - 10)(\mathrm{x} + 1) = 0$ $\mathrm{x} = 10$ (positive roots only)

		9	$-15 = -8\ln(3x) + 7$ $-22 = -8\ln(3x)$ $\frac{11}{4} = \ln(3x)$ $e^{\frac{11}{4}} = 3x$ $x = \frac{1}{3}e^{\frac{11}{4}}$
10			$1 + \int_0^4 3^x \ln 3dx = 1 + (3^4 - 3^0) = 81$ $\log_3 81 = 4$
	10	10	$x^{x^{x^{\cdots}}} = 5$ $x = 5^{\frac{1}{5}}, \sqrt[5]{5}$
11	11	11	$\left(2 * 2 * 3 * 3 * 3^{\frac{1}{2}}\right)^8 = (2 * 3 * 3 * 6)^4 \left(6^{\frac{1}{2}}\right)^{x^{\frac{3}{2}}}$ $\left(6^2 * 3^{\frac{1}{2}}\right)^8 = (6^2 * 3)^4 \left(6^{\frac{1}{2}}\right)^{x^{\frac{3}{2}}}$ $6^{16}3^4 = 6^8 3^4 \left(6^{\frac{1}{2}}\right)^{x^{\frac{3}{2}}} \rightarrow 6^8 = 6^{\frac{1}{2}x^{\frac{3}{2}}} \rightarrow 8 = \frac{1}{2}x^{\frac{3}{2}}$ $16 = x^{\frac{3}{2}} \rightarrow 256 = x^3 \rightarrow x = 4\sqrt[3]{4}$
12	12	12	Population is equal to $10(15.625^t)$, where t is equal to 3 days. To find the population after 4 days, divide 4 by 3. $\text{Pop} = 10\left(15.625^{\frac{4}{3}}\right) = 10\left(15.625^{\frac{1}{3}}\right)^4$ $\text{Pop} = 10(2.5^4) = 10\left(\frac{5}{2}\right)^4 = 10\left(\frac{625}{16}\right) = 390\frac{5}{8}$ Since “fractions of an ant” don’t exist, the population is 390

<table>
<tr><td>13</td><td>13</td><td>13</td><td>Simplifying:
$2\log_b x = 2\log_b((1-a^2) - \log_b\left(\frac{1}{a} - a\right)^2$
$2\log_b x = 2\log_b((1-a^2) - 2\log_b\left(\frac{1-a^2}{a}\right)$
$2\log_b x = 2\log_b\left(1-a^2 \middle/ \frac{1-a^2}{a}\right) = 2\log_b a$
$x = a$
By inspection, it is implied that a is restricted to $-1 < a < +1$ AND $a \neq 0$. However, accept x = a as a correct answer.</td></tr>
<tr><td>14</td><td>14</td><td></td><td>$Let\ a = \log_3 \sqrt{243\sqrt{81\sqrt[3]{3}}}$
$a = \log_3 \sqrt{3^5 * 3^2 * 3^{\frac{1}{6}}} = \log_3\left(3^{\frac{43}{6}}\right)^{\frac{1}{2}}$
$a = \log_3 3^{\frac{43}{12}}$
$a = \frac{43}{12}$

$Let\ b = \log_2 \sqrt[4]{64} + \log_e e^{-10}$
$b = \log_2 (2^6)^{\frac{1}{4}} - 10 = \frac{3}{2}\ - 10$
$b = -8\frac{1}{2}$

Therefore, $\frac{a}{b} = -\frac{43}{102}$</td></tr>
<tr><td></td><td></td><td>14</td><td>$7^2 7^{(-2x+6)}(7^3)^{-4x} = (7^2)^{x-4}$
$7^{2-2x+6-12x} = 7^{2x-8}$
$8 - 14x = 2x - 8$
$-16x = -16$
$x = 1$</td></tr>
</table>

15			$f(x) = \ln e^{2\ln e^{\left(\frac{3}{x}\right)\ln e^{\sqrt{x}}}} = 2\ln e^{\left(\frac{3}{x}\right)\ln e^{\sqrt{x}}}$ $f(x) = 2\left(\left(\frac{3}{x}\right)\ln e^{\sqrt{x}}\right) = 6\left(\frac{1}{x}\right)\sqrt{x} = 6x^{-\frac{1}{2}}$ $f'(x) = -3x^{-\frac{3}{2}}$ $f'(2) = -3(2)^{-\frac{3}{2}} = -3(8)^{-\frac{1}{2}} = -\frac{3}{2\sqrt{2}}$ $f'(2) = -\frac{3}{4}\sqrt{2}$
	15	15	$\log_{81} 3^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}\cdots\right)}$ $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} \cdots = \frac{1}{2}$ $\log_{81} 3^{\frac{1}{2}} = \frac{1}{8}$