2015 – 2016 Log1 Contest Round 1
Theta Geometry

Name: ________________

	4 points each	
1	A rectangle has a width of $2\frac{3}{8}$ inches and a length that exceeds the width by $1\frac{3}{4}$ inches. Find the perimeter, in inches, of the rectangle.	
2	A triangle has sides of length 4, 5, and 7 inches. Find the area, in square inches, enclosed by the triangle.	
3	The two diagonals of a rhombus have lengths that differ by 3 inches. If the rhombus encloses an area of 9 square inches, find the length, in inches, of the longer diagonal of the rhombus.	
4	The ratio of the numerical values of a sphere's surface area to its volume is $\frac{2}{3}$. Find the volume of that sphere.	
5	If the area enclosed by a circle is 1 square foot, how many feet are in that circle's circumference?	

	5 points each	
6	The positive difference between two complementary angles is 16°. Find the degree measure of the larger of those two angles.	
7	What is the degree measure of the smaller angle that the hour and minute hands make on a clock when the time is 6:54?	
8	In triangle *ABC*, the angle bisector from vertex *A* intersects side *BC* at point *D*. If $\lvert AB\rvert=4$, $\lvert AC\rvert=8$, and $\lvert BC\rvert=6$, find the area enclosed by triangle *ABD*.	
9	A regular hexagon has sides of length $\sqrt{3}$. From one of the vertices of this hexagon, the three diagonals of this hexagon are drawn. Find the sum of the lengths of these three diagonals.	
10	The sum of the number of vertices and edges of a dodecahedron is 50. Find the number of vertices of this dodecahedron.	

6 points each		
11	How many regular polygons have interior angles whose degree measure has an integer value?	
12	Find the least positive integer n such that the total number of diagonals in a regular n-gon is greater than the degree measure of an interior angle of that n-gon.	
13	Find the area enclosed by the ellipse whose equation is $2x^2+3y^2-4x+6y=19$.	
14	Nine points on a circle in a plane are spaced evenly. Three points are chosen at random, and the three inscribed angles made with those points are drawn. What is the probability that the sum of those three angles is exactly 180°?	
15	A triangle's three side lengths are consecutive integers. The product of the numerical values of the lengths of the radii of the triangle's inscribed and circumscribed circles equals the length of the longest side of the triangle. Find the length of that longest side.	

2015 – 2016 Log1 Contest Round 1
Alpha Geometry

Name: ________________

4 points each		
1	A rectangle has a width of $2\frac{3}{8}$ inches and a length that exceeds the width by $1\frac{3}{4}$ inches. Find the perimeter, in inches, of the rectangle.	
2	A triangle has sides of length 4, 5, and 7 inches. Find the area, in square inches, enclosed by the triangle.	
3	The two diagonals of a rhombus have lengths that differ by 3 inches. If the rhombus encloses an area of 9 square inches, find the length, in inches, of the longer diagonal of the rhombus.	
4	A convex quadrilateral has vertices at the points $(-2,1)$, $(3,4)$, $(4,-2)$, and $(-1,0)$. Find the area enclosed by this quadrilateral.	
5	If the area enclosed by a circle is 1 square foot, how many feet are in that circle's circumference?	

5 points each		
6	The positive difference between two complementary angles is 16°. Find the degree measure of the larger of those two angles.	
7	What is the degree measure of the smaller angle that the hour and minute hands make on a clock when the time is 6:54:24? Write your answer as a decimal number of degrees.	
8	In triangle *ABC*, the angle bisector from vertex *A* intersects side *BC* at point *D*. If $\lvert AB\rvert=4$, $\lvert AC\rvert=8$, and $\lvert BC\rvert=6$, find the length of segment *AD*.	
9	A regular hexagon has sides of length $\sqrt{3}$. From one of the vertices of this hexagon, the three diagonals of this hexagon are drawn. Find the sum of the lengths of these three diagonals.	
10	The sum of the number of vertices and edges of a dodecahedron is 50. Find the number of edges of this dodecahedron.	

6 points each		
11	How many regular polygons have interior angles whose degree measure has an integer value?	
12	Find the least positive integer n such that the total number of diagonals in a regular n-gon is greater than the degree measure of an interior angle of that n-gon.	
13	Find the area of the region enclosed by the x-axis, $x=0$, $x=4$, and $f(x)=\begin{cases} x+1, \text{ if } 0\le x<1 \\ 3-x, \text{ if } 1\le x<2 \\ 1+\sqrt{-x^2+6x-8}, \text{ if } 2\le x\le 4 \end{cases}$.	
14	Nine points on a circle in a plane are spaced evenly. Three points are chosen at random, and the three inscribed angles made with those points are drawn. What is the probability that the sum of those three angles is exactly 180°?	
15	A triangle has sides whose lengths are integers and are in an increasing arithmetic progression. The product of the numerical values of the lengths of the radii of the triangle's inscribed and circumscribed circles equals 14. Find the length of the shortest side of the triangle.	

2015 – 2016 Log1 Contest Round 1
Mu Geometry

Name: ________________

	4 points each	
1	A rectangle has a width of $2\frac{3}{8}$ inches and a length that exceeds the width by $1\frac{3}{4}$ inches. Find the perimeter, in inches, of the rectangle.	
2	A triangle has sides of length 4, 5, and 7 inches. Find the area, in square inches, enclosed by the triangle.	
3	What is the area enclosed by the polar graph with equation $r = 7\cos\theta$?	
4	A convex quadrilateral has vertices at the points $(-2,1)$, $(3,4)$, $(4,-2)$, and $(-1,0)$. Find the area enclosed by this quadrilateral.	
5	If the area enclosed by a circle is 1 square foot, how many feet are in that circle's circumference?	

	5 points each	
6	The positive difference between two complementary angles is $16^\circ 13'32''$. Find the measure of the larger of those two angles in degrees, minutes, and seconds.	
7	What is the degree measure of the smaller angle that the hour and second hands make on a clock when the time is 6:54:24? Write your answer as a decimal number of degrees.	
8	Find the area of the region enclosed by the graphs of $f(x) = 2x^2 - 3x + 2$, $y = 0$, $x = 1$, and $x = 3$.	
9	The sum of the number of vertices and edges of a dodecahedron is 50. Find the number of edges of this dodecahedron.	
10	How many regular polygons have interior angles whose degree measure has an integer value?	

6 points each		
11	Find the least positive integer n such that the total number of diagonals in a regular n-gon is greater than the degree measure of an interior angle of that n-gon.	
12	Find the area of the region enclosed by the x-axis, $x=0$, $x=4$, and $f(x)=\begin{cases} x+1, \text{ if } 0\le x<1 \\ 3-x, \text{ if } 1\le x<2 \\ 1+\sqrt{-x^2+6x-8}, \text{ if } 2\le x\le 4 \end{cases}$.	
13	Nine points on a circle in a plane are spaced evenly. Three points are chosen at random, and the three inscribed angles made with those points are drawn. What is the probability that the sum of those three angles is exactly 180°?	
14	A triangle has sides whose lengths are integers and are in an increasing arithmetic progression. The product of the numerical values of the lengths of the radii of the triangle's inscribed and circumscribed circles equals 14. Find the length of the shortest side of the triangle.	
15	A circle has radius of length 3. A chord of length $4\sqrt{2}$ divides the circle into two regions, the larger of which has area that can be written in the form $\dfrac{18\arcsin\left(\frac{1}{3}\right)+9\pi+A}{2}$, where A is a real number. Find the value of A^2.	

2015 – 2016 Log1 Contest Round 1
Theta Geometry

Name: ________________

4 points each		
1	A rectangle has a width of $2\frac{3}{8}$ inches and a length that exceeds the width by $1\frac{3}{4}$ inches. Find the perimeter, in inches, of the rectangle.	13
2	A triangle has sides of length 4, 5, and 7 inches. Find the area, in square inches, enclosed by the triangle.	$4\sqrt{6}$
3	The two diagonals of a rhombus have lengths that differ by 3 inches. If the rhombus encloses an area of 9 square inches, find the length, in inches, of the longer diagonal of the rhombus.	6
4	The ratio of the numerical values of a sphere's surface area to its volume is $\frac{2}{3}$. Find the volume of that sphere.	$\frac{243\pi}{2}$
5	If the area enclosed by a circle is 1 square foot, how many feet are in that circle's circumference?	$2\sqrt{\pi}$

5 points each		
6	The positive difference between two complementary angles is 16°. Find the degree measure of the larger of those two angles.	53°
7	What is the degree measure of the smaller angle that the hour and minute hands make on a clock when the time is 6:54?	117°
8	In triangle *ABC*, the angle bisector from vertex *A* intersects side *BC* at point *D*. If $\lvert AB\rvert=4$, $\lvert AC\rvert=8$, and $\lvert BC\rvert=6$, find the area enclosed by triangle *ABD*.	$\sqrt{15}$
9	A regular hexagon has sides of length $\sqrt{3}$. From one of the vertices of this hexagon, the three diagonals of this hexagon are drawn. Find the sum of the lengths of these three diagonals.	$6+2\sqrt{3}$
10	The sum of the number of vertices and edges of a dodecahedron is 50. Find the number of vertices of this dodecahedron.	20

	6 points each	
11	How many regular polygons have interior angles whose degree measure has an integer value?	22
12	Find the least positive integer n such that the total number of diagonals in a regular n-gon is greater than the degree measure of an interior angle of that n-gon.	20
13	Find the area enclosed by the ellipse whose equation is $2x^2+3y^2-4x+6y=19$.	$4\sqrt{6}\pi$
14	Nine points on a circle in a plane are spaced evenly. Three points are chosen at random, and the three inscribed angles made with those points are drawn. What is the probability that the sum of those three angles is exactly 180°?	1
15	A triangle's three side lengths are consecutive integers. The product of the numerical values of the lengths of the radii of the triangle's inscribed and circumscribed circles equals the length of the longest side of the triangle. Find the length of that longest side.	8

2015 – 2016 Log1 Contest Round 1
Alpha Geometry

Name: ________________

	4 points each	
1	A rectangle has a width of $2\frac{3}{8}$ inches and a length that exceeds the width by $1\frac{3}{4}$ inches. Find the perimeter, in inches, of the rectangle.	13
2	A triangle has sides of length 4, 5, and 7 inches. Find the area, in square inches, enclosed by the triangle.	$4\sqrt{6}$
3	The two diagonals of a rhombus have lengths that differ by 3 inches. If the rhombus encloses an area of 9 square inches, find the length, in inches, of the longer diagonal of the rhombus.	6
4	A convex quadrilateral has vertices at the points $(-2,1)$, $(3,4)$, $(4,-2)$, and $(-1,0)$. Find the area enclosed by this quadrilateral.	18
5	If the area enclosed by a circle is 1 square foot, how many feet are in that circle's circumference?	$2\sqrt{\pi}$

	5 points each	
6	The positive difference between two complementary angles is 16°. Find the degree measure of the larger of those two angles.	53°
7	What is the degree measure of the smaller angle that the hour and minute hands make on a clock when the time is 6:54:24? Write your answer as a decimal number of degrees.	119.2°
8	In triangle *ABC*, the angle bisector from vertex *A* intersects side *BC* at point *D*. If $\lvert AB\rvert=4$, $\lvert AC\rvert=8$, and $\lvert BC\rvert=6$, find the length of segment *AD*.	$2\sqrt{6}$
9	A regular hexagon has sides of length $\sqrt{3}$. From one of the vertices of this hexagon, the three diagonals of this hexagon are drawn. Find the sum of the lengths of these three diagonals.	$6+2\sqrt{3}$
10	The sum of the number of vertices and edges of a dodecahedron is 50. Find the number of edges of this dodecahedron.	30

	6 points each	
11	How many regular polygons have interior angles whose degree measure has an integer value?	22
12	Find the least positive integer n such that the total number of diagonals in a regular n-gon is greater than the degree measure of an interior angle of that n-gon.	20
13	Find the area of the region enclosed by the x-axis, $x=0$, $x=4$, and $f(x)=\begin{cases} x+1, \text{ if } 0\le x<1 \\ 3-x, \text{ if } 1\le x<2 \\ 1+\sqrt{-x^2+6x-8}, \text{ if } 2\le x\le 4 \end{cases}$.	$\frac{10+\pi}{2}$
14	Nine points on a circle in a plane are spaced evenly. Three points are chosen at random, and the three inscribed angles made with those points are drawn. What is the probability that the sum of those three angles is exactly 180°?	1
15	A triangle has sides whose lengths are integers and are in an increasing arithmetic progression. The product of the numerical values of the lengths of the radii of the triangle's inscribed and circumscribed circles equals 14. Find the length of the shortest side of the triangle.	6

2015 – 2016 Log1 Contest Round 1
Mu Geometry

Name: ________________

	4 points each	
1	A rectangle has a width of $2\frac{3}{8}$ inches and a length that exceeds the width by $1\frac{3}{4}$ inches. Find the perimeter, in inches, of the rectangle.	13
2	A triangle has sides of length 4, 5, and 7 inches. Find the area, in square inches, enclosed by the triangle.	$4\sqrt{6}$
3	What is the area enclosed by the polar graph with equation $r = 7\cos\theta$?	$\frac{49\pi}{4}$
4	A convex quadrilateral has vertices at the points $(-2,1)$, $(3,4)$, $(4,-2)$, and $(-1,0)$. Find the area enclosed by this quadrilateral.	18
5	If the area enclosed by a circle is 1 square foot, how many feet are in that circle's circumference?	$2\sqrt{\pi}$

	5 points each	
6	The positive difference between two complementary angles is $16^\circ 13'32''$. Find the measure of the larger of those two angles in degrees, minutes, and seconds.	$53^\circ 6'46''$
7	What is the degree measure of the smaller angle that the hour and second hands make on a clock when the time is 6:54:24? Write your answer as a decimal number of degrees.	63.2°
8	Find the area of the region enclosed by the graphs of $f(x) = 2x^2 - 3x + 2$, $y = 0$, $x = 1$, and $x = 3$.	$\frac{28}{3}$
9	The sum of the number of vertices and edges of a dodecahedron is 50. Find the number of edges of this dodecahedron.	30
10	How many regular polygons have interior angles whose degree measure has an integer value?	22

6 points each		
11	Find the least positive integer n such that the total number of diagonals in a regular n-gon is greater than the degree measure of an interior angle of that n-gon.	20
12	Find the area of the region enclosed by the x-axis, $x=0$, $x=4$, and $f(x)=\begin{cases} x+1, \text{ if } 0\le x<1 \\ 3-x, \text{ if } 1\le x<2 \\ 1+\sqrt{-x^2+6x-8}, \text{ if } 2\le x\le 4 \end{cases}$.	$\frac{10+\pi}{2}$
13	Nine points on a circle in a plane are spaced evenly. Three points are chosen at random, and the three inscribed angles made with those points are drawn. What is the probability that the sum of those three angles is exactly 180°?	1
14	A triangle has sides whose lengths are integers and are in an increasing arithmetic progression. The product of the numerical values of the lengths of the radii of the triangle's inscribed and circumscribed circles equals 14. Find the length of the shortest side of the triangle.	6
15	A circle has radius of length 3. A chord of length $4\sqrt{2}$ divides the circle into two regions, the larger of which has area that can be written in the form $\frac{18\arcsin\left(\frac{1}{3}\right)+9\pi+A}{2}$, where A is a real number. Find the value of A^2.	32

2015 – 2016 Log1 Contest Round 1
Geometry Solutions

Mu	Al	Th	Solution
1	1	1	The length of the rectangle is $2\frac{3}{8}+1\frac{3}{4}=\frac{19}{8}+\frac{7}{4}=\frac{33}{8}$, and since the width is $\frac{19}{8}$, the perimeter is $2\left(\frac{19}{8}+\frac{33}{8}\right)=2\left(\frac{52}{8}\right)=\frac{52}{4}=13$.
2	2	2	The semi-perimeter of the triangle is $\frac{4+5+7}{2}=8$, so using Hero's formula, the enclosed area is $\sqrt{8(8-4)(8-5)(8-7)}=\sqrt{8\cdot 4\cdot 3\cdot 1}=4\sqrt{6}$.
	3	3	Let x be the longer diagonal length. Then $\frac{x(x-3)}{2}=9\Rightarrow 0=x^2-3x-18=(x-6)(x+3)$ $\Rightarrow x=6$ since x is a length.
3			$r=7\cos\theta$ is a circle with diameter of length 7, so the enclosed area is $\pi\left(\frac{7}{2}\right)^2=\frac{49\pi}{4}$.
		4	$\frac{2}{3}=\frac{4\pi r^2}{\frac{4}{3}\pi r^3}=\frac{3}{r}\Rightarrow r=\frac{9}{2}$, so the volume of the sphere is $\frac{4}{3}\pi\left(\frac{9}{2}\right)^3=\frac{243\pi}{2}$.
4	4		The points are listed in order of how they would be connected to form a convex quadrilateral, so using the shoelace method, $\begin{array}{cccc} & -2 & 1 & \\ 3 & 3 & 4 & -8 \\ 16 & 4 & -2 & -6 \\ 2 & -1 & 0 & 0 \\ 0 & -2 & 1 & -1 \\ 21 & & & -15 \end{array}\Rightarrow A=\frac{1}{2}\lvert 21-(-15)\rvert=\frac{1}{2}(36)=18$.
5	5	5	$\pi r^2=1\Rightarrow r=\frac{\sqrt{\pi}}{\pi}$, so the circumference of the circle is $2\pi\left(\frac{\sqrt{\pi}}{\pi}\right)=2\sqrt{\pi}$.
	6	6	Let x be the larger of the two angles in degrees. Then $x-(90^\circ-x)=16^\circ\Rightarrow 2x-90^\circ$ $=16^\circ\Rightarrow 2x=106^\circ\Rightarrow x=53^\circ$.
6			Let x be the larger of the two angles in degrees. Then $x-(90^\circ-x)=16^\circ 13'32''$ $\Rightarrow 2x-90^\circ=16^\circ 13'32''\Rightarrow 2x=106^\circ 13'32''=106^\circ 12'92''\Rightarrow x=53^\circ 6'46''$.
		7	$\theta=\frac{1}{2}\lvert 60\cdot 6-11\cdot 54\rvert=\frac{1}{2}(234^\circ)=117^\circ$
	7		$\theta=\frac{1}{2}\left\lvert 60\cdot 6-11\cdot 54-\frac{11}{60}\cdot 24\right\rvert=\frac{1}{2}(238.4^\circ)=119.2^\circ$

7			Since 6:00, the number of seconds that have elapsed to that time is $60 \cdot 54 + 24 = 3264$, so the hour hand has moved $\frac{3264}{3600} = \frac{68}{75}$ of the way from 6:00 to 7:00. Since the movement of the hour hand is 30° for each hour passed, the hour hand has moved $\frac{68}{75} \cdot 30^\circ = 27.2^\circ$. The second hand is 6 seconds behind reaching 30 seconds (which is at the same position as 6 hours for the hour hand), so the second hand is $\frac{6}{60} \cdot 360^\circ = 36^\circ$ from back of that position, making the angle between the hour and second hands $27.2^\circ + 36^\circ = 63.2^\circ$.
		8	The angle bisector divides *BC* into two pieces in the same ratio at the other two sides, so $\frac{BD}{DC} = \frac{4}{8} \Rightarrow BD = 2$ and $DC = 4$ (since $BC = 6$). However, since triangles *ABD* and *ACD* use the same altitude to side *BC*, the areas of those two triangles are also in the ratio of 1:2. By Hero's formula, since the semi-perimeter of triangle *ABC* is 9, the total enclosed area of triangle *ABC* is $\sqrt{9(9-4)(9-6)(9-8)} = 3\sqrt{15}$. This means that the area of triangle *ABC* is one-third of that, or $\sqrt{15}$.
	8		The angle bisector divides *BC* into two pieces in the same ratio at the other two sides, so $\frac{BD}{DC} = \frac{4}{8} \Rightarrow BD = 2$ and $DC = 4$ (since $BC = 6$). Therefore, the length of the angle bisector is $\sqrt{4 \cdot 8 - 2 \cdot 4} = \sqrt{24} = 2\sqrt{6}$.
8			Since the graph of *f* is always above the *x*-axis, the area is $\int_1^3 \left(2x^2 - 3x + 2\right) dx$ $= \left(\frac{2}{3}x^3 - \frac{3}{2}x^2 + 2x\right)\Big\|_1^3 = \left(18 - \frac{27}{2} + 6\right) - \left(\frac{2}{3} - \frac{3}{2} + 2\right) = \frac{28}{3}$.
	9	9	The longest diagonal is just twice the length of a side, so this is $2\sqrt{3}$. The other two diagonals are equal in length, and due to the relationship of sides in the special right triangle, the length of each of the other two diagonals is $2 \cdot \sqrt{3} \cdot \frac{\sqrt{3}}{2} = 3$, so the total length is $3 + 3 + 2\sqrt{3} = 6 + 2\sqrt{3}$.
9	10	10	Any dodecahedron has 12 faces, so using Euler's formula, $V - E + 12 = 2 \Rightarrow V - E = -10$. Additionally, $V + E = 50$ was given, so solving this system, $V = 20$ and $E = 30$.
10	11	11	For an interior angle to have an integer number of degrees in its measure, an exterior angle must also. Since the measure of an exterior angle is $\frac{360^\circ}{n}$, we just need to find the number of positive integral divisors of 360 that are greater than or equal to 3. Since $360 = 2^3 \cdot 3^2 \cdot 5$, the number of positive integral divisors is $(3+1)(2+1)(1+1)$ $= 24$. Excluding 1 and 2, there are 22 possibilities.
11	12	12	We are trying to solve the inequality $\frac{n(n-3)}{2} > 180 - \frac{360}{n}$, and by inspection, the least value of *n* is $n = 20$.
		13	$2x^2 + 3y^2 - 4x + 6y = 19$ is equivalent to $\frac{(x-1)^2}{12} + \frac{(y-1)^2}{8} = 1$, so the area enclosed by the ellipse is $\pi \cdot \sqrt{12} \cdot \sqrt{8} = 4\sqrt{6}\pi$.

12	13		The region described results in the following shapes: For $0 \le x < 1$ and $1 \le x < 2$, the regions are trapezoids with bases of length 1 and 2 and height 1, so the total area for these two regions is $2 \cdot \frac{1}{2}(1+2)(1) = 3$. For $2 \le x \le 4$, region is a 1 by 2 rectangle with half a disk of radius 1 sitting on top of it (you can see this by completing the square underneath the radical), so the are for this region is $2 \cdot 1 + \frac{1}{2}\pi(1)^2 = 2 + \frac{\pi}{2}$. This makes the total area $3 + 2 + \frac{\pi}{2} = \frac{10+\pi}{2}$.
13	14	14	If three points were to be chosen, being on the circle, no three of them would be collinear. Therefore, create a triangle with those three points as its vertices. The three angles of the triangle are the three inscribed angles, so they always sum to 180°. Therefore, the probability is 1.
		15	First, an observation: since the radius of the inscribed circle is the area of the triangle divided by its semi-perimeter, and since the radius of the inscribed circle is the product of the three sides of the triangle divided by 4 times its area, the product we desire equals the product of the three sides of the triangle divided by twice the perimeter of the triangle. Now, let *x* be the middle length side of the triangle. Then we are trying to find the solution to $x+1 = \frac{(x-1)x(x+1)}{2((x-1)+x+(x+1))} = \frac{(x-1)x(x+1)}{2(3x)} = \frac{(x-1)(x+1)}{6}$. The solutions to this equation are $x=-1$ (since $x+1$ would be equal to 0) or $x=7$ (since $\frac{x-1}{6}$ would be equal to 1). Obviously the first answer can't be the correct one, so the second one is, making the longest side have length 8 (the triangle has sides of length 6, 7, and 8, which is a valid triangle).
14	15		First, an observation: since the radius of the inscribed circle is the area of the triangle divided by its semi-perimeter, and since the radius of the inscribed circle is the product of the three sides of the triangle divided by 4 times its area, the product we desire equals the product of the three sides of the triangle divided by twice the perimeter of the triangle. Now, let *x* be the middle length side of the triangle, and let *k* be the positive common difference of the arithmetic progression. Then we are trying to find the solution to $14 = \frac{(x-k)x(x+k)}{2((x-k)+x+(x+k))} = \frac{(x-k)x(x+k)}{2(3x)} = \frac{(x-k)(x+k)}{6} \Rightarrow (x-k)(x+k) = 84$. Further, due to the constraints in the problem, *x* and *k* must be positive integers, and since $x-k$ and $x+k$ differ by an even number and have an even product, both must be even. Both $x-k$ and $x+k$ must be positive as well, so the only possibilities are: 1) $x-k=2, x+k=42 \Rightarrow x=22, k=20$, OR 2) $x-k=6, x+k=14 \Rightarrow x=10, k=4$. The first case yields side lengths of 2, 22, and 42 (which is not a triangle), and the second case yields side lengths of 6, 10, and 14 (which is a triangle). Therefore, the shortest side length of the triangle is 6.

15			Since the chord has length $4\sqrt{2}$, by the Pythagorean theorem, it must be a distance $\sqrt{3^2-2\sqrt{2}^2}=1$ from the center of the circle. For ease in working the problem, center the circle at the origin. Then we are trying to find the value of $2\int_{-1}^{3}\sqrt{9-x^2}dx$. Using right triangle substitution to find the antiderivative, this integral becomes $\left(9\arcsin\frac{x}{3}+x\sqrt{9-x^2}\right)\Big\vert_{-1}^{3}=\left(\frac{9\pi}{2}+0\right)-\left(9\arcsin\left(-\frac{1}{3}\right)-2\sqrt{2}\right)=\frac{9\pi}{2}-9\arcsin\left(-\frac{1}{3}\right)+2\sqrt{2}$ $=\frac{9\pi}{2}+9\arcsin\frac{1}{3}+2\sqrt{2}=\frac{18\arcsin\left(\frac{1}{3}\right)+9\pi+4\sqrt{2}}{2}$, so $A=4\sqrt{2}\Rightarrow A^2=32$.