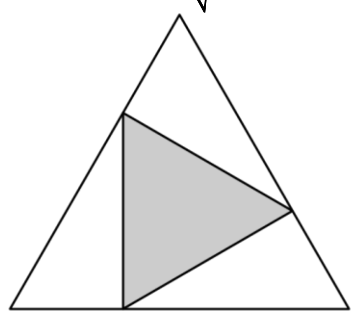
1. D
2. B
3. E
4. C
5. E
6. A
7. C
8. B
9. D
10. C
11. C
12. C
13. D
14. D
15. E
16. B
17. B
18. C
19. A
20. B
21. C
22. C
23. A
24. D
25. E
26. C
27. A
28. B
29. C
30. E

1. D A horizontal stretch means that the vertical length of the circle stays the same, yet the horizontal length will increase by some factor k. Since the area is increased by a factor of 3 and only the horizontal length changed, this means that this length must also have increased by a factor of 3. Thus the new length is (4)(3) = 12.
2. B The dog can roam for three-quarters of the area of a radius 2 circle outside his house. Inside his house, he can roam for half of a radius 1 circle. Adding these areas together gives $\frac{3}{4}\pi 2^2 + \frac{1}{2}\pi 1^2 = \frac{7}{2}\pi$
3. E Billy can wander through out the entirety of inside the cube since 9 is greater than the length of the half diagonal, $3\sqrt{3}$. This gives us a volume of $6^3 = 216$. Outside the cube, Billy can roam 8 seven-eighth spheres of radius 3, which equates to 7 of these spheres, giving us volume $7\left(\frac{4}{3}\right)\pi 3^3 = 468\pi$. The total volume is thus $468\pi + 216$
4. C An icosahedron has 20 faces, all of which are equilateral triangles with 3 edges each. We are looking for the number of edges, so we multiply 20 by 3 to get 60 edges. But this overcounts since each edge of the icosahedron consists of two triangle edges meeting, so we must divide by 2 to get 30 edges. Each has length 12 inches, so 30 — 12 = 360 of total edge length.
5. E The side lengths look dicult to work with, but if we try out the Pythagorean theorem, indeed, $\left(4\sqrt{3}\right)^2 + \left(\sqrt{13}\right)^2 = \left(\sqrt{61}\right)^2$, so the area is just $\frac{1}{2}\left(4\sqrt{3}\right)\left(\sqrt{13}\right) = 2\sqrt{39}$
6. A Since, $\frac{360-45}{360} = \frac{7}{8}$, we just need to calculate seven-eighths of a normal cylinder's surface area, then add the exposed inside area back in.
The original surface area is $2\pi(8)^2 + 2\pi(8)(2) = 160\pi$, seven-eighth of which is 140π. Then the inner exposed area consists of two rectangles with combined area $2(2)(8) = 32$. The total area is then $140\pi + 32$.
7. C Draw a diagram first. We are looking for the outside perimeter of this configuration. Since each center lies on the other circles' perimeters, connect each of the three centers. Since each side has length 1, this makes an equilateral triangle.
We can apply the same principle to determine how much arc of each of the circles is on the outside. Now, connect the centers of ω_1 and ω_2, as well as their point of intersection which is not inside ω_3. Note that this triangle is also equilateral since each side is a radius of length 1.
By symmetry, we can do this for the other two triangles formed by connecting intersection points. For each circle, 3 equilateral triangles are formed, which means $3(60) = 180°$ of arc is exposed. Since there are then 3 half circles that make up the outside perimeter of the configuration, the total perimeter is $3\left(\frac{1}{2}\right)(2\pi) = 3\pi$
8. B This question is asking for us to find the perimeter when we combine the intersections of each of the circles taken two at a time. Which looks somewhat like a flower with three pointy petals, but we only want the outside perimeter of the flower. Each of the 3 petals is made up of 2 arcs. Based on our findings of equilateral triangles in the previous solution, we know that each of these arcs is 60°. With 6 arcs, the total is 2π

9. D First, we solve for the side lengths of each of the faces by using the formula for the area of an equilateral triangle. We find that $s = 3\sqrt{2}$. Now, to find the volume of the octahedron, we just need the height of each of the two pyramids that make it up. We take a diagonal cross-section out to find a 45-45-90 triangle with sides $3\sqrt{2} - 3\sqrt{2} - 6$, the height of which is 3. Now we plug this information to find the volume is $2\left(\frac{1}{3}\right)\left(3\sqrt{2}\right)^2(3) = 36$

10. C Let θ be the angle between the sides of lengths 7 and 10. We have that $28 = \frac{1}{2} \cdot 7 \cdot 10 \cdot \sin(\theta)$. Rearranging, we can find that $\sin(\theta) = \frac{4}{5}$. Since θ is acute, we can draw a 3-4-5 right triangle to see that $\cos(\theta) = \frac{3}{5}$. By the law of cosines, we can compute that the third side of the triangle has length $\sqrt{7^2 + 10^2 - 2 \cdot 7 \cdot 10 \cdot \frac{3}{5}} = \boxed{\sqrt{65}}$.

11. C



Let the side length ΔABC be s. Its area is given by $\frac{s^2\sqrt{3}}{4}$. Each of the small white triangles has area given by $\frac{1}{2} \cdot \frac{s}{3} \cdot \frac{2s}{3} \cdot \sin(60°) = \frac{s^2\sqrt{3}}{18}$. The area of ΔDEF is thus $\frac{s^2\sqrt{3}}{4} - 3 \cdot \frac{s^2\sqrt{3}}{18} = \frac{s^2\sqrt{3}}{12}$. Our desired ratio is thus $\frac{s^2\sqrt{3}}{12} / \frac{s^2\sqrt{3}}{4} = \boxed{\frac{1}{3}}$

12. C First, we calculate the area of the top and bottom bases, 36π and 81π respectively. As for the slanted surface area, we need to look at a vertical cross section of the cookie. This should look like a trapezoid. We extend the sides of the trapezoid until they create a triangle, from which can subtract to get our desired area. Since the cookie has height 4 and the difference of the bases is 3, the hypotenuse (side of the trapezoid) has length 5. Using similar triangles, this tells use that the big triangle has a length 15 hypotenuse with a base of 9. From here, we subtract the slanted areas to get $\pi(15)(9) - \pi(6)(10) = 75\pi$.
Adding it to the base areas gives a total of 192π.

13. D One could use the Shoelace Formula here, but beware the given order of the coordinates.
Another method is to draw the points and note that above the x-axis, there is a trapezoid, and below, there is a right triangle. Computing the areas, we get

$$\frac{1}{2}(5+7)(3) + \frac{1}{2}(7)(5) = \frac{71}{2}$$

14. D With perimeter as a constraint, the shape that optimizes the area is a circle, so 30 is the perimeter, giving us $r = \frac{15}{\pi}$, and area of $\pi\left(\frac{15}{\pi}\right)^2 = \frac{225}{\pi}$

15. E Let the side length of the square portion of Stanley's house be s. The height of the right triangle is $\frac{1}{2}s$, and the total height of his house is then $\frac{3}{2}s$. Therefore, $s = \frac{2}{3}(2+\sqrt{2})$. The total outer perimeter of his house can be calculated as $s(3+\sqrt{2}) = \frac{2}{3}(3+\sqrt{2})(2+\sqrt{2})$.

16. B Divide the pentagon radially into five isosceles triangles with legs of length 1. Each has a central angle of $72°$. Draw in the altitude to any of the triangles, so the central angle is halved to $36°$, but we have to draw two right triangles. Using basic trigonometry, the smaller base of each right triangle is $\sin 36°$, and there are 10 of these that form the perimeter for a total of $10 \sin 36°$.

17. B Without loss of generalization, assume that the circle has radius 1. We will compute the ratio of side lengths and then square it.
Immediately, this tells us that the smaller side length is length 1, since the circumradius of a hexagon equals its side length. Now, for the larger side length, we note that the radius of the circle is the apothem of an equilateral triangle within the large hexagon, so its side length is $\frac{2}{\sqrt{3}}$. Our desired ratio is then $\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$.

18. C We just set the formulas equal to each other and solve
$$\frac{1}{3}\pi r^2 \cdot r = \pi r^2 \to r = 3$$

19. A Using Heron's formula, we can determine the area of the triangle to be $\sqrt{9(3)(2)(4)} = 6\sqrt{6}$. Now, just need to find the circumradius, which is
$$R = \frac{(5)(6)(7)}{4(6\sqrt{6})} = \frac{35\sqrt{6}}{24}$$
The circumference is then $2\pi\left(\frac{35\sqrt{6}}{24}\right) = \frac{35\sqrt{6}}{12}$.

20. B To solve this type of problem, remember that the shortest distance between two points is always a straight line, but here it doesn't appear that this is possible since we are constrained to travel along the surface. However, if we take any two faces that Andrew travels along, we can imagine cutting them out of the prism and laying them at so we have one larger rectangle which Andrew just has to go across from corner to corner.
With this approach in mind there are only three pairs of faces that Andrew can go across, so we will compute each and compare: $\sqrt{(3+4)^2+5^2} = \sqrt{74}$, $\sqrt{(5+4)^2+3^2} = \sqrt{90}$, and $\sqrt{(5+3)^2+4^2} = \sqrt{80}$. $\sqrt{74}$ is the shortest.

21. C Using the equal tangent rule, $BB' = 5$ tells us that $BX = 5$, and similarly that $CX = CC' = 3$. Putting these together, we have $BC = 8$.
Now, since BC is tangent to the circle at X, we know that ΔXOB is right. Given $m\angle XOB = 30°$, we can determine that $m\angle XBO = 60°$, which symmetrically means that $m\angle B'BO = 60°$. Finally, we know that $m\angle ABX = 60°$, so the area is $\frac{1}{2}(8)(6)\sin 60° = 12\sqrt{3}$

22. C We need to find the surface area of the top of the milk, the outside of the cone cup, and the inside of the cone cup above the milk. First, we solve for the radius of the

milk portion of the cone as $100\pi = \frac{1}{3}(12)\pi r^2 \rightarrow r = 5$. Thus, the milk surface area is $\pi 5^2 = 25\pi$.
Next, we will find twice the outside area of the entire cone, and subtract the portion touching milk. By similar triangles, the total cone radius is 10, meaning the slant height has length 26, while the smaller slant height is half. This gives us $2\pi(10)(26) - \pi(5)(13) = 455\pi$, adding in the 25π to get 480π

23. A The information about the lengths of the cevians with respect to the intersection point is basically telling us that Q is the centroid and that UU' and TT' are medians. One notable property of medians is that they divide the triangle into 6 triangles of equal area. Quadrilateral $QU'VT'$ is made up of two of these triangles, giving it arear $2(4) = 8$.

24. D The outer circle has radius 15, while the inner circle has radius 5 (determined by using the circumference formula). From here, we are looking for the area bounded by the two circles, which is just $\pi 15^2 - \pi 5^2 = 200\pi$

25. E The volume of the boxes form a geometric series of the form $1^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{4}\right)^3 + \cdots$. The common ratio is $\frac{1}{8}$, and the entire volume is $\frac{1}{1-\frac{1}{8}} = \frac{8}{7}$.

26. C We can consider just the northern face of the building and then multiply by 4 to get the other 3 faces. They make a geometric series again, with the common ratio of $\frac{1}{4}$. So the 4 directions together has surface area $\frac{4}{1--\frac{1}{4}} = \frac{16}{3}$.
However, we can't discount the faces on the top. Looking directly down the structure, all the tops combine to just make a square with area 1, which makes the total area $\frac{19}{3}$.

27. A When we see these three equations, we are perhaps reminded of a cubic polynomial or Vieta's formulas. We know a formula that involves only side lengths of a triangle that gives us its area, Herron's Formula. If we expand this formula and group like terms, we get a formula very similar:
$$\sqrt{s(s-a)(s-n)(s-d)} = \sqrt{s(s^3 - (a+n+d)s^2 + (ad+nd+an)s - and}$$
We have all the information to compute this, as $s = \frac{d+a+n}{3}$. Plugging everything in to get the area to be $6\sqrt{2}$

28. B Regardless of the distance of the track, Connor is running at twice the speed of Carol, so when they meet, it will be at the two-thirds point around the circle, as Connor's arc will be twice Carol's. This means that they will meet at a $120°$ angle from their starting point. Now we just need to determine the length of the side opposite this angle in the triangle made up this angle, and two radii. We can break the triangle into two 30-60-90 triangles, to determine that this length is $185\sqrt{3}$

29. C $\frac{1}{3}\left(\frac{3(6^2)\sqrt{3}}{2}\right)(5) = 90\sqrt{3}$

30. E $\frac{4}{3}\pi(3^3) = 36\pi$