Theta

Circles and Polygons

Test #112

Directions:

1. Fill out the top section of the Round 1 Google Form answer sheet and select **Theta- Circles and Polygons** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

NOTA denotes "None of the Above."

1. What is the area of a regular octagon with side length 8?
    A. $128+128\sqrt{2}$ B. $64+64\sqrt{2}$ C. $192+128\sqrt{2}$ D. 256   E. NOTA

2. Find the area of the convex quadrilateral with the following vertices: (1,2). (2,7), (4,5), and (−6,1).

    A. 12   B. 23   C. 24   D. 46   E. NOTA

3. Quadrilateral WXYZ is inscribed in a circle with radius 5. If WX = 3, ZY = 5, and ZX = 10, then what is the measure of arc ZWX in degrees?
    A. 120   B. 180   C. 90   D. 60   E. NOTA

4. Find the radius of the circle that passes through the points (1, −2), (5,4), and (10,5).
    A. $\sqrt{82}$ B. $\sqrt{65}$ C. $\sqrt{29}$ D. $2\sqrt{13}$ E. NOTA

5. Two congruent circles intersect in a way such that the center of one circle lies on the other circle and vice versa. If the radius of the circles is R, then in terms of R, what is the length between the two intersection points?
    A. R   B. 2R   C. $R\sqrt{3}$ D. $2R\sqrt{3}$ E. NOTA

6. Given square WIGY and a point J inside the square such that WIJ is an equilateral triangle, what is the measure of angle GJI(in degrees)?
    A. 75   B. 90   C. 120   D.   135   E. NOTA

7. A sphere is inscribed in a cone of height 8 and radius 6. Find the volume of the sphere.

    A. $\dfrac{9\pi}{2}$ B. $32\pi\sqrt{3}$ C. 32π D. 36π E. NOTA

8. A right triangle has an area of 180 and a hypotenuse of 36. What is the sum of the lengths of the legs?

    A. 49 B. $12\sqrt{14}$ C. $6\sqrt{46}$ D. $6\sqrt{41}$ E. NOTA

9. A triangle has vertices at (0,0), (2,6), and (6,6). A circle passing through the circumcenter of this triangle and the point (8,8). Find the area of the smallest circle satisfying the conditions.

    A. 13π B. 26π C. 52π D. 104π E. NOTA

10. Triangle ABC is drawn with side BC equal to $4\sqrt{2}$. DE is drawn with D on AB, E on AC, and DE parallel to BC. If DE = 4 and ABC has an area of 48, then what is the area of ADE?

    A. $24\sqrt{2}$ B. $24\sqrt{3}$ C. 24 D. 18 E. NOTA

11. An equiangular hexagon has alternating side lengths 6 and 3. What is the area of the hexagon?

    A. $36\sqrt{3}$ B. 54 C. 36 D. $\frac{117\sqrt{3}}{4}$ E. NOTA

12. A circle is inscribed in a square. A square is then inscribed in that circle, which then has a circle inscribed in it, and so on. If the first square has an area of 36, what is the total area of all the circles?

    A. 9π B. 18π C. 27π D. 36π E. NOTA

13. If a wheel with radius 7 makes 5 revolutions a second, how far does it travel in 3 minutes?
    A. 12600π B. 210π C. 6300π D. 4200π E. NOTA

14. Let $ABCD$ be a trapezoid with $\overline{AB} \parallel \overline{CD}$ and $AB = 1$. Point P lies on $\overline{CD}$ such that the area of $\triangle ABP$ is $20/21$ the area of trapezoid $ABCD$. Find CD.
    A. 1/20   B. 1/21   C. 1/40   D. 1/42   E. NOTA

15. Points A, B, C, and D lie on a circle in that order such that $AB = 6$, $BC = 8$, and $m\angle B = 90°$. Find the maximum possible area of quadrilateral $ABCD$.
    A. 48   B. 49   C. 99/2   D. 50   E. NOTA

16. Three circles of radius 1 are pairwise externally tangent. An equilateral triangle circumscribes all three circles such that each side is tangent to two circles. Find the area of this equilateral triangle.
    A. $6 + 8\sqrt{3}$ B. $6 + 4\sqrt{3}$ C. $6 + 3\sqrt{3}$ D. $6 + 7\sqrt{3}$ E. NOTA

17. Three points are chosen uniformly at random on a circle. What is the probability that the triangle formed by connecting these points is obtuse?
    A. 1/2   B. 2/3   C. 3/4   D. 5/6   E. NOTA

18. Two circles with radii 6 and 3 are drawn so that their centers are 41 units apart. What is the length of one of the common internal tangents?
    A. 50   B. $2\sqrt{418}$ C. $\sqrt{1762}$ D. 40   E. NOTA

19. Which point does not lie on Euler's Line?
    A. Circumcenter B. Incenter   C. Centroid   D. Orthocenter E. NOTA

20. A triangle with sides 13, 14, and 15 is drawn, and a circle is inscribed in it. What is the area of the circle?

    A. 8π B. 64π C. 4π D. 16π E. NOTA

21. A circle is drawn, and a tangent and a secant to the circle are drawn from point A, which is outside of the circle. If the tangent line intersects the circle at point B, and the secant line intersects the circle at points C and D, with $AC < AD$, then what is the length of CD, given that $AB = 12$ and $AC = 9$?

    A. 3   B. 16   C. 7   D. 25   E. NOTA

22. Two concentric circles are drawn with radii 6 and 4. An external tangent line is drawn to the inner circle, and it intersects the outer circle at points Y and Z. What is the length of YZ?

    A. 4   B. $\sqrt{5}$ C. $2\sqrt{5}$ D. $4\sqrt{5}$ E. NOTA

23. Square ABCD has a side length of 8. Point E is drawn on the outside of square ABCD so that CDE is an equilateral triangle. What is the square of the length of EA?

    A. $128+64\sqrt{3}$ B. $64+32\sqrt{3}$ C. $116+64\sqrt{3}$ D. 256   E. NOTA

24. Two circles with radii 7 and 2 are drawn with their centers 12 units apart. What is the length of one of the common external tangents?

    A. $\sqrt{119}$ B. 12   C. 13   D. 15   E. NOTA

25. Right triangle ABC with a right angle at B is drawn. If the length from A to the midpoint of BC is $4\sqrt{2}$, and the length from C to the midpoint of AB is $4\sqrt{3}$, then what is the length of hypotenuse AC?

    A. $4\sqrt{5}$ B. 4   C. 8   D. 12   E. NOTA

26. A triangle with sides 10, 17, and 21 is drawn. What is the radius of the circumscribed circle of this triangle?

    A. 12   B. $\frac{85}{8}$ C. $\frac{25}{2}$ D. 10.5   E. NOTA

27. The equation of the tangent line that intersects the circle $x^2 + y^2 - 12x + 8y + 7 = 0$ at the point $(3,2)$ is in the form $Ax + By = C$, where A, B, and C are relatively prime integers with $A > 0$. Compute $A + B + C$.

    A. -2 B. 5   C. 6   D. 11   E. NOTA

28. A square is inscribed in a triangle with side lengths 3, 4, 5 so that exactly one vertex of the square lies on the hypotenuse of the triangle. What is the side length of the square?

    A. $\frac{12}{7}$ B. 4   C. 3   D. $\frac{16}{7}$ E. NOTA

29. If the sum of the diagonals of a rhombus is 9 and the sum of the squares of the diagonals is 57, what is the area of the rhombus?

    A. 6   B. 9   C. 12   D. 18   E. NOTA

30. A circle with radius r is drawn. N additional circles with radius r are drawn so that:
    - Each of the N circles is tangent to the original circle
    - Each of the N circles is tangent to two of the N circles.
    - No two of the N circles intersect at multiple points.

    What is the value of N?

    A. 3   B. 4   C. 5   D. 6   E. NOTA