2011 – 2012 Log1 Contest Round 3
Theta Individual

Name: _____

	4 points each	
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: ____ Mean: ____
2	Fred, in New York and accustomed to using the Fahrenheit temperature scale, checked his thermometer and he saw that it was 45°. He calls his friend, Sergei, in Russia and accustomed to the Celsius temperature scale, and lets him know what the temperature in New York was without saying its unit. If Sergei assumed that the temperature in New York was 45°C, what would this temperature be in degrees Fahrenheit? Unit is optional.	
3	Find the sum of the number of faces, vertices, and edges of a pyramid whose base is in the shape of a regular pentagon	
4	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	
5	What is the sum $1 + 4 + 9 + 16 + \cdots + 100$?	

	5 points each	
6	Solve for x: $512^{x+5} = 8^{x^2-3}$	
7	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	
8	Simplify the expression below. $\frac{1}{2\times 5}+\frac{1}{5\times 8}+\frac{1}{8\times 11}+\frac{1}{11\times 14}+\frac{1}{14\times 17}$	
9	If a@b means $\frac{\binom{a}{b}-\binom{3b}{a}}{a^b}$, evaluate 6@2.	
10	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	

6 points each		
11	Find the coefficient of the x^6 term in the binomial expansion of $5x\,(3x-2)^7$.	
12	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	
13	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	
14	$If\ A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	
15	How many positive factors does the expression $20^2 11^2$ have?	

2011 – 2012 Log1 Contest Round 3
Alpha Individual

Name: _____

	4 points each	
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: ____ Mean: ____
2	Fred, in New York and accustomed to using the Fahrenheit temperature scale, checked his thermometer and he saw that it was 45°. He calls his friend, Sergei, in Russia and accustomed to the Celsius temperature scale, and lets him know what the temperature in New York was without saying its unit. If Sergei assumed that the temperature in New York was 45°C, what would this temperature be in degrees Fahrenheit? Unit is optional.	
3	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	
4	Solve for x: $512^{x+5} = 8^{x^2-3}$	
5	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	

	5 points each	
6	Simplify the expression below. $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\frac{1}{11\times14}+\frac{1}{14\times17}$	
7	The symbol $\binom{x}{y}$ means the number of ways one can choose y items from x distinguishable items. What is $\binom{2}{0}+\binom{3}{1}+\binom{4}{2}+\binom{5}{3}+\binom{6}{4}$?	
8	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	
9	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	
10	Dave and Molly are playing a game where they take turns spinning a spinner. The spinner has a one-third chance of coming up "WIN" and two-thirds chance of "PASS". Dave goes first and they take turns until one of them spins "WIN". What is the probability that Dave wins?	

6 points each		
11	If $A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	
12	How many positive factors does the expression $20^2 11^2$ have?	
13	Find the number of common prime factors of 2002 and 1729.	
14	Convert this equation from polar to rectangular form. $r = 4 \sin \theta$	
15	Evaluate $\cos(75°) \times \sqrt{\sin(30°)}$	

2011 – 2012 Log1 Contest Round 3
Mu Individual

Name: _____

	4 points each	
1	Fred, in New York and accustomed to using the Fahrenheit temperature scale, checked his thermometer and he saw that it was 45°. He calls his friend, Sergei, in Russia and accustomed to the Celsius temperature scale, and lets him know what the temperature in New York was without saying its unit. If Sergei assumed that the temperature in New York was 45°C, what would this temperature be in degrees Fahrenheit? Unit is optional.	
2	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	
3	Solve for x: $512^{x+5} = 8^{x^2-3}$	
4	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	
5	Simplify the expression below. $$\frac{1}{2\times 5}+\frac{1}{5\times 8}+\frac{1}{8\times 11}+\frac{1}{11\times 14}+\frac{1}{14\times 17}$$	

	5 points each	
6	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	
7	Find the coefficient of the x^6 term in the binomial expansion of $5x\,(3x-2)^7$.	
8	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	
9	There are 5 coins in a box. 2 have a 40% chance of landing on heads and the other 3 are fair coins. If you pick two at random and flip them, what is the probability that they will both be tails? Express as a percentage.	
10	How many positive factors does the expression $20^2 11^2$ have?	

	6 points each	
11	What is the smallest positive three-digit number that has a remainder of 2 when divided by 3, a remainder of 4 when divided by 5 and a remainder of 6 when divided by 7?	
12	Convert this equation from polar to rectangular form. $$r = 4\sin\theta$$	
13	Evaluate $\cos(75°) \times \sqrt{\sin(30°)}$	
14	Given the function $7xe^{2y} = 2$, determine the value of $\frac{dy}{dx}$ when $x = 5$.	
15	Calculate the volume of an object that is produced when the function $f(x) = x^2 + 3x + 2$ on the domain $x = [0, 4]$ is revolved around the x-axis.	

2011 – 2012 Log1 Contest Round 3
Theta Individual

Name: _____

	4 points each	
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: 10 Mean: 13
2	Fred, in New York and accustomed to using the Fahrenheit temperature scale, checked his thermometer and he saw that it was 45°. He calls his friend, Sergei, in Russia and accustomed to the Celsius temperature scale, and lets him know what the temperature in New York was without saying its unit. If Sergei assumed that the temperature in New York was 45°C, what would this temperature be in degrees Fahrenheit? Unit is optional.	113
3	Find the sum of the number of faces, vertices, and edges of a pyramid whose base is in the shape of a regular pentagon	22
4	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	1
5	What is the sum $1 + 4 + 9 + 16 + \cdots + 100$?	385

	5 points each	
6	Solve for x: $512^{x+5} = 8^{x^2-3}$	{6,-3}
7	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	4:42
8	Simplify the expression below. $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\frac{1}{11\times14}+\frac{1}{14\times17}$	$\frac{5}{34}$
9	If a@b means $\frac{\binom{a}{b}-\binom{3b}{a}}{a^b}$, evaluate 6@2.	$\frac{7}{18}$
10	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	900,900

6 points each		
11	Find the coefficient of the x^6 term in the binomial expansion of $5x\,(3x-2)^7$.	102060
12	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	$\frac{1}{5}$
13	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	$\frac{19}{21}$
14	$If\ A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	$A_{22} = -87$
15	How many positive factors does the expression $20^2 11^2$ have?	45

2011 – 2012 Log1 Contest Round 3
Alpha Individual

Name: _____

	4 points each	
1	Find the median and mean for the following set of numbers: {8,10,16,3,8,6,12,4,20,13,11,14,5}	Median: 10 Mean: 13
2	Fred, in New York and accustomed to using the Fahrenheit temperature scale, checked his thermometer and he saw that it was 45°. He calls his friend, Sergei, in Russia and accustomed to the Celsius temperature scale, and lets him know what the temperature in New York was without saying its unit. If Sergei assumed that the temperature in New York was 45°C, what would this temperature be in degrees Fahrenheit? Unit is optional.	113
3	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	1
4	Solve for x: $512^{x+5} = 8^{x^2-3}$	{6,-3}
5	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	4:42

	5 points each	
6	Simplify the expression below. $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\frac{1}{11\times14}+\frac{1}{14\times17}$	$\frac{5}{34}$
7	The symbol $\binom{x}{y}$ means the number of ways one can choose y items from x distinguishable items. What is $\binom{2}{0}+\binom{3}{1}+\binom{4}{2}+\binom{5}{3}+\binom{6}{4}$?	35
8	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	900,900
9	A bag contains 2 blue, 3 red and 5 green balls. Bill wants to know what the probability will be of picking a blue ball on his second draw. Previously drawn balls are not returned to the bag.	$\frac{1}{5}$
10	Dave and Molly are playing a game where they take turns spinning a spinner. The spinner has a one-third chance of coming up "WIN" and two-thirds chance of "PASS". Dave goes first and they take turns until one of them spins "WIN". What is the probability that Dave wins?	$\frac{3}{5}$

6 points each		
11	$If\ A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$, find the value of A_{22}, the entry in the second row and second column of A.	$A_{22} = -87$
12	How many positive factors does the expression $20^2 11^2$ have?	45
13	Find the number of common prime factors of 2002 and 1729.	2
14	Convert this equation from polar to rectangular form. $r = 4\sin\theta$	$x^2 + y^2 = 4x$ or equivalent
15	Evaluate $\cos(75°) \times \sqrt{\sin(30°)}$	$\frac{\sqrt{3}-1}{4}$ or equivalent

2011 – 2012 Log1 Contest Round 3
Mu Individual

Name: _____

	4 points each	
1	Fred, in New York and accustomed to using the Fahrenheit temperature scale, checked his thermometer and he saw that it was 45°. He calls his friend, Sergei, in Russia and accustomed to the Celsius temperature scale, and lets him know what the temperature in New York was without saying its unit. If Sergei assumed that the temperature in New York was 45°C, what would this temperature be in degrees Fahrenheit? Unit is optional.	113
2	The seventh element of an arithmetic sequence is 22 and the eleventh element is 36. What is the first element?	1
3	Solve for x: $512^{x+5} = 8^{x^2-3}$	{6,-3}
4	If a digital clock starts at midnight, 12:00, and loses one second every hour, what time will the clock read in three years? Assume there are 365 days in a year.	4:42
5	Simplify the expression below. $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\frac{1}{11\times14}+\frac{1}{14\times17}$	$\frac{5}{34}$

	5 points each	
6	How many combinations of 13 light bulbs lined up in a straight row will there be if you have 4 red, 4 green, 3 yellow and 2 blue?	900,900
7	Find the coefficient of the x^6 term in the binomial expansion of $5x(3x-2)^7$.	102060
8	9 points are arranged in a 3 row by 3 column pattern. What is the probability of randomly selecting three distinct points that form a triangle?	19/21
9	There are 5 coins in a box. 2 have a 40% chance of landing on heads and the other 3 are fair coins. If you pick two at random and flip them, what is the probability that they will both be tails? Express as a percentage.	29.1%
10	How many positive factors does the expression $20^2 11^2$ have?	45

	6 points each	
11	What is the smallest positive three-digit number that has a remainder of 2 when divided by 3, a remainder of 4 when divided by 5 and a remainder of 6 when divided by 7?	104
12	Convert this equation from polar to rectangular form. $r = 4\sin\theta$	$x^2+y^2=4x$ or equivalent
13	Evaluate $\cos(75°)\times\sqrt{\sin(30°)}$	$\frac{\sqrt{3}-1}{4}$ or equivalent
14	Given the function $7xe^{2y}=2$, determine the value of $\frac{dy}{dx}$ when $x=5$.	$-\frac{1}{10}$
15	Calculate the volume of an object that is produced when the function $f(x)=x^2+3x+2$ on the domain x = [0, 4] is revolved around the x-axis.	$\frac{14672\pi}{15}$

2011 – 2012 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
	1	1	Median: 10 Mean: $130/10 = 13$ {3,4,5,6,8,8,10,11,12,13,14,16,20}
1	2	2	Convert Celsius to Fahrenheit $F = \frac{9}{5}C + 32 \;\; F = \frac{9}{5}(45) + 32 \;\; F = \frac{9}{5}(45) + 32 \;\; F = 113$
		3	This pyramid has 6 vertices. Each edge on the base pentagon corresponds to exactly one lateral face on the pyramid. Thus there are 6 faces on this pyramid (including the base). Each vertex on the base pentagon corresponds to exactly one lateral edge on the pyramid. Thus there are 5 lateral edges on the pyramid and 5 edges on the base pentagon. The sum is 22.
2	3	4	The difference between the 11th and 7th elements is 4 times the common difference. The 7th and 1st elements differ by 6 times the common difference. The 1st element is then 22 – (6/4)(36-22) = 22-21 = 1
		5	The formula is n(n+1)(2n+1)/6 = (10)(11)21/6 = 385.
3	4	6	$512^{x+5} = 8^{x^2-3} = (2^9)^{x+5} = (2^3)^{x^2-3}$ $9(x+5) = 3(x^2-3)$ $3(x+5) = (x^2-3)$ $3x + 15 = x^2 - 3$ $0 = x^2 - 3x - 18$ $0 = (x-6)(x+3)$ $x = \{6, -3\}$
4	5	7	There are 3*365*24 = 26280 hours in exactly three year. Thus, 26280 seconds are lost. This is equivalent to 438 minutes or 7 hours, 18 minutes. Exactly three years later, instead of reading midnight, the clock will be slow by this amount, equivalent to 4:42.
5	6	8	Knowing that $\frac{C}{AB} = \left(\frac{C}{B-A}\right) x \left(\frac{1}{A} - \frac{1}{B}\right)$ $\left(\frac{1}{3}\right) x \left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{3}\right) x \left(\frac{1}{5} - \frac{1}{8}\right) + \cdots + \left(\frac{1}{3}\right) x \left(\frac{1}{14} - \frac{1}{17}\right)$ Expanding this out, only the first and last terms remain. All the middle terms cancel out. Therefore, $\left(\frac{1}{3}\right) x \left(\frac{1}{2} - \frac{1}{17}\right) \rightarrow \frac{1}{3}\left(\frac{15}{34}\right) \rightarrow \frac{5}{34}$
		9	Evaluate. $\frac{\binom{6}{2} - \binom{3x2}{6}}{6^2} = \frac{15-1}{36} = \frac{14}{36} = \frac{7}{18}$
	7		One can evaluate each combination separately but there is a formula sometimes referred to as "hockey-stick" which has this sum equal $\binom{7}{4} = 35$. In other words, $\binom{n}{r} = \sum_{j=o}^{r} \binom{n+j-r-1}{j}$
6	8	10	13! / (4! 4! 3! 2!) = 900,900

7		11	For the factor $(3x-2)^7$, the coefficient of the x^5 term is $3^5(-2)^2\binom{7}{5} = 243(4)(21) = 20412$. This number is then multiplied by 5 to attain the final result of 102060
	9	12	$\lfloor P(R1 or G1)\ AND\ P(B2) \rfloor\ or\ \lfloor P(B1) AND\ P(B2) \rfloor$ $\left\lfloor \frac{8}{10} x \frac{2}{9} \right\rfloor + \left\lfloor \frac{2}{10} x \frac{1}{9} \right\rfloor = \frac{18}{90} = \frac{1}{5}$
8		13	There are $\binom{9}{3} = 84$ ways to choose 3 points out of a 3x3 lattice. Of these 84 points, the only possible combinations that CANNOT form a triangle are those that are collinear. These would be the 3 columns, 3 rows and 2 diagonal sets of points. Thus there are 76 possible combinations that can form a triangle. The probability is $\frac{76}{84} = \frac{19}{21}$
	10		In the first turn, Dave has a $1/3$ chance of winning. In order for Molly to win, Dave must pass and then Molly "win" with probability $(2/3)(1/3) = 2/9$. There is a $5/9$ chance of someone winning in the first round and $4/9$ chance that it goes to round two. One can treat this as an infinite sequence or note that Dave's probability of winning is $(1/3)/(5/9) = (3/5)$, so the desired chance is 3/5.
9			Create a choice table: H 50 50 50 40 40 T 50 50 50 60 60 Define the following probabilities: P(TT) = probability of throwing two tails regardless of the type of coin. $P(T_{50}T_{50})$ = probability of drawing 2 regular coins AND throwing two tails. $\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{40}$ $P(T_{60}T_{60})$ = probability of drawing 2 biased coins AND throwing two tails. $\left(\frac{2}{5}\right)\left(\frac{1}{4}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{9}{250}$ $P(T_{50}T_{60})$ = probability of drawing a biased and a regular coin AND throwing two tails. $\left(\frac{6}{10}\right)\left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{9}{50}$ There are 6 out of 10 possible ways to draw a biased and a regular coin. $P(TT) = P(T_{50}T_{50}) + P(T_{60}T_{60}) + P(T_{50}T_{60})$ $P(TT) = \frac{3}{40} + \frac{9}{250} + \frac{9}{50} = \frac{75+36+180}{1000} = \frac{291}{1000}$
	11	*14*	$Let\ C = \begin{bmatrix} 3 & -1 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -6 & -8 \end{bmatrix}$ $Then\ C = \begin{bmatrix} 3(0)-1(-6) & 3(4)-1(-8) \\ -2(0)+9(-6) & -2(4)+9(-8) \end{bmatrix}$ $Thus\ C = \begin{bmatrix} 6 & 20 \\ -54 & -80 \end{bmatrix}$ Evaluating $A = \begin{bmatrix} 2 & -5 \\ 4 & -7 \end{bmatrix} + \begin{bmatrix} 6 & 20 \\ -54 & -80 \end{bmatrix} = \begin{bmatrix} 8 & 15 \\ -50 & -87 \end{bmatrix}$ For this problem, one really only needs: $-7 + (-2)(4) + 9(-8) = -87$
10	12	15	In terms of prime factors $20^2 11^2 = 2^4 5^2 11^2$. Therefore, the total number of factors is $(4+1)(2+1)(2+1) = 45$
	13		One can find the greatest common divisor using the Euler method. 2002=1729(1) + 273, 1729=273(6)+91 and 273=91(3). This means the GCF=91 with prime factors of 7 and 13.
11			If 1 is added to the number, it is divisible by 3, 5 and 7 or 3*5*7=105. Therefore the number is 105-1 = 104.

12	14		Since $r^2 = x^2 + y^2$ and $r = 4\,Sin\theta$, if we let $x = r\,Sin\,\theta$, then $r^2 = 4r\,Sin\theta = 4x$. Thus, $x^2 + y^2 = 4x$ Other possibilities $(x-2)^2 + y^2 = 4$ or $x^2 - 4x + y^2 = 0$
13	15		The answer will depend on whether a sum formula or half angle formula is used to evaluate cos(75). I. $\cos(75) = \cos(45+30) = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$, so the answer is $\frac{\sqrt{6}-\sqrt{2}}{4}\frac{\sqrt{2}}{2} = \frac{\sqrt{3}-1}{4}$ II. Let $A = \cos(75)\sqrt{\sin(30)}$ $A = \sin(15)\sqrt{\sin(30)} = \sqrt{\frac{1-\cos(30)}{2}}\sqrt{\frac{1}{2}}$ Therefore, $A = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{4}} = \sqrt{\frac{2-\sqrt{3}}{8}} = \frac{1}{2}\sqrt{\frac{2-\sqrt{3}}{2}} = \frac{1}{2}\sqrt{1-\frac{\sqrt{3}}{2}}$
14			Implicitly differentiating, $\frac{d}{dx}(7xe^{2y}) = \frac{d}{dx}(2)$ $7xe^{2y}(2)\frac{dy}{dx} + e^{2y}(7) = 0$ $14xe^{2y}\frac{dy}{dx} = -7e^{2y}$ $14x\frac{dy}{dx} = -7$ $\frac{dy}{dx} = -\frac{7}{14x} = -\frac{7}{14(5)} = -\frac{1}{10}$
15			Using the disc method, $V = \int_0^4 \pi f(x)^2 dx$ $V = \int_0^4 \pi\,(x^2+3x+2)^2 dx$ $V = \pi\int_0^4 (x^4+6x^3+13x^2+12x+4)\,dx$ $V = \pi\left[\frac{1}{5}x^5 - \frac{3}{2}x^4 + \frac{13}{3}x^3 + 6x^2 + 4x \;\Big|_0^4\right]$ $V = \pi\left[\frac{1024}{5} + \frac{768}{2} + \frac{832}{3} + \frac{192}{2} + \frac{32}{2}\right]$ $V = \pi\left[\frac{6144+11520+8320+2880+480}{30}\right]$ $V = \pi\left[\frac{29344}{30}\right]$ $V = \pi\left[\frac{14672}{15}\right]$