Theta

Triangles

Test #111

Directions:

1. Fill out the top section of the Round 1 Google Form answer sheet and select **Theta-Triangles** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select “E) NOTA” and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like “which of the following could be” or “what is one solution of”, an answer choice providing one of the possibilities is considered to be correct. Do not select “E) NOTA” in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select “E) NOTA” in that instance.

10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

The answer choice E. NOTA denotes that 'none of these answers' are correct. The notation $\overline{AB}$ denotes a segment with endpoints A and B, while the notation AB denotes the length of such a segment unless otherwise specified. Diagrams *are* to scale. Problems are not necessarily in increasing order of difficulty, so don't be afraid to skip around. Good luck and have fun!

1. Legosi has a triangle with two sides of lengths 20 and 22. Find the number of possible integer lengths for the third side of his triangle.
   A. 41 B. 39 C. 42 D. 40 E. NOTA

2. Find the area of a triangle with vertices at $(0,0)$, $(20,0)$, and $(20,22)$.
   A. 220 B. 200 C. 242 D. 440 E. NOTA

3. Which of the following describes a triangle with side lengths 5, 6, and 8?
   A. Obtuse B. Right C. Acute D. Degenerate E. NOTA

4. A triangle with side lengths 4, 8, and x has area 6. Find the area of a triangle with side lengths 16, 8, and $2x$.
   A. 24 B. 48 C. 6 D. 12 E. NOTA

5. Suppose the degree measures of the interior angles of a triangle form an arithmetic sequence. What is the middle term of this sequence?
   A. $30°$ B. $90°$ C. $45°$ D. $60°$ E. NOTA

6. Find the area of an equilateral triangle with side length 6.
   A. $6\sqrt{3}$ B. $12\sqrt{3}$ C. $18\sqrt{3}$ D. $9\sqrt{3}$ E. NOTA

7. Triangle ABC has $AB = 4$, $BC = 5$, and $AC = 6$. What is the distance between the midpoint of $\overline{AB}$ and the midpoint of $\overline{AC}$?
   A. 3 B. 2 C. $5/2$ D. $3/2$ E. NOTA

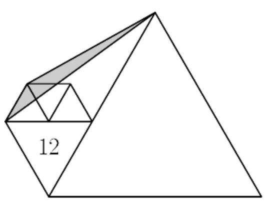
8. The measures of the exterior angles of a triangle are in the ratio $4 : 5 : 6$. What is the ratio of the measures of their corresponding interior angles?
   A. $11 : 10 : 9$ B. $7 : 5 : 3$ C. $6 : 5 : 4$ D. $5 : 4 : 3$ E. NOTA

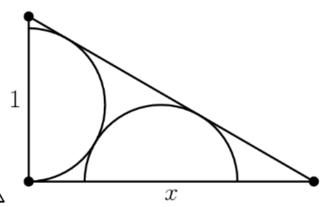
9. Triangle ABC has $AB = 10$, $BC = 5$, and $\sin(A) = 2/5$. Find the positive difference between the two possible lengths of $\overline{AC}$.
A. 8 B. 6 C. $2\sqrt{21}$ D. 4 E. NOTA

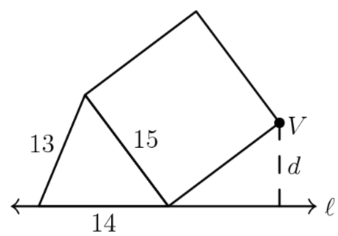
10. Triangle ABC has $m\angle B = 90°$ and $\tan(A) = \sqrt{2}$. Compute $\sin(C)$.
A. $1/3$ B. $\sqrt{2}/3$ C. $\sqrt{3}/3$ D. $\sqrt{6}/3$ E. NOTA

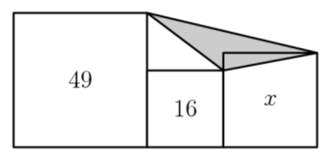
11. Equilateral triangle ABC has points D, E, and F on $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ respectively such that D is $1/3$ of the way from A to B, E is $1/3$ of the way from B to C, and F is $1/3$ of the way from C to A. What is the ratio of the area of ΔDEF to the area of ΔABC?
A. $\frac{1}{9}$ B. $\frac{2}{5}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. NOTA

Use the following four diagrams for problems 12-15, from left to right.









12. Five equilateral triangles are oriented as shown in the diagram. One of the triangles is labeled with its area. What is the area of the shaded triangle?
A. 9 B. 6 C. 12 D. 3 E. NOTA

13. Two semicircles of equal radius are inscribed in a right triangle as shown in the diagram. The lengths of the legs of the right triangle are labeled. Find x.
A. $5/3$ B. $\sqrt{2}$ C. $\sqrt{3}$ D. $3/2$ E. NOTA

14. A square is leaning up against a 13-14-15 triangle, as shown in the diagram. Find the distance d from vertex V of the square to line ℓ, which contains the side of length 14 of the triangle.
A. 12 B. 9 C. 10 D. 8 E. NOTA

15. Three squares are oriented as shown in the diagram, and each square is labeled with its area. If the shaded triangle is isosceles, find x.
A. $\frac{25}{2} + 2\sqrt{34}$ B. $\frac{3}{2} + \frac{\sqrt{41}}{2}$ C. $\frac{25}{2} + \frac{3\sqrt{41}}{2}$ D. $2 + \frac{\sqrt{34}}{2}$ E. NOTA

16. Find the area of the triangle bounded by the lines $y = x$, $y = -x$, and $y = 2x - 6$.
A. 6 B. 24 C. 3 D. 12 E. NOTA

17. In triangle ABC, let X and Y denote the midpoints of $\overline{AB}$ and $\overline{AC}$. If the area of quadrilateral $BXYC$ is 30, find the area of triangle ABC.
A. 45 B. 60 C. 40 D. 50 E. NOTA

18. Let $ABCD$ be a trapezoid with $\overline{AB} \parallel \overline{CD}$ and $AB = 1$. Point P lies on $\overline{CD}$ such that the area of ΔABP is $20/21$ the area of trapezoid $ABCD$. Find CD.
A. $1/20$ B. $1/21$ C. $1/40$ D. $1/42$ E. NOTA

19. Triangle ABC has vertices $A(-2,6)$, $B(3,5)$, $C(0,-4)$ in the coordinate plane. Let $X = (10,6)$, and let D, E, and F be the midpoints of $\overline{AX}$, $\overline{BX}$, and $\overline{CX}$, respectively. Find the area of ΔDEF.
A. 24 B. 3 C. 12 D. 6 E. NOTA

20. Points A, B, C, and D lie on a circle in that order such that $AB = 6$, $BC = 8$, and $m\angle B = 90°$. Find the maximum possible area of quadrilateral $ABCD$.
A. 48 B. 49 C. $99/2$ D. 50 E. NOTA

21. Rectangle $ABCD$ has $AB = 3$ and $BC = 4$. Find the distance between the incenters of ΔABC and ΔADC.
A. $\sqrt{13}$ B. $\sqrt{10}$ C. 2 D. $\sqrt{5}$ E. NOTA

22. An acute triangle has two sides of length 7 and 10 and an area of 28. Find the length of its third side.
A. $\sqrt{93}$ B. $\sqrt{37}$ C. $\sqrt{65}$ D. $\sqrt{107}$ E. NOTA

23. Triangle ABC has $AB = 5, BC = 8$, and $m\angle B = 40°$. Let I be the incenter of ΔABC. Find the measure of $\angle AIC$.
A. $120°$ B. $100°$ C. $130°$ D. $110°$ E. NOTA

24. Three circles of radius 1 are pairwise externally tangent. An equilateral triangle circumscribes all three circles such that each side is tangent to two circles. Find the area of this equilateral triangle.
A. $6+8\sqrt{3}$ B. $6+4\sqrt{3}$ C. $6+3\sqrt{3}$ D. $6+7\sqrt{3}$ E. NOTA

25. Equilateral triangle ABC has points D, E, and F on $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ respectively such that D is $1/3$ of the way from A to B, E is $1/3$ of the way from B to C, and F is $1/3$ of the way from C to A. What is the ratio of the area of ΔDEF to the area of ΔABC?
A. $1/9$ B. $2/5$ C. $1/3$ D. $1/2$ E. NOTA

26. How many distinct triangles have angles with degree measures that are all multiples of 10? Here we consider triangles to be 'not distinct' if and only if they are similar.
A. 27 B. 20 C. 66 D. 36 E. NOTA

27. Triangle ABC has $m\angle A = 50°$, $m\angle B = 60°$, and $m\angle C = 70°$. Let D lie on $\overline{BC}$ such that $\angle BAD \cong \angle CAD$. If line AD intersects the perpendicular bisector of $\overline{BC}$ at P, find the measure of $\angle BPC$.
A. $140°$ B. $120°$ C. $130°$ D. $110°$ E. NOTA

28. Three points are chosen uniformly at random on a circle. What is the probability that the triangle formed by connecting these points is obtuse?
A. $1/2$ B. $2/3$ C. $3/4$ D. $5/6$ E. NOTA

29. Triangle ABC has $AB = 13$, $BC = 14$, and $AC = 15$. A square $PQRS$ in inscribed inside ΔABC such that P and Q are on $\overline{BC}$, R is on $\overline{AC}$, and S is on $\overline{AB}$. Find the side length of square $PQRS$.
A. $84/13$ B. $72/13$ C. $98/13$ D. $96/13$ E. NOTA

30. Right triangle ABC has $AB = 3$, $BC = 4$, and $AC = 5$. The altitude to the hypotenuse of ΔABC has length S_1 and splits ΔABC into two smaller right triangles. The altitudes to the hypotenuses of these two triangles have a sum of lengths S_2 and further divide ΔABC into four smaller right triangles. The altitudes to the hypotenuses of these four triangles have a sum of lengths S_3 and further divide ΔABC into eight right triangles. S_4, S_5, and so on are defined by continuing this process. Compute S_{20}/S_{22}.
A. $7/5$ B. $5/7$ C. $49/25$ D. $25/49$ E. NOTA