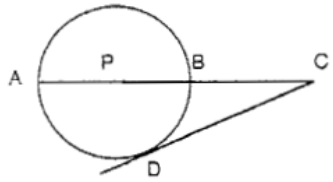
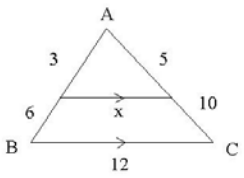
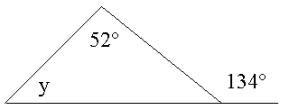
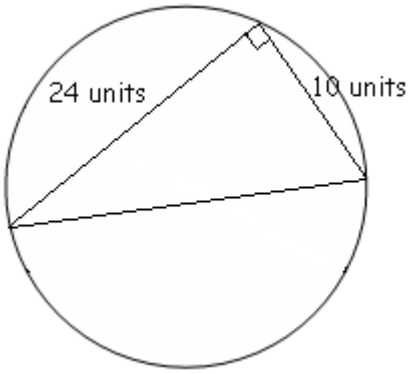
**2008 – 2009 Log1 Contest Round 2**
**Theta Geometry**

Name: _____

Leave answers in terms of $\pi$. Non-integer rational numbers should be given as a reduced fraction. Units are not needed.

| **4 points each** | | |
|---|---|---|
| 1 | What is the perimeter of a square with a diagonal of $\sqrt{2}$ units? | |
| 2 | A trapezoid has an area of 36 square units, a height of 8 units and a base of 3 units. Find the length of the remaining base. | |
| 3 | Cube A has a volume of 343 cubic units. Cube B has a volume of 27 cubic units. Cube C has a volume of 64 cubic units. Cube D has a volume of 125 cubic units. Find the sum of the surface areas of Cubes A, B, C, and D. | |
| 4 | Triangle ABC is given such that $\overline{AB}$ and $\overline{BC}$ have lengths of 4 units. If $\angle CAB$ has an angle measure of $54°$, what is the measure, in degrees, of $\angle ABC$? | |
| 5 | What is the y-coordinate of the y-intercept of $f(x) = (x-24)(x+9)$? | |

| **5 points each** | | |
|---|---|---|
| 6 | Determine the sum of the interior angles of a regular dodecagon in degrees. | |
| 7 | $\overline{CD}$ is tangent to circle P as shown. $\overline{AB}$ is a diameter of circle P. CD=6, BC=4. Find the radius of circle P.   | |
| 8 | A pyramid is constructed with a square base and all edges equal to 2. What is the volume of the pyramid? | |
| 9 | Find the value of $xy$, disregarding units.     | |
| 10 | A ship starts at point A and travels 39 km North. It then goes 52 km West and stops. How far, in km, is the ship from point A? | |

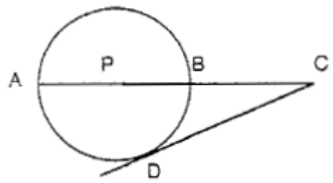
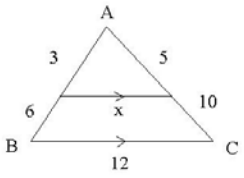
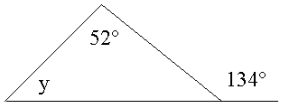
| 6 points each | | |
|---|---|---|
| 11 | How many diagonals can be placed in a convex octagon? | |
| 12 | A circle of radius 1 is tangent to two perpendicular lines. What is the radius of the largest circle that can be placed between the circle and corner and tangent to the lines and the larger circle? | |
| 13 | $\left\{\begin{matrix} x \geq 4 \\ y \geq -5 \\ y \leq -x+7 \end{matrix}\right\}$<br><br>Determine the area of the shape enclosed by these restrictions. | |
| 14 | If the volume of a sphere is doubled, what is the ratio of the new radius to the old one in the form A:B? | |
| 15 | What is the area of this circle?<br>  <br> | |

**2008 – 2009 Log1 Contest Round 2**
**Alpha Geometry**

Name: _____

Leave answers in terms of $\pi$. Non-integer rational numbers should be given as a reduced fraction. Units are not needed.

| **4 points each** | | |
|---|---|---|
| 1 | What is the perimeter of a square with a diagonal of $\sqrt{2}$ units? | |
| 2 | A trapezoid has an area of 36 square units, a height of 8 units and a base of 3 units. Find the length of the remaining base. | |
| 3 | Cube A has a volume of 343 cubic units. Cube B has a volume of 27 cubic units. Cube C has a volume of 64 cubic units. Cube D has a volume of 125 cubic units. Find the sum of the surface areas of Cubes A, B, C, and D. | |
| 4 | Triangle ABC is given such that $\overline{AB}$ and $\overline{BC}$ have lengths of 4 units. If $\angle CAB$ has an angle measure of $54°$, what is the measure, in degrees, of $\angle ABC$? | |
| 5 | What is the area, in square centimeters, of a regular octagon with sides of length 10 centimeters? | |

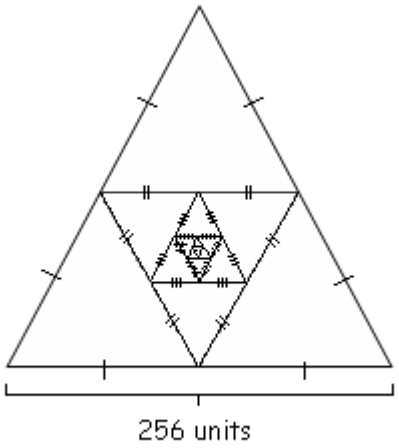
| **5 points each** | | |
|---|---|---|
| 6 | Determine the sum of the interior angles of a regular dodecagon in degrees. | |
| 7 | $\overline{CD}$ is tangent to circle P as shown. $\overline{AB}$ is a diameter of circle P. CD=6, BC=4. Find the radius of circle P.   | |
| 8 | A pyramid is constructed with a square base and all edges equal to 2. What is the volume of the pyramid? | |
| 9 | Find the value of $xy$, disregarding units.     | |
| 10 | What is the radius of the following circle? $4x^2 + 4y^2 - 16x - 64y = 52$ | |

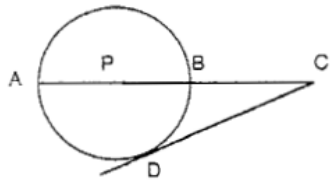
| 6 points each | | |
|---|---|---|
| 11 | How many diagonals can be placed in a convex octagon? | |
| 12 | A circle of radius 1 is tangent to two perpendicular lines. What is the radius of the largest circle that can be placed between the circle and corner and tangent to the lines and the larger circle? | |
| 13 | $\left\{\begin{matrix} x \geq 4 \\ y \geq -5 \\ y \leq -x+7 \end{matrix}\right\}$<br><br>Determine the area of the shape enclosed by these restrictions. | |
| 14 | If the volume of a sphere is doubled, what is the ratio of the new radius to the old one in the form A:B? | |
| 15 | A circle is completely divided into n distinct sectors. The angles of these sectors form an arithmetic progression. If the smallest angle is $12^\circ$, and the largest is $68^\circ$, determine the value of n. | |

**2008 – 2009 Log1 Contest Round 2**
**Mu Geometry**

Name: _____

Leave answers in terms of $\pi$. Non-integer rational numbers should be given as a reduced fraction. Units are not needed.

| **4 points each** | | |
|---|---|---|
| 1 | What is the perimeter of a square with a diagonal of $\sqrt{2}$ units? | |
| 2 | A trapezoid has an area of 36 square units, a height of 8 units and a base of 3 units. Find the length of the remaining base. | |
| 3 | Cube A has a volume of 343 cubic units. Cube B has a volume of 27 cubic units. Cube C has a volume of 64 cubic units. Cube D has a volume of 125 cubic units. Find the sum of the surface areas of Cubes A, B, C, and D. | |
| 4 | If the pattern continues, what is the sum of all of the perimeters as the number of triangles approaches ∞?<br>  <br> | |
| 5 | What is the area, in square centimeters, of a regular octagon with sides of length 10 centimeters? | |

| **5 points each** | | |
|---|---|---|
| 6 | Determine the sum of the interior angles of a regular dodecagon in degrees. | |
| 7 | $\overline{CD}$ is tangent to circle P as shown. $\overline{AB}$ is a diameter of circle P. CD=6, BC=4. Find the radius of circle P.<br>  <br> | |
| 8 | A pyramid is constructed with a square base and all edges equal to 2. What is the volume of the pyramid? | |
| 9 | What is the volume of when the region between $f(x) = x$ and $g(x) = x^2$ is rotated around the x-axis? | |
| 10 | What is the radius of the following circle? $4x^2 + 4y^2 - 16x - 64y = 52$ | |

| 6 points each | | |
|---|---|---|
| 11 | How many diagonals can be placed in a convex octagon? | |
| 12 | A circle of radius 1 is tangent to two perpendicular lines. What is the radius of the largest circle that can be placed between the circle and corner and tangent to the lines and the larger circle? | |
| 13 | $\left\{\begin{matrix} x \geq 4 \\ y \geq -5 \\ y \leq -x+7 \end{matrix}\right\}$<br><br>Determine the area of the shape enclosed by these restrictions. | |
| 14 | A ladder is leaning on the side of a house with the foot 7 feet from the wall and the top 24 feet from the ground. The ladder starts to slip and the bottom moves away from the wall at a constant rate of 6 inches/second. At what rate, in radians per second, is the angle between the ladder and the wall changing when the bottom is 20 feet from the wall? | |
| 15 | A circle is completely divided into n distinct sectors. The angles of these sectors form an arithmetic progression. If the smallest angle is $12^\circ$, and the largest is $68^\circ$, determine the value of n. | |

**2008 – 2009 Log1 Contest Round 2**
**Geometry Answers**

[ ] represent optional units

| Theta Answers | |
|---|---|
| 1 | 4 [units] |
| 2 | 6 [units] |
| 3 | 594 [$units^2$] |
| 4 | $\angle ABC$ = 72[°] |
| 5 | -216 |
| 6 | 1800[°] |
| 7 | Radius of circle P = 2.5 |
| 8 | $\frac{4\sqrt{2}}{3}$ |
| 9 | 328 |
| 10 | 65 km |
| 11 | 20 |
| 12 | $3-2\sqrt{2}$ |
| 13 | 32 [$units^2$] |
| 14 | $\sqrt[3]{2}:1$ |
| 15 | $169\pi$ [$units^2$] |

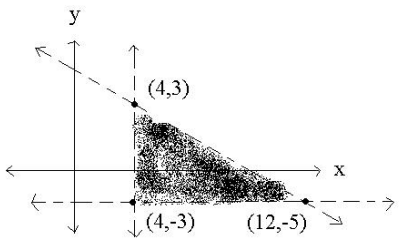
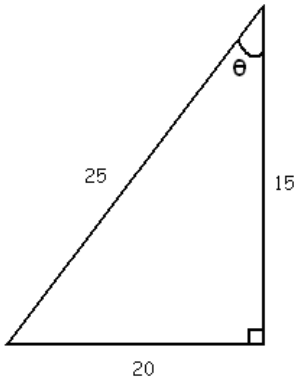
| Alpha Answers | |
|---|---|
| 1 | 4 [units] |
| 2 | 6 [units] |
| 3 | 594 [$units^2$] |
| 4 | $\angle ABC$ = 72[°] |
| 5 | $200+200\sqrt{2}$ [$cm^2$] |
| 6 | 1800[°] |
| 7 | Radius of circle P = 2.5 |
| 8 | $\frac{4\sqrt{2}}{3}$ |
| 9 | 328 |
| 10 | 9 |
| 11 | 20 |
| 12 | $3-2\sqrt{2}$ |
| 13 | 32 [$units^2$] |
| 14 | $\sqrt[3]{2}:1$ |
| 15 | 9 |

| Mu Answers | |
|---|---|
| 1 | 4 [units] |
| 2 | 6 [units] |
| 3 | 594 [$units^2$] |
| 4 | 1536 [units] |
| 5 | $200+200\sqrt{2}$ [$cm^2$] |
| 6 | 1800[°] |
| 7 | Radius of circle P = 2.5 |
| 8 | $\frac{4\sqrt{2}}{3}$ |
| 9 | $\frac{2}{15}\pi$ |
| 10 | 9 |
| 11 | 20 |
| 12 | $3-2\sqrt{2}$ or $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ |
| 13 | 32 [$units^2$] |
| 14 | $\frac{1}{30}$ [radians/ sec] |
| 15 | 9 |

**2007 – 2008 Log1 Contest Round 2**
**Applications Solutions**

| Th | Al | Mu | Solution |
|---|---|---|---|
| 1 | 1 | 1 | The diagonal of a square is equal to $x\sqrt{2}$ units, where x equals the side length of the square. Thus the side length is 1 unit, making the perimeter 4 units. |
| 2 | 2 | 2 | $A = \frac{1}{2}h(b_1 + b_2)$<br>$36 = \frac{1}{2}(8)(3 + b_2)$<br>$b_2 = 6$ |
| 3 | 3 | 3 | Surface area of Cube<br>$A = 6(7)^2 = 294$<br>$B = 6(3)^2 = 54$<br>$C = 6(4)^2 = 96$<br>$D = 6(5)^2 = 150$<br>$\therefore 294 + 54 + 96 + 150 = 594$ |
| 4 | 4 | | $x = 180 - 2(54)$<br>$x = 180 - 108$<br>$x = 72^\circ$ |
| | | 4 | The individual perimeters of each triangle form a geometric sequence:<br>$3(256 + 128 + 64 + 32 + 16 + 8 + ....)$<br>$= 3(\frac{t_1}{1 - r})$<br>$= 3(\frac{256}{1 - \frac{1}{2}})$<br>$= 3(512)$<br>$= 1536$ |
| 5 | | | To find the y-intercept, evaluate when x=0.<br>= (-24)(9)<br>= -216 |
| | 5 | 5 | Sketch the octagon and extend the sides to form a square. The area of the octagon is the area of the square minus the area of the four resultant triangles.<br>$A = (10 + 10\sqrt{2})^2 - 2 \bullet (5\sqrt{2})^2 = 200 + 200\sqrt{2}$ |
| 6 | 6 | 6 | $S = 180(n - x)$<br>$S = 180(12 - 2)$<br>$X = 1800$ |
| 7 | 7 | 7 | $(CD)^2 = (AC) \bullet (BC)$<br>$6^2 = (AC) \bullet 4$<br>$AC = 9$<br><br>$\frac{9 - 4}{2} = 2.5$ |
| 8 | 8 | 8 | The volume of a pyramid is one-third the product of the area of the base times the height. The area of the base is 4. The height can be found by dropping a perpendicular from the top to the base, from there perpendicular to one side of the base and back the top. The hypotenuse is the height of an equilateral triangle with side 2. This is $\sqrt{3}$. The height of the pyramid is then $\sqrt{2}$. The volume is $\frac{1}{3}4\sqrt{2}$. |

| | | | |
|---|---|---|---|
| 9 | 9 | | Set up a proportion to solve for x.<br>$\frac{3}{x}=\frac{9}{12}$ $y=180-52-46$ $xy=4\bullet 82$<br>$x=4$ $y=82^\circ$ $xy=328$ |
| | | 9 | $y_1=x$<br>$y_2=x^2$<br>$V=\pi\int_0^1(y_1^2-y_2^2)dx$<br>$=\pi\int_0^1(x^2-x^4)dx$<br>$=\pi(\frac{1}{3}x^3-\frac{1}{5}x^5)\Big\vert_0^1$<br>$=\pi(\frac{1}{3}-\frac{1}{5})$<br>$=\frac{2}{15}\pi$ |
| 10 | | | The distances form a 3-4-5 right triangle with legs of 3(13) and 4(13), so the hypotenuse/distance from point A is 5(13), or 65. |
| | 10 | 10 | Complete the square:<br>$x^2+y^2-4x-16y=13$<br>$x^2-4x+4+y^2-16y+64=13+4+64=81=9^2$ |
| 11 | 11 | 11 | Number of diagonals:<br>$=\frac{n(n-3)}{2}$<br>$=\frac{8(5)}{2}$<br>$=20$ |
| 12 | 12 | 12 | From the center of the larger circle drop a perpendicular to one of the lines and to the center of the smaller circle. Draw a perpendicular from the center of the smaller circle to the first perpendicular. Call the radius of the smaller circle, x. One gets a right triangle with two sides equal to 1-x and a hypotenuse of 1+x.<br>$(1-x)^2+(1-x)^2=(1+x)^2$<br>$x^2-6x+1=0$<br>$x=3\pm 2\sqrt{2}$<br>Only the smaller value is between 0 and 1.<br>Or consider the line drawn from the center of the larger circle (1,1) through the center of the small circle (x,x) to the origin (intersection of the lines). The whole length is $\sqrt{2}$ but it can be divided into the radius of the larger circle 1, smaller circle x, and from the center of the smaller circle to the origin $x\sqrt{2}$ . So $\sqrt{2}=1+x+x\sqrt{2}$ . |

| | | | |
|---|---|---|---|
| 13 | 13 | 13 |  <br>$A = \frac{1}{2}bh$<br>$A = \frac{1}{2}(8)(8)$<br>$A = 32$ |
| 14 | 14 | | Volume of Sphere: $\frac{4\pi}{3}r^3$<br>Double of Sphere: $\frac{8\pi}{3}r^3$<br>Radius of Sphere: r<br>Radius of Doubled Sphere: R<br>$\frac{4\pi}{3}R^3 = \frac{8\pi}{3}r^3$<br>$R^3 = 2r^3$<br>$\frac{R^3}{r^3} = 2$<br>$\frac{R}{r} = \sqrt[3]{2}$<br>$R : r = \sqrt[3]{2} : 1$ |
| | | 14 |  <br>$\sin\theta = \frac{x}{25}$<br>$\cos\theta\frac{d\theta}{dt} = \frac{1}{25}\frac{dx}{dt}$<br>$\cos\theta\frac{d\theta}{dt} = \frac{1}{50}$<br>$\frac{15}{25}\frac{d\theta}{dt} = \frac{1}{50}$<br>$\frac{d\theta}{dt} = \frac{1}{30}$ |

| | | | |
|---|---|---|---|
| 15 | | | An inscribed right triangle's hypotenuse is always the length of the diameter of the circle. Also the triangle is a 5-12-13 right triangle, albeit by a factor of two.<br>$r = \frac{hypotenuse}{2}$<br>$= \frac{13(2)}{2}$<br>$= 13$<br>$A = \pi r^2$<br>$= 169\pi$ |
| | 15 | 15 | $\sum total = n \bullet average = n \bullet \frac{first + last}{2}$<br>$n = \frac{360° \bullet 2}{12° + 68°} = 9$ |