

How Can Mathematics Help - mathematics at work -

“What is the real cost of a speeding ticket?”, “How can a computer recognize a person’s voice?”, “How can an insurance company use loss data to identify possible fraud?”. These are some of the math questions the participants in the Mathematics in Industry Institute for Teachers (MIIT) at Worcester Polytechnic Institute will ask their students this fall. The message to the students is that advanced mathematics can be used to develop and improve products from cars and airplanes to inkjet printers, or can provide a better understanding of the stock market or insurance.

Mathematicians, who work in industry, came to the Institute to give examples of the exciting math they use and the new research areas that were motivated by real-world applications. Bob LaBarre, principal mathematician at the United Technologies Research Center talked about the necessity to have students “problem solvers” instead of “problem doers.” Solvers understand why the solution is right and they can explain why the solution works. Derek Kane described his work at DEKA Research and Development on projects ranging from the iBot, to the new Segway human transporter, to sensor design, to water purification systems. He argued that the best person to explore new areas for a company is one trained in mathematics – mathematicians know how to learn. Bruce Kearnan from John Hancock Life Insurance described the working actuary as an “insurance engineer.” Actuaries design and build financial and insurance instruments. He argued that the ability to explain mathematics to the customer is one key to success in industrial math.

Sheila Tobias, a strong advocate for “mathematical mental health” as the ability to learn the math you need, when you need it focused on the stage of “creative floundering” that is crucial in any real-world application. She helped the teachers understand the process needed to overcome the panic that comes with open-ended real-world problems.

More than forty teachers worked for five days to develop versions of real industrial projects that will be used in their own classrooms. An abridged version of one project that focuses on decision making in the automobile insurance business is attached here. A much longer version, as well as additional projects, are available on the web at www.wpi.edu/+CIMS/teachers.

The Institute, organized by the Center for Industrial Mathematics and Statistics (CIMS) at WPI, is supported for the next three years by the GE Fund as part of its *Math Excellence* initiative. The GE Fund’s main goal is to promote a stronger, more diverse pipeline in quantitative disciplines, including mathematics and engineering. GE believes that real applications in math classes will engage students, especially women and minority students, so that they are ready and able to study math and science and engineering in college.

For more information about the Institute see www.wpi.edu/+CIMS/teachers .

You're in the Driver's Seat

The Problem: You are provided with some sample data obtained from First Stage Insurance Agency. Among the information given is SDIP (Safe Driver Insurance Plan) levels, the number of policies at each SDIP level, and the number of these policies that reported losses. Your job is to create a contingency table (a sample is given), make a scatter plot of the data, determine a line of best fit, and evaluate the accuracy of your line.

Areas of Application:

- Percentages
- Graphing & scatter plots
- Data analysis & statistics
- Linear regression & correlation coefficients
- Exponential growth & decay

Material Included:

- List of assumptions for SDIP premium costs
- Data table of SDIP levels, number of policies at each level, number of policies that reported losses, and total monetary losses incurred at each SDIP level
- Additional support activities
- Suggested approach and solutions

Background Information & Assumption:

1. SDIP levels are a rating on a driver's performance. The lower the SDIP number, the safer the driver.
2. All new Massachusetts drivers are assigned to SDIP 15.
3. Base cost of a step 15 policy is \$700.
4. Premium costs increase or decrease depending SDIP rating
 - a. 4% discount per step below SDIP 15
 - b. 7% increase per step above SDIP 15
5. Rates are compounded annually.

Classroom Time:

Left to individual Teachers

Assessment:

Left to individual Teachers

Line of Best Fit

SDIP Level	Number of Policies	Number of Policies with Claims	Percent with Claims	Number of Policies without Claims	Percent without Claims
9	202204	14681			
10	50745	4273			
11	9258	802			
12	5399	510			
13	27847	2631			
14	20215	2022			
15	18660	2000			
16	4248	492			
17	4264	526			
18	3785	499			
19	2738	353			
20	1784	243			
21	1460	186			
22	1042	126			
23	887	115			
24	669	103			
25	485	81			
26	390	69			
27	288	44			
28	228	25			
29	178	26			
30	134	19			
31	103	24			
32	70	12			
33	55	8			
34	67	16			
35	153	30			

Solution for Line of Best Fit Data Table:

SDIP Level	Number of Policies	Number of Policies with Claims	Percent with Claims	Number of Policies without Claims	Percent without Claims
9	202204	14681	7.26%	187523	92.74%
10	50745	4273	8.42%	46472	91.58%
11	9258	802	8.66%	8456	91.34%
12	5399	510	9.45%	4889	90.55%
13	27847	2631	9.45%	25216	90.55%
14	20215	2022	10.00%	18193	90.00%
15	18660	2000	10.72%	16660	89.28%
16	4248	492	11.58%	3756	88.42%
17	4264	526	12.34%	3738	87.66%
18	3785	499	13.18%	3286	86.82%
19	2738	353	12.89%	2385	87.11%
20	1784	243	13.62%	1541	86.38%
21	1460	186	12.74%	1274	87.26%
22	1042	126	12.09%	916	87.91%
23	887	115	12.97%	772	87.03%
24	669	103	15.40%	566	84.60%
25	485	81	16.70%	404	83.30%
26	390	69	17.69%	321	82.31%
27	288	44	15.28%	244	84.72%
28	228	25	10.96%	203	89.04%
29	178	26	14.61%	152	85.39%
30	134	19	14.18%	115	85.82%
31	103	24	23.30%	79	76.70%
32	70	12	17.14%	58	82.86%
33	55	8	14.55%	47	85.45%
34	67	16	23.88%	51	76.12%
35	153	30	19.61%	123	80.39%

Data Analysis

SDIP Level	Number of Policies	Number of Policies with Claims	Total Losses (in Dollars)
9	202204	14681	\$6,588,539.18
10	50745	4273	\$2,546,494.35
11	9258	802	\$627,581.04
12	5399	510	\$381,969.60
13	27847	2631	\$1,936,968.51
14	20215	2022	\$1,734,026.76
15	18660	2000	\$2,960,560.00
16	4248	492	\$587,605.44
17	4264	526	\$718,289.82
18	3785	499	\$567,991.74
19	2738	353	\$523,696.68
20	1784	243	\$386,637.30
21	1460	186	\$285,679.26
22	1042	126	\$188,836.20
23	887	115	\$112,156.05
24	669	103	\$141,164.59
25	485	81	\$139,203.36
26	390	69	\$139,020.51
27	288	44	\$97,581.00
28	228	25	\$32,485.75
29	178	26	\$44,242.12
30	134	19	\$80,595.15
31	103	24	\$69,745.92
32	70	12	\$37,427.88
33	55	8	\$3,855.52
34	67	16	\$65,466.24
35	153	30	\$72,738.90

Potential questions:

- 1) What is the base price of a policy at SDIP 15?
(Calculations and graphs here are based on a premium cost of \$700 for SDIP 15. See the Interactive Excel Worksheet for additional base policy costs.)
- 2) What is the percent increase in policy cost above SDIP 15?
What is the percent decrease in policy cost below SDIP 15?
(Calculations and graphs here are based on 7% and 4% respectively.)
- 3) What is the cost per policy in dollars based on the SDIP rating and percent increase or decrease per step away from 15?
- 4) How frequently do various SDIP levels have claims?
- 5) What is the average pay-out per claim? Per policy?
- 6) What is the total revenue, net profit, average profit per SDIP level?

Actual Price of a Ticket

Getting pulled over for going 40 mph in a 30 mph speed zone usually results in a fifty dollar ticket. Is that all that ticket really costs you? Unfortunately, if this is your second ticket (the first ticket in Massachusetts does not affect your SDIP rating) for a minor moving violation (stop sign, speeding, turning left), the price of the ticket is just the beginning. This incident affects your insurance premium due to a change in your SDIP rating. The second ticket for a minor moving violation, not only results in a ticket, but an increase of two levels in your SDIP rating.

Referring to the chart below, one can determine the impact in overall cost for receiving a second \$50 ticket for a minor moving violation.

SDIP Level	Cost per Policy
9	\$547.93
10	\$570.76
11	\$594.54
12	\$619.32
13	\$645.12
14	\$672.00
15	\$700.00
16	\$749.00
17	\$801.43
18	\$857.53
19	\$917.56
20	\$981.79
21	\$1,050.51
22	\$1,124.05
23	\$1,202.73
24	\$1,286.92
25	\$1,377.01
26	\$1,473.40
27	\$1,576.53
28	\$1,686.89
29	\$1,804.97
30	\$1,931.32
31	\$2,066.51
32	\$2,211.17
33	\$2,365.95
34	\$2,531.57
35	\$2,708.78

As an example, a new driver is automatically placed at a SDIP level of 15. Upon receiving a second \$50 ticket for a minor moving violation, the SDIP level would be 17. Therefore, the annual cost of the policy would go from \$700.00/year for SDIP level 15 to \$801.43/year for SDIP level 17, a difference of \$101.43. If the driver receives no additional tickets during that year, the driver would move to SDIP level 16, and would pay \$749.00 for that year instead of \$672.00 for SDIP level 14, the level that the driver would have been if he/she had not received a ticket. The difference in the second year is \$77.00. Therefore, over the two year period following the ticket, the driver has had to pay an additional two point penalty of \$178.43 (\$101.43 + \$77.00) plus \$50 for the ticket for a total cost of \$228.43.

What would be the total cost for a driver at SDIP level 26 for a second \$50 minor moving violation ticket?
(Ans: \$413.02 + \$50 = \$463.02)

If the driver has a minor accident, the SDIP level is increased by three levels. What would be the total policy increase in cost for a driver SDIP level 20 who was involved in a minor accident? (Assume no additional tickets or accidents over the next three years.)
(Ans: \$620.41)

If the driver above who had that minor accident later received a second \$50 ticket for a minor moving violation two years after the accident, what would be the total increase in his insurance premiums before returning to his SDIP level 20?
(Ans: \$1,983.68)