

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. Simplify:  $\frac{10^{2009} + 10^{2011}}{10^{2010} + 10^{2010}}$

- A) 0.01      B) 0.2      C) 1      D) 5      E) NOTA

2. What is the unit's digit of the sum  $\sum_{k=1}^{10} (k!)^2$ ?

- A) 9      B) 7      C) 5      D) 3      E) NOTA

3. How many ordered triples  $(x, y, z)$  of positive integers satisfy  $(x^y)^z = 64$ ?

- A) 2      B) 4      C) 8      D) 9      E) NOTA

4. Suppose  $\frac{2}{A} = \sqrt{\frac{9B}{A} - 2B^2}$ , where defined, for real numbers  $A$  and  $B$ . If  $A = MB^N$ , where  $M$  and  $N$  are real numbers, find the sum of all possible values of  $M + N$ .

- A)  $\frac{1}{2}$       B)  $-3$       C)  $\frac{9}{2}$       D)  $\frac{5}{2}$       E) NOTA

5. If  $k^{\log_2 5} = 16$ , what is the value of  $k^{(\log_2 5)^2}$ ?

- A) 625      B) 256      C) 128      D) 32      E) NOTA

6. Evaluate  $\left(x + \frac{1}{x}\right)^2$  when  $x = \frac{\sqrt{42}}{7}$ .

- A)  $\frac{85}{42}$       B)  $\frac{169}{42}$       C)  $\frac{127}{42}$       D) 4      E) NOTA

7. If  $A = \frac{(\log_3 1 - \log_3 4)(\log_3 9 - \log_3 2)}{(\log_3 1 - \log_3 9)(\log_3 8 - \log_3 4)}$ , evaluate  $3^A$ .

- A)  $\frac{1}{3}$       B)  $\frac{81}{16}$       C)  $\frac{1}{9}$       D)  $\frac{9}{2}$       E) NOTA

8. Find the sum of the real values of  $x$  that satisfy the equation  $3 + 3\log_3(x^3 + 1) = 3^2$ .

- A) 0                      B) 2                      C) 3                      D) 4                      E) NOTA

9. Evaluate:  $\sqrt{2014 \cdot 2012 \cdot 2010 \cdot 2008 + 16}$

- A) 4,044,116      B) 4,044,146      C) 4,044,096      D) 4,044,121      E) NOTA

10.  $e^2 - 5e + 6$  is closest in value to which of the following?

- A) -0.4                      B) -0.2                      C) 0                      D) 0.2                      E) 0.4

11. Find the sum of the digits of the product  $2^{2009} \cdot 5^{2011}$  when written in decimal form.

- A) 10                      B) 7                      C) 11                      D) 1                      E) NOTA

12. How many of the elements of the set  $\{7^1, 7^2, 7^3, \dots, 7^n, \dots, 7^{2011}\}$  have a unit's digit of 3?

- A) 670                      B) 503                      C) 402                      D) 335                      E) NOTA

13. The solutions to the equation  $x^2 - ax + b = 0$  are the squares of the solutions to the equation  $x^2 - cx + d = 0$ . Express  $a$  in terms of  $c$  and  $d$ .

- A)  $a = c^2 - 2d$       B)  $a = \sqrt{c^2 - 4d}$       C)  $a = c^2 + d^2$       D)  $a = \sqrt{2c^2 - 8d}$       E) NOTA

14. Evaluate:  $\sqrt{5 + \sqrt{5 + \sqrt{5 + \dots}}} + \frac{5}{1 + \frac{5}{1 + \frac{5}{1 + \dots}}}$

- A) 5                      B)  $\sqrt{21}$                       C) 0                      D)  $\frac{1}{4}$                       E) NOTA

15. Find the sum, where  $i = \sqrt{-1}$ :  $\sum_{n=1}^{14} (1 + (-1)^n i)^n$

- A)  $-25 - 127i$       B)  $-25 + 127i$       C)  $-129 - 77i$       D)  $-129 + 77i$       E) NOTA

16. Find the domain of the function  $y = \ln\left(\frac{x}{x^2 - 1}\right)$ .

- A)  $(1, \infty)$     B)  $(-\infty, -1) \cup (1, \infty)$     C)  $(-1, 1) \cup (1, \infty)$     D)  $(-1, 0) \cup (1, \infty)$     E) NOTA

17. Given that  $\log 2 = .301$ , how many digits are in the solution to the equation

$$\log_5(\log_4(\log_3(\log_2 x))) = 0?$$

- A) 21                      B) 23                      C) 25                      D) 27                      E) NOTA

18. Find the inverse of the function  $f(x) = \log_5(3x) - 1$ .

- A)  $f^{-1}(x) = \frac{5}{3}(5^x)$     B)  $f^{-1}(x) = \frac{1}{3}(5^x + 1)$     C)  $f^{-1}(x) = \log_5\left(\frac{1}{3x}\right) - 1$   
D)  $f^{-1}(x) = \frac{1}{\log_5(3x) - 1}$     E) NOTA

19. A bacteria culture with initial population 10 grows exponentially to a population of 160 after 8 hours. Find the bacteria population one hour later, rounded to the nearest whole number.

- A) 198                      B) 226                      C) 274                      D) 320                      E) NOTA

20. Find the sum of the  $y$ -values of the points of intersection of the graphs of  $y = \log_6 x^2$  and  $y = \log_6(x + 6)$ .

- A) 2                      B)  $\log_6 12$                       C) 1                      D)  $\log_6 9$                       E) NOTA

21. Let  $M$  be the number of lattice points on the line segment whose endpoints are

$\left(-\frac{7}{3}, \frac{5}{9}\right)$  and  $\left(\frac{173}{3}, \frac{185}{9}\right)$ . Find the smallest positive integer  $K$  such that

$\log_2(M - K)$  is an integer

- A) 1                      B) 2                      C) 3                      D) 4                      E) NOTA

22. Find the coefficient of  $a^2b^{-3}$  in the expansion of  $\left(a^2b^{-1} - b^{\frac{1}{2}}a^{-3}\right)^6$

- A) 15                      B) -15                      C) 20                      D) -20                      E) NOTA

23. Find the largest value of  $\log_x 3$  that satisfies the equation  $\log_x 3 + \log_3 x = 3$ .

- A)  $\frac{5+\sqrt{5}}{2}$       B)  $\frac{5-\sqrt{5}}{2}$       C)  $\frac{3+\sqrt{5}}{2}$       D)  $\frac{3-\sqrt{5}}{2}$       E) NOTA

24. Evaluate:  $\left( \frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}i \right)^{2011}$

- A)  $\frac{\sqrt{6}-\sqrt{2}}{4} - \frac{\sqrt{6}+\sqrt{2}}{4}i$       B)  $\frac{\sqrt{6}+\sqrt{2}}{4} - \frac{\sqrt{6}-\sqrt{2}}{4}i$       C)  $-\frac{\sqrt{6}-\sqrt{2}}{4} + \frac{\sqrt{6}+\sqrt{2}}{4}i$   
 D)  $-\frac{\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{4}i$       E) NOTA

25. Simplify:  $\sqrt{\sum_{n=0}^2 (n+\sqrt{2})^2}$

- A)  $3+\sqrt{2}$       B)  $3+2\sqrt{2}$       C)  $3+3\sqrt{2}$       D)  $3+4\sqrt{2}$       E) NOTA

26. Evaluate:  $\sqrt{8 + \frac{8}{\sqrt{8 + \frac{8}{\sqrt{8 + \frac{8}{\dots}}}}}}$

- A)  $-1+\sqrt{5}$       B)  $-2$       C)  $1+\sqrt{5}$       D)  $2$       E) NOTA

27. Find the value of the sum  $\sum_{n=1}^{2011} \lceil \sqrt{2n+0.25} - 0.5 \rceil$ , where  $\lceil x \rceil$  represents the least integer greater than or equal to  $x$ .

- A) 85,596      B) 89,565      C) 85,029      D) 88,998      E) NOTA

28. What integer is closest to  $(\sqrt{3} + \sqrt{2})^6$ ?

- A) 968      B) 970      C) 972      D) 974      E) NOTA

29. Find the value(s) of  $x$  that satisfy the equation  $\log x + \log(x - 2) = \log(x^2 - 2x)$ .

- A)  $x < 0$       B)  $0 < x < 2$       C)  $x > 2$       D) all real numbers      E) NOTA

30. Find the sum of the solutions to the equation  $3^{4x} - 3^{2x + \log_3 12} + 27 = 0$ .

- A) 0      B)  $\frac{1}{2}$       C) 1      D)  $\frac{3}{2}$       E) NOTA