

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. In statistics, one can approximate a line of best fit of n data points by generating a slope

$$m = \frac{n \sum_{i=1}^n (x_i y_i) - \left(\sum_{i=1}^n x_i \right) \left(\sum_{i=1}^n y_i \right)}{n \sum_{i=1}^n (x_i^2) - \left(\sum_{i=1}^n x_i \right)^2}$$

and a y -intercept $b = \bar{y} - m\bar{x}$, where (x_i, y_i) are the data

points, \bar{y} is the average of the y -values of the data points, and \bar{x} is the average of the x -values of the data points. If a data set consists of the three points $(1,2)$, $(3,9)$, and $(5,13)$, what is the value of $b - m$?

- A) -3 B) -2.25 C) -0.25 D) 2.75 E) NOTA

2. For what values of x is $x^4 + 8x^3 + 7x^2 - 72x - 144 > 0$?

- A) $(-\infty, -4) \cup (3, \infty)$ B) $(-\infty, -3) \cup (3, \infty)$ C) $(-\infty, -4) \cup (-3, 3)$ D) $(-\infty, -3)$ E) NOTA

3. Define a function mod as follows: for positive integers x and n , $x \text{ mod } n = x - n \left\lfloor \frac{x}{n} \right\rfloor$, where $\lfloor a \rfloor$ represents the greatest integer $b \leq a$. Evaluate $(17 \text{ mod } 4) - (20511 \text{ mod } 3) \left((449 \text{ mod } 7) - (9901 \text{ mod } 5) \right)$.

- A) 14 B) 10 C) 4 D) 1 E) NOTA

4. Define a function λ as follows: for positive integer n , $\lambda(n)$ = the smallest positive exponent k such that $x^k \text{ mod } n = 1 \text{ mod } n$ for all $x < n$ relatively prime to n and mod is the same function as defined in the previous problem. Evaluate $\lambda(16)$.

- A) 1 B) 2 C) 4 D) 8 E) NOTA

5. Let x and y satisfy $0 \leq x, y < 1$ and let a and b satisfy $a, b \geq 1$. Consider the function F , defined as $F(x, y) = x - y$, $F(a, y) = F(a - 1, y) + 1$, $F(x, b) = F(x, b - 1) + 1$, and $F(a, b) = F(a - 1, b - 1) + 2$ for values of x, y, a , and b as defined previously. Evaluate $F\left(\frac{25}{4}, \frac{16}{3}\right)$.

- A) $\frac{133}{12}$ B) $\frac{131}{12}$ C) $\frac{45}{4}$ D) $\frac{43}{4}$ E) NOTA

6. Which of the following statements is true regarding $w(x) = -3x^4 + 7x^3 - 4x^2 + 5x - 8$?

- I) w could have at least one negative root
- II) If w has at least one positive root, then it also has at least one non-real root
- III) w could have all non-real roots

A) II only B) III only C) I & II only D) II & III only E) NOTA

7. Suppose f satisfies $f(x) + 2f\left(\frac{1}{1-x}\right) = x$. Evaluate $f(2)$.

A) $\frac{2}{3}$ B) $\frac{4}{5}$ C) $\frac{5}{6}$ D) $\frac{7}{5}$ E) NOTA

8. Define $h(n) = \prod_{i=3}^n \log_{i-1} i$ for integer $n \geq 3$. Evaluate $\sum_{k=2}^{10} h(2^k)$.

A) 45 B) 54 C) 55 D) 72 E) NOTA

9. What is the range of the inverse of the real-valued function $B(x) = \frac{x}{x-3}$?

A) $(-\infty, \infty)$ B) $(-\infty, 3) \cup (3, \infty)$ C) $(-\infty, 1) \cup (1, \infty)$ D) $(-\infty, 1) \cup (3, \infty)$ E) NOTA

10. Which of the following statements is true concerning function Q , defined by the property $Q(x+y) = Q(x) \cdot Q(y)$, and that there exists an x such that $Q(x) \neq 0$?

- I) $Q(0) = 0$ and $Q(0) = 1$ are both possible
- II) $Q(-2) \cdot Q(2) = 1$
- III) $Q(x)$ is defined for all real values x

A) I only B) II only C) II & III only D) I, II, & III E) NOTA

11. What is the sum of the solutions of the equation $(x+3)^3 + 2(x+3)^2 - 8(x+3) = 0$?

A) -11 B) -7 C) -4 D) -1 E) NOTA

12. If polynomial $M(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d , and e are real numbers, has roots of $i, 1+2i$, and -3 , evaluate $(b-a)(c-e)$.

A) 39 B) 30 C) 12 D) 9 E) NOTA

13. If the solution to the inequality $|17 - 2x| > 4$ is $x < a$ or $x > b$, evaluate $a + b$.

- A) 34 B) 17 C) 10.5 D) 7.5 E) NOTA

14. Which of the following describes the graph of $4x^2 - 25y^2 - 24x - 50y + 11 = 0$?

- A) circle B) ellipse C) hyperbola D) parabola E) NOTA

15. What is the length of the latus rectum of the parabola with equation $x = \frac{1}{3}y^2 - 2y + 1$?

- A) 3 B) $\frac{11}{4}$ C) $\frac{3}{4}$ D) $\frac{1}{3}$ E) NOTA

16. Let $C(x)$ = the number of circular permutations of a set with x elements, and let

$P(x)$ = the number of 2-element groups that can be formed from a set of x elements.

Evaluate $C(P(4))$.

- A) 12! B) 11! C) 6! D) 5! E) NOTA

17. Evaluate $P(C(4))$, where P and C are defined as in the previous problem.

- A) 1 B) 15 C) 138 D) 276 E) NOTA

18. If $g(x)$ is an exponential function, then $\ln(g(x))$ is related to x in what way?

- A) inversely B) logarithmically C) quadratically D) linearly E) NOTA

19. The formula to convert from degrees Celsius, C , to degrees Fahrenheit, F , is the function $F(C) = 1.8C + 32$. Find the sum of the digits of the absolute value of the temperature that is the same on both scales.

- A) 4 B) 6 C) 9 D) 12 E) NOTA

20. Which of the following functions is odd?

- I) $y = \ln\sqrt{x}$ II) $y = 2^{-x^2+1}$ III) $y = x^3 - x^2 + 1$

- A) I only B) III only C) I & II only D) II & III only E) NOTA

21. Refer to question 19. To convert to Celsius from Kelvin temperatures, subtract 273 from the Kelvin temperature. If the graph giving Celsius temperature in terms of Fahrenheit is graphed, what type of shift would give Kelvin temperature in terms of Fahrenheit?

- A) horizontal shift right by 273 units B) horizontal shift left by 273 units
C) vertical shift up by 273 units D) vertical shift down by 273 units E) NOTA

22. Define S by $S(x) = \begin{cases} 1, & \text{if } x < 2 \\ x^2 + 2x + 3, & \text{if } 2 \leq x < 7 \\ x^3 - 7, & \text{if } x \geq 7 \end{cases}$. Evaluate $S(-1) + S(2) + S(7)$.

- A) 78 B) 338 C) 346 D) 348 E) NOTA

23. If the solution to the inequality $\frac{x+1}{x-3} < 2$ is $x < a$ or $x > b$, evaluate $b - a$.

- A) 0 B) 3 C) 4 D) 7 E) NOTA

24. Given two functions $u(x) = ae^{bx}$ and $v(x) = c \ln(dx)$ for positive a , b , c , and d , then when does $u(v(x)) = v(u(x))$ for all positive values x ?

- A) never B) if and only if $bc = 1$ C) if and only if $ad = 1$ D) if and only if $bc = ad$
E) NOTA

25. Given $f(2x) = \log_2 x$, find an expression equivalent to $f(x)$.

- A) $\frac{1}{2} \log_2 x$ B) $2 \log_2 x$ C) $1 + \frac{1}{2} \log_2 x$ D) $-1 + \log_2 x$ E) NOTA

26. The roots of $H(x) = x^3 - 12x^2 + 37x + G$ form an arithmetic progression. Find the value of G .

- A) 20 B) -20 C) 10 D) -10 E) NOTA

27. What is the equation of the oblique asymptote of the graph of $S(x) = \frac{x^2 - x - 2}{x + 2}$?

- A) $y = 0$ B) $y = 3 - x$ C) $y = x - 3$ D) $y = x + 3$ E) NOTA

28. Find all values of a such that the quadratic equation $x^2 + (a-3)x + a = 0$ has two distinct positive real solutions.

- A) $a > 0$ B) $0 < a < 1$ C) $a < 1$ D) $a > 9$ E) NOTA

29. If $f(x) = \frac{x+1}{x}$ for $x \neq 0$ and $f(g(x)) = x$ for $x \neq 1$, then $g\left(1 - \frac{1}{x}\right) = ?$

- A) $x-1$ B) $\frac{1}{x}$ C) $-x$ D) $x(x+1)$ E) NOTA

30. If $y = \log_b \left(\frac{1 + \sqrt{1-x^2}}{x} \right)$, where $0 < x \leq 1$, $b > 0$, and $b \neq 1$, then what is the value of $y^{-1}(0)$?

- A) $\frac{1}{2}$ B) 1 C) 2 D) $y^{-1}(0)$ is undefined E) NOTA