ANALYTIC GEOMETRY – ALPHA
Mu Alpha Theta National Convention 2003

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Who is considered the father of analytic geometry?
a) Apollonius b) Euclid c) Euler d) Poincare e) NOTA

2. Find the eccentricity, e, of a parabola whose equation is $y^2 = 4x$.
a) $\frac{1}{2}$ b) 1 c) 2 d) 8 e) NOTA

3. What is the distance between the point $(1,1,1)$ and the plane $2x + y - 2z + 4 = 0$?
a) $\frac{5}{3}$ b) $\frac{8}{3}$ c) 3 d) 1 e) NOTA

4. What is the value of the eccentricity of the ellipse with equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$?
a) $\frac{\sqrt{65}}{9}$ b) $\frac{\sqrt{65}}{4}$ c) $\frac{\sqrt{5}}{3}$ d) $\frac{\sqrt{5}}{9}$ e) NOTA

5. What are the slopes of the asymptotes of the hyperbola with equation $\frac{x^2}{4} - \frac{y^2}{9} = 1$?
a) $\pm\frac{2}{3}$ b) $\pm\frac{4}{9}$ c) $\pm\frac{81}{16}$ d) $\pm\frac{9}{4}$ e)NOTA

6. Find the equation of a parabola with its vertex at the origin and focus at $(2,0)$.
a) $y^2 = 8x$ b) $y = 4x^2$ c) $y^2 = 4x$ d) $y = \frac{x^2}{8}$ e) NOTA

7. What is the shortest distance between the sphere $x^2 + y^2 + z^2 = 9$ and the point, represented in spherical coordinates, $\left(5, \frac{\pi}{2}, \frac{\pi}{6}\right)$? *(Hint: this sphere is centered about the origin)*
a) 2 b) $2\sqrt{3}$ c) $\sqrt{6}$ d) $5 - 3\sqrt{2}$ e) NOTA

8. What is the ratio of the area of an ellipse, $A_{ellipse}$, to the area of its circumscribed circle, A_{circle}? (i.e., find $\frac{A_{ellipse}}{A_{circle}}$ if the semi-major axis of the ellipse is 'a', the semi-minor axis is 'b', and a diameter of the circle is concurrent with the major axis of the ellipse.)
a) $\frac{a}{b}$ b) $\frac{b}{a}$ c) $\frac{2b}{a}$ d) $\frac{\pi b}{a}$ e) NOTA

9. What positive (counterclockwise) angle of rotation is needed to eliminate the xy term from the second degree equation $\sqrt{3}x^2 + 6xy + 3\sqrt{3}y^2 + 2x - 3y + 4 = 0$?

a) 60° b) 45° c) 90° d) 30° e) NOTA

10. How many 'petals' does the graph of the curve $r = 4\cos\theta \sin\theta$ have?

a) 2 b) 3 c) 4 d) 6 e) NOTA

11. Let $c_1, c_2, ..., c_5$ be each of the fifth roots of –2. Find $\sum_{i=1}^{5} |c_i|$.

a) 10 b) $5\sqrt[5]{2}$ c) 2 d) $\sqrt[5]{2}$ e) NOTA

12. What is the area of the rectangle found by joining the two latera recta of the ellipse with equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$?

a) $\frac{16\sqrt{5}}{3}$ b) $\frac{8\sqrt{5}}{3}$ c) $\frac{16\sqrt{13}}{3}$ d) $\frac{8\sqrt{13}}{3}$ e) NOTA

13. What is the distance between the centers of the circles with equations $x^2 + y^2 - 2x - 2y - 7 = 0$ and $x^2 + y^2 + 6x + 4y + 12 = 0$?

a) 1 b) 3 c) 4 d) 5 e) NOTA

14. A sphere with radius $\sqrt{3}$ has a cube inscribed in it. Inside this cube is inscribed another sphere and inside that another cube and so on…(to 'infinity'). What is the sum of the radii of all the spheres in this intricate figure?

a) $\frac{3\sqrt{3}}{3\sqrt{3}-1}$ b) $\frac{3}{\sqrt{3}-1}$ c) $\frac{\sqrt{3}}{\sqrt{3}-1}$ d) 3 e) NOTA

15. What is the vertex of the parabola with equation $y = x^2 + 6x + 11$?

a) (-6,11) b) (-3,2) c) (3,38) d) (0,11) e) NOTA

16. Which of the following is the equation of a line tangent to the circle $x^2 + y^2 = 8$ and perpendicular to the line $x + y - 3 = 0$?

a) $y = x + 4$ b) $y = x + 2\sqrt{2}$ c) $y = x - 2\sqrt{2}$ d) $y = x$ e) NOTA

17. Find $\det\begin{vmatrix} \sin x & i\sin x & -1 \\ i & \cos x & i \\ -\sin x & i & -\sin x \end{vmatrix}$, when $x = \frac{3\pi}{2}$.

a) i b) 1 c) 0 d) -1 e) NOTA

18. If r_{circum} represents the radius of a circumscribed circle and r_{in} represents the radius of an inscribed circle, find $\frac{r_{circum}}{r_{in}}$ for a triangle with sides of length 3,4, and 5.

a) $\frac{2}{5}$ b) $\frac{5}{12}$ c) 5 d) 10 e) NOTA

19. What is the area of the convex polygon formed by the vertices at (0,0), (2,2), (5,3), (4,-3), (1,-4) ?

a) 6 b) 18 c) 44 d) 22 e) NOTA

20. What is the smallest period for T, $T > 0$, of the cycloid given by the parametric equations $x = 2(t - \sin t)$ and $y = 2(1 - \cos t)$?

a) $\frac{\pi}{2}$ b) π c) 2π d) 4π e) NOTA

21. What is the smallest angle (to the nearest 10^{th} of a degree) formed at the intersection of the lines $3x - 2y - 3 = 0$ and $x - 5y + 3 = 0$?

a) 45.0 b) 65.3 c) 56.3 d) 30.0 e) NOTA

22. Which vector is perpendicular to both $\langle 1,2,3 \rangle$ and $\langle 4,5,6 \rangle$?

a) $\langle -1,-2,-1 \rangle$ b) $\langle -1,2,1 \rangle$ c) $\langle 1,-2,1 \rangle$ d) $\langle -1,2,-1 \rangle$ e) NOTA

23. A triangle defined by the coordinates (1,10), (1,5), and (4,1), with area A, is transformed by the matrix $\begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$ to a new triangle with area A'. What is the ratio of the area of the new triangle to that of the old? *(i.e., find* $\frac{A'}{A}$*)*

a) $\frac{5}{2}$ b) 5 c) 10 d) 1 e) NOTA

24. What is the volume enclosed by the elliptical cylinder $\frac{x^2}{4} + \frac{y^2}{9} + 0z^2 = 1$ and the planes $z = 0$ and $z = 3$?

a) 9π b) 3π c) 18π d) 24π e) NOTA

25. Which conic section is represented by the equation $4x^2 - 4xy + y^2 + 12x - 6y + 9 = 0$?

a) ellipse b) 2 lines c) hyperbola d) 1 line e) NOTA

26. If the distance from the focus to the directrix of a parabola is p, what is the length of the latus rectum of the same parabola?

a) p b) $\frac{p}{2}$ c) $2p$ d) $4p$ e) NOTA

27. Which of the following represents an expression for a parabola in polar coordinates?

i) $r = \frac{2}{3+\sin\theta}$ ii) $r = \frac{2}{3-\sin\theta}$ iii) $r = \frac{2}{1+\cos\theta}$

iv) $r = \frac{2}{1-\sin\theta}$ v) $r = \frac{2}{\sin\theta - 1}$

a) iii only b) i, ii only c) iii, iv, v only d) iv, v only e) NOTA

28. A circle has a chord of length 12 that is tangent to a smaller, concentric circle. Find the area between the two circles.

a) 6π b) 9π c) 24π d) 36π e) NOTA

29. What is the sum of the number of faces of one each of the five Platonic solids (*Hint: a Platonic solid is a regular polyhedron)*?

a) 24 b) 50 c) 36 d) 48 e) NOTA

30. What is the fewest number of points needed to define a general second degree equation? *[Hint: a general equation of second degree (conic section) can be written as* $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ *]*

a) 3 b) 4 c) 6 d) 8 e) NOTA