Alpha Number Theory Topic Test

Note: For each problem, where there is no choice (e), assume (e) none of the above.

1. Evaluate $\cos^2 1^\circ + \cos^2 2^\circ + \cos^2 3^\circ + \cdots + \cos^2 90^\circ$.
   a) 44.5 b) 52.5 c) 255 d) not possible to compute

2. Narcissistic numbers are numbers whose values can be expressed using only the digits in the numbers and operations to include four basic operations, powers, factorials, nth roots, etc. Some narcissistic numbers follow:
   $2427 = 2^1 + 4^2 + 2^3 + 7^4$
   $81 = (8+1)^2$
   $355 = 3(5!) - 5$
   Which of these is narcissistic?
   a) 121 b) 135 c) 217 d) 273

3. A two – digit number is a perfect square and has exactly nine positive integral factors. What is the number? a) 25 b) 36 c) 49 d) 64

4. Find the units digit of $3^{2002} - 2^{2002}$. a) 1 b) 3 c) 5 d) 7

5. How many four-digit positive integers have four different nonzero digits whose sum is 10?
   a) 16 b) 24 c) 64 d) 144

6. How many digits are in 5^{5^5}? a) 25 b) 1725 c) 2185 d) 3125

7. For which of the following sets of positive integers a, b, c will $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$?
   a) 2, 3, 4 b) 2, 3, 6 c) 2, 2, 4 d) 3, 6, 9

8. Which is largest: a) 63_8 b) 110100_2 c) 56_{10} d) 301_4

9. A school hallway has a long row of lockers. Every sixth locker contains a package of chewing gum, with the 6th locker being the first to have chewing gum, every eighth locker (the first one is locker #8) contains a hockey stick, and every ninth locker (locker #9 is the first) contains a mirror. Which is the first locker to contain all three items?
   a) 18 b) 24 c) 64 d) 72

10. If $\sqrt{19^2 + 19^2 + 19^2 + \ldots + 19^2} = 19^2$, how many times must 19^2 appear in the radicand?
    a) 19 b) 20 c) 361 d) 380

11. What is the smallest prime number that is a factor of the sum of 3^{15} and 5^{17}?
    a) 2 b) 3 c) 5 d) 17

12. Write the following as the ratio of two integers. $\dfrac{\sqrt{4+2\sqrt{3}} - \sqrt{28+10\sqrt{3}}}{15}$
    a) $-\frac{4}{15}$ b) $\frac{8}{15}$ c) $-\frac{1}{5}$ d) $\frac{1}{3}$

13. A digital root is the value of the sum of the digits of a number until only one digit remains. For example, for the number 625, first add 6+2+5 = 13. Now since 13 has 2 digits, add those digits; that is, 1+3=4. So the digital root of 625 is 4. What is the digital root is of 6^6? (Hint: Consider the first 5 terms of the sequence of 6^{th} powers and their digital roots.)
a) 0 b) 1 c) 6 d) 9

14. Find two 4-digit numbers from the digits 2 through 9, using each digit exactly once, so that the absolute value of the difference of the two numbers is as small as possible.
a) 6234 & 5987 b) 5942 & 6843 c) 4576 & 3894 d) 9642 & 8735

15. How many positive factors does 1000 have? a) 35 b) 27 c) 22 d) 16

16. Do two distinct integers exist with the property that their sum is equal to their product?
a) yes b) no

17. The Fibonacci Middle School Math Club is preparing for their first competition. The entrance rules of the competition call for teams of three, where each team must include at least one girl and one boy. No student can be on more than one team. Of the 44 members that will be able to attend the competition, there are eighteen more girls than boys. What is the maximum number of teams the club can enter in the competition? a) 13 b) 18 c) 26 d) 31

18. If $]3[= 47,\ ]10[= 138,\ ]1[= 39$ and if $\boxed{1} = 5,\ \boxed{20} = 43,\ \boxed{99} = 201$, find n so that $\boxed{]n[} =]\boxed{n}[$. a) 2 b) 4 c) 6 d) 8

19. Choose a prime number between 120 and 130. a) 121 b) 123 c) 125 d) 127 e) 129

20. Each of the letters A, B, C and D represents a different integer in the set {1, 2, 3, 4} but not necessarily in that order. If $\frac{A}{B} - \frac{C}{D} = 1$, then the sum of A and C is ?
a) 3 b) 4 c) 5 d) 6 e) 7

21. In triangle ABC $3\sin A + 4\cos B = 6$ and $4\sin B + 3\cos A = 1$. Find the measure for $\angle C$ in degrees. a) 30 b) 60 c) 90 d) 120 e) 150

22. For what positive value of x, is x + 2 the same as the reciprocal of x - 2?
a) $\sqrt{3}$ b) $\sqrt{5}$ c) $\sqrt{6}$ d) $\sqrt{7}$

23. Each letter in the puzzle below represents a different digit. All digits in "NINE" are odd. What is the least possible value for N?
a) 1 b) 3 c) 5 d) 7

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 FOUR
+F IVE
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 NINE
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24. Find the smallest integer greater than 1000 that is divisible by 5 and 13 but not 4.
a) 1040 b) 1105 c) 1625 d) 4225

25. The digits 3, 4, 5 and 6 are used to form 24 four-digit positive integers. Each digit is used once in each integer. If the integers are listed in increasing order, what is the thirteenth number in the list? a) 4563 b) 5346 c) 5436 d) 5643

26. An integer that is greater than 5 and less than 15 is chosen at random and denoted by x. What is the probability that x^2 - 29 is prime?
a) $\frac{1}{6}$ b) $\frac{5}{9}$ c) $\frac{2}{9}$ d) $\frac{1}{3}$

27. $2\left(1-\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)+4\left(1-\frac{1}{4}\right)+\ldots+10\left(1-\frac{1}{10}\right)=$
a) 45 b) 49 c) 50 d) 54 e) 55

28. What is the sum of the digits of the decimal form of the product $2^{1999}\cdot 5^{2001}$?
a) 1 b) 2 c) 5 d) 7 e) 10

29. Find the sum of all prime numbers between 1 and 100 that are simultaneously 1 greater than a multiple of 4 and 1 less than a multiple of 5.
a) 118 b) 137 c) 158 d) 187 e) 245

30. What is the smallest integral multiple of 30 that has exactly thirty-six divisors?
a) 750 b) 960 c) 1260 d) 1560

31. The rightmost nonzero digit of the product $n(n+1)(n+2)(n+3)(n+4)$ is odd. What is the some of the digits of the smallest possible value of n?
a) 3 b) 4 c) 6 d) 10

32. The fourteen digits in a credit card number are represented by the boxes below. If the sum of any three consecutive digits is 18, what is the value of X?

			7				X				8		

a) 2 b) 3 c) 4 d) 5 e) 6

33. If A, B, and C are distinct digits, how many pairs of three digit numbers can satisfy the condition that both the numbers ABC and CBA are divisible by 7?
a) 0 b) 1 c) 2 d) 3

34. How many odd numbers are in the 100th row of Pascal's triangle?
a) 7 b) 8 c) 9 d) 10

35. Two points are randomly and simultaneously selected from the 4 by 5 grid of twenty lattice points, where $1\le x\le 5,\ 1\le y\le 4$. What is the probability that the distance between them is a rational number? a) $\frac{3}{7}$ b) $\frac{7}{19}$ c) $\frac{37}{70}$ d) $\frac{36}{95}$