Logs & Exponents - Solutions

FAMAT State Convention - 2002

1. $\log_{21} 42 = \dfrac{\log 42}{\log 21} = 1.2276702487 \approx 1.2277$ - B.

2. $15 \times 2.1^{\frac{x}{3}} = 74.6 \Rightarrow 2.1^{\frac{x}{3}} = \dfrac{74.6}{15} \Rightarrow \dfrac{x}{3} = \dfrac{\log\left(\frac{74.6}{15}\right)}{\log 2.1} \Rightarrow x = 3 \times \dfrac{\log\left(\frac{74.6}{15}\right)}{\log 2.1} \Rightarrow x \approx 6.4861$ - C.

3. $AB = \log_b 9 - \log_b 8 = \log_b \dfrac{9}{8}$. $CD = \log_b 900 - \log_b 800 = \log_b \dfrac{900}{800} = \log_b \dfrac{9}{8}$. - B.

4. $f(t) = 542 \times \left(\dfrac{580}{542}\right)^{10} \approx 1067.30$ - C

5. $2001^{2001} = 10^{\log\left(2001^{2001}\right)} = 10^{2001 \times \log 2001} = 10^{6605.7954244} = 10^{.7954244} \times 10^{6605} = 6.234 \times 10^{6605}$ - D.

6. $\log_a\left[\left(\dfrac{9000}{a^2}\right)^3\right] = 3 \times \log_a\left(2^3 \times 3^2 \times 5^3 \div a^2\right) =$ - E.

$3 \times \left(3\log_a 2 + 2\log_a 3 + 3\log_a 5 - 2\log_a a\right) = 3 \times \left(3x + 2y + 3z - 2 \times 1\right)$

7. $\log_b \sqrt[4]{t^3} = \dfrac{3}{4} \times \log_b t = \dfrac{3}{4} \times 1.44 = 1.08 = A.$

8. $2^{10000} + 2^{10000} = 2 \times 2^{10000} = 2^{1+10000} = 2^{10001}$ - C.

9. $1999^{-1999} = 10^{\log\left(1999^{-1999}\right)} = 10^{-1999 \times \log 1999} = 10^{-6598.3247754} = 10^{0.67522455788} \times 10^{-6599} = 4.734 \times 10^{-6599}$ C

10. $A(t) = A_0 \times \left(\dfrac{11.1}{12.4}\right)^{\left(\frac{t}{74hr}\right)} \Rightarrow .5 = 1 \times \left(\dfrac{11.1}{12.4}\right)^{\left(\frac{half-life}{74hr}\right)}$

$\dfrac{half-life}{74hr} = \dfrac{\log .5}{\log\left(\frac{11.1}{12.4}\right)} \Rightarrow half-life = 74hr \times \dfrac{\log .5}{\log\left(\frac{11.1}{12.4}\right)} \approx 463.1hrs$ - B

11. An exponential function's graph decreases only when $0 < b < 1$. - C

12. $D(t) = (83-29)\left(\dfrac{54.5-29}{83-29}\right)^{\frac{t}{9}} \Rightarrow$ temp of coffee was: $29 + D(-3) = 29 + 69.344 = 98.3$ - B.

13. $\log_a 10 = \log_a \sqrt{10}^2 = 2 \times \log_a \sqrt{10} = 2 \times \dfrac{\log_b \sqrt{10}}{\log_b a} = 2 \times \dfrac{\log_b \sqrt{10}}{5} = .4\log_b \sqrt{10}$ - D.

14. $$1.5 \times \frac{(1-.5^n)}{.5} = 2.994140625 \Rightarrow (1-.5^n) = \frac{2.994140625}{3} \Rightarrow$$
$$.5^n = 1 - \frac{2.994140625}{3} \Rightarrow n = \frac{\log\left(1-\frac{2.994140625}{3}\right)}{\log .5} = 9$$ - A.

15. See solution to #1 - B.

16. $\log_{12} x = 4 \Rightarrow x = 12^4 = 20736$. - D.

17. $\log_{a^3} \sqrt{a} = \frac{1}{3} \times \log_a a^{\frac{1}{2}} = \frac{1}{3} \times \frac{1}{2} \times \log_a a = \frac{1}{6} \times 1 = \frac{1}{6}$ - D.

18. $\log_2(x-2) + \log_2(x-6) = 5 \Rightarrow \log_2[(x-2)(x-6)] = 5 \Rightarrow 2^5 = (x-2)(x-6)$
$\Rightarrow 0 = x^2 - 8x - 20 \Rightarrow x = -2,10$ Since the argument of a log cannot be negative, $x = 10$ - A

19. $65432^{9876} = 10^{\log(65432^{9876})} = 10^{9876 \times \log(65432)} = 10^{47560.743968}$ $\therefore$ 47561 digits - D

20. q2p(◀)⇒◀ ⊗ ◀ = ⧉, q3p(◀)⇒⧉ ⊗ ◀ = ▼, q4p(◀)⇒▼ ⊗ ◀ = ▶,
q5p(◀)⇒▶ ⊗ ◀ = ▲, q6p(◀)⇒▲ ⊗ ◀ = □, q7p(◀)⇒□ ⊗ ◀ = ◀,
q8p(◀)⇒◀ ⊗ ◀ = ⧉, q9p(◀)⇒⧉ ⊗ ◀ = ▼, q10p(◀)⇒▼ ⊗ ◀ = ▶,
q11p(◀)⇒▶ ⊗ ◀ = ▲, q12p(◀)⇒▲ ⊗ ◀ = □, q13p(◀)⇒□ ⊗ ◀ = ◀, …
q2p = q8p = q14p = q(6*n*+2)p = ⧉; q3p = q9p = q15p = q(6*n*+3)p = ▼
now, 987 = (6(164) + 3) so the answer is B.

21. $A = 50\left(1+\frac{.0525}{4}\right)^{(4\times 10)} = \$84.23$ - B

22. The coefficient 4 in 4*x* denotes repeated addition. Just as the exponent in x^4 denotes repeated multiplication. - C

23. $(2^{6972593}-1)(2^{6972593-1}) = 2^{6972593} \times 2^{6972592} - 2^{6972593} = 2^{13945185} - 2^{6972593}$
$2^{6972593}$ when subtracted from $2^{13945185}$ won't change the number of digits in the final answer.
$2^{13945185} = 10^{\log(2^{13945185})} = 10^{(13945185 \times \log 2)} = 10^{4197918.9801}$ $\therefore$ it will have 4197919 digits - C

24. See solution to #1. - B

25. $\log_7(9-x) + \log_7(8-x) = \log_7 2 \Rightarrow \log_7(9-x)(8-x) = \log_7 2 \Rightarrow (9-x)(8-x) = 2$
$\Rightarrow x^2 - 17 + 72 = 2 \Rightarrow x^2 - 17 + 70 = 0 \Rightarrow x \in \{7,10\}$ Since the argument of a log cannot be negative, we discard 10 and keep only the 7. - A

26. $\ln \pi = \log_e \pi = \frac{\log_{10}\pi}{\log_{10} e} = \frac{1}{\log_{10} e} \times \log_{10}\pi = \underline{\log_e 10} \times \log_{10}\pi$ - C

27. Log(100!) = log1+log2+log3+log4+…+log98+log99+log100 = 157.97000365 - C

28. $P = B\frac{\frac{i}{n}}{\left(1-\left(1+\frac{i}{n}\right)^{(-n\times t)}\right)} = 30000\frac{\frac{.08}{12}}{\left(1-\left(1+\frac{.08}{12}\right)^{(-5\times 12)}\right)} = 608.29182865$ - B

29. $\log_b x = \frac{\log_c x}{\log_? b}$, *c* belongs there - C.

30. $7^{\log_{49} 343} = 7^{\log_{(7^2)}(7^3)} = 7^{\frac{1}{2}\times 3\times \log_7 7} = 7^{\frac{3}{2}}$ - B