Alpha Individual Test

FAMAT State Convention 2002

Solutions:

1. If $x = \sqrt{5 - \sqrt{5 - \ldots}}$ then $x^2 = 5 - x$. Using the quadratic formula and knowing that the value of x is positive, we get $x = \frac{-1+\sqrt{21}}{2}$. Choice **A**.

2. $2^{2x} \bullet 2^x = 2^{3x}$. Choice **D**.

3. $\frac{2\sin x \cos x}{\sin x} = \frac{a}{b}$. $2\cos x = \frac{a}{b}$. $\cos x = \frac{a}{2b}$. Choice **B**.

4. Since $\sin^2 x + \cos^2 x = 1$, choices A and B are true. By definition of Tangent and Secant, choices C and D are true. All are true. Choice **E**.

5. $\tan 54 = \frac{10-0.2t}{5}$ gives that it will take 15.59045199 minutes. This makes the time 12:15:35. Remember that 0.59 of of minute is not equal to 59 minutes. Choice **A**.

6. The slope of the two points is 2 so the perpendicular line has slope $\frac{-1}{2}$. Substituting the point into the line gives $x + 2y = 101$ and $-4 + 2k = 101$ and $k = 52.5$. Choice **C**.

7. Using the law of cosines, $81 = 16 + 36 - 2(4)(6)\cos A$ which solves to $\cos A = \frac{-29}{48}$. Choice **D**.

8. Using the identity $2\sin x \cos x = \sin(2x)$ we get $y = \frac{1}{2}\sin 2x$ which has period π. Choice **B**.

9. $\cos\theta(2\cos\theta - 1) = 0$ gives $\cos\theta = 0$ or $\cos\theta = \frac{1}{2}$. This gives solutions $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$ and with denominator 6, $\frac{3\pi}{6}, \frac{9\pi}{6}, \frac{2\pi}{6}, \frac{10\pi}{6}$. So a=2, b=3, c=9, d=10. The sum b+d = 13, which is choice **B**.

10. $\left(\sqrt[4]{3}\right)^2 + k^2 = \sqrt{5+2\sqrt{6}}$ by the Pyth. Th. So $(\sqrt{3} + k^2)^2 = 5 + 2\sqrt{6}$. So $k^2 = \sqrt{2}$ and $k = \sqrt[4]{2}$. $k^6 = 2^{\frac{6}{4}} = 2^{\frac{3}{2}} = 2\sqrt{2}$. Choice **C**.

11. Using Heron's formula, the area of the triangle is $\sqrt{7(7-5)(7-6)(7-3)}$. Which gives area $2\sqrt{14}$ and so $2\sqrt{14} = \frac{1}{2}(6)h$ and $h = \frac{2\sqrt{14}}{3}$. Choice **D**.

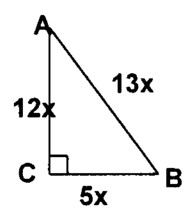
12. The length BD is half of AE. So $2\cos\theta = \sin\theta$ and $\tan\theta = 2$ in quadrant I. So $\theta^2 - \theta$ was approximately 0.119, choice **C**.

13. To get f(3) we substitute x=4. $\sqrt{12} = 2\sqrt{3}$, choice **B**.

14. The surface area will be the original surface area (6 times 64) minus the area of an isosceles right triangle of hypotenuse 1, on three faces. This is $3(\frac{1}{2} \bullet \frac{1}{\sqrt{2}} \bullet \frac{1}{\sqrt{2}})$ or $\frac{3}{4}$. Plus the area of the equilateral triangle face: $\frac{side^2}{4}\sqrt{3} = \frac{1}{4}\sqrt{3}$. Getting a common denominator 4, gives $\frac{1536-3+\sqrt{3}}{4} = \frac{1533+\sqrt{3}}{4}$. Choice **B**.

15. The angle between the vectors is 100°. Using the law of cosines gives $900 + 25 - 2(30)(5)\cos 100^\circ \approx 31.3$ mph. Choice **D**.

16. $\frac{\frac{12x}{13x}}{\frac{5x}{13x}} = \frac{12}{5}$ which is choice **A**.



17. $\cos(a+b) = \cos a \cos b - \sin a \sin b$. $c\sqrt{1-d^2} - d\sqrt{1-c^2}$. Choice **C**.

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Solutions (page two of two):

18. Using the Pythagorean Theorem, we get $4\log x + (\log x)^2 = 12$ or $(\log x)^2 + 4\log x - 12 = 0$ and so $(\log x + 6)(\log x - 2) = 0$ $\log x = -6$ or $\log x = 2$. Using the second answer , $\cos\theta = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{6}}{3}$ = choice **C**.

19. The volume of the dog is the volume of the cylinder $V = \pi r^2(\Delta h) = \pi(16)(3) = 48\pi$ which is choice **C**. The height of the water after the dog leaves it is irrelevant.

20. Let x be the larger side of the small rectangle and y be the smaller side of the small rectangle. $5x + 8y = 72$ and from the picture we see $3x = 6y$ (opposite sides are congruent). Substituting gives $5(2y) + 8y = 72$ and y=4, x=8. The perimeter of a small rectangle is 24. Choice **B**.

21. This is the ambiguous case of SSA and this triangle has two solutions. Using the law of sines, we get $\frac{\sin C}{12} = \frac{\sin 40}{8}$ and $\sin C \approx 0.96418$ which may be the angle 74.62 or 105.38 degrees. When C is 105.38 degrees, B= 36.4 deg., choice **A**.

22. $(\cos x + \cos y)^2 = \frac{1}{4}$ and $(\sin x - \sin y)^2 = \frac{1}{9}$ $\cos^2 x + 2\cos x\cos y + \cos^2 y = \frac{1}{4}$ and $\sin^2 x - 2\sin x \sin y + \sin^2 y = \frac{1}{9}$. Add and use the fact that $\cos^2 x + \sin^2 x = 1$ to get $2 + 2(\cos x\cos y - \sin x\sin y) = \frac{13}{36}$. Solving for the parentheses gives $\cos(x+y) = \frac{-59}{72}$ which is choice **B**.

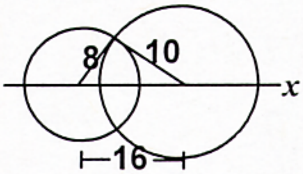
23. The sum of 8x and 3x is 11x which equals 44. So x=4 and black beads are 8(4)=32 which is choice **D**.

24. $\log_2(\frac{x}{9}) = 4$ and $16 = \frac{x}{9}$ so x=9(16). $2\sqrt{x} = 2 \bullet 3 \bullet 4 = 24 =$ choice **A**.

25. f is the parabola with vertex (1,0) and which opens to the left such that $\frac{1}{4a} = 2$ and so $a = \frac{1}{8}$ and the equation is $-\frac{1}{8}y^2 = (x-1)$. If y=4 then x= -1. Choice **D**.

26. Complete the square to get $(x-2)^2 + (y+3)^2 = 16$. The minimum value of f is the distance from the center (2, -3) to the point (10, 12) minus the length of the radius 4. The distance is 17, using the distance formula, minus 4 is 13. Choice **C**.

27. The height to the side of length 16 is requested. Get the area of the triangle using Heron's formula. This is approx. 32.726. Set this equal to $\frac{1}{2}(16)x$, and x is appromately 4.1, or choice **A**.



28. Using $A = \frac{1}{2}bh$, $\frac{1}{2}(10)h = 20$, $h = 4$. So the point (K, 4) lies on the line x-4y=2. K is then 18, and we want the distance from (0,0) to (18,4) which is approximately 18.4. Choice **C**.

29. Since $x < 0$, $\sqrt{2x}$ and $\sqrt{8x}$ are not real and thus we cannot use $\sqrt{a} \bullet \sqrt{b} = \sqrt{ab}$. Thus choice A is false. The product of two imaginary numbers is negative, so $-4|x|$ is the only possible answer. NOT $-4x$ since x is negative, and this product will be positive. Choice **D**.

30. The slope of the line is $\tan 30^\circ = \frac{1}{\sqrt{3}}$ and so the equation through (0,0) is $x - \sqrt{3}\,y = 0$. Since $x = \sqrt{3}\,y$ we get $6(\sqrt{3}\,y) - 3(3y^2) = y$ and this solves to y=0 or y=$\frac{6\sqrt{3}-1}{9}$, so $27n$ gives choice **A**.