Mu Trigonometry Topic Test 2001 National Convention

(1) $56° \cdot \frac{\pi}{180°} = \frac{14\pi}{45}$ (A)

(2) $\frac{43\pi}{18} \cdot \frac{180°}{\pi} = 430°$ (D)

(3) $\sin 0 - \cos 30° + \tan\frac{\pi}{4} - \sec 60° + \csc\frac{\pi}{2} - \cot 120° = 0 - \frac{\sqrt{3}}{2} + 1 - 2 + 1 - \left(-\frac{1}{\sqrt{3}}\right) = -\frac{\sqrt{3}}{6}$ (D)

(4) $1 + \tan^2 57° = \sec^2 57° = \csc^2(90° - 57°) = \csc^2 33°$ (C)

(5) $\sin\left(\cos\frac{\pi}{2}\right) = \sin 0 = 0$ (D)

(6) $2(\sin^3 y \cos y + \cos^3 y \sin y) = 2\sin y \cos y(\sin^2 y + \cos^2 y) = \sin 2y$ (B)

(7) $(\cos 73° \cos 13° + \sin 73° \sin 13°) + (\sin 37° \cos 8° + \cos 37° \sin 8°) =$
$\cos(73° - 13°) + \sin(37° + 8°) = \frac{1}{2} + \frac{\sqrt{2}}{2}$ (A)

(8) $\frac{1}{\cot w} + \frac{1}{\tan w} = \frac{\tan w + \cot w}{\tan w \cdot \cot w} = \tan w + \cot w = \frac{143}{69} \Rightarrow m + n = 212$ (B)

(9) Law of sines $\frac{BC}{\sin A} = \frac{AB}{\sin C} \therefore BC = \frac{\sin A}{\sin C} AB = \frac{\sin 79°}{\sin 36°} = \frac{\sin 101°}{\cos 54°}$ (B)

(10) $f'(\theta) = -\csc\theta \cot\theta = -\left(\frac{25}{24}\right)\left(-\frac{7}{24}\right) = \frac{175}{576}$ (C)

(11) $K = \frac{1}{2}ab\sin C$ Therefore, our desired ratio (since, a, b stay constant) is
$\frac{\sin 2C}{\sin C} = 2\cos C = 2\left(\frac{55}{73}\right) = \frac{110}{73}$ (C)

(12) Period is $\frac{2\pi}{6} = \frac{\pi}{3}$ (D)

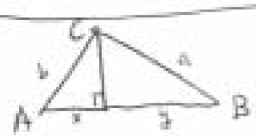
(13) $\cos(-u) + \sin(-v)\cos(u) = \cos u - \sin v \cos u = \frac{5}{13} - \frac{4}{5} \cdot \frac{5}{13} = \frac{1}{13}$ (B)

(14) The radius of the cross-section is $\sqrt{25 - 25\cos^2 37°} = 5\sin 37°$, so the area is
$25\pi \sin^2 37° = 25\pi\left(\frac{1 - \cos 74°}{2}\right)$ (A)

(15) 

$\tan\theta = \frac{x}{75000} \therefore \sec^2\theta \frac{d\theta}{dt} = \frac{1}{75000} \frac{dx}{dt}$ @ $x = 37{,}500$, $\sec^2\theta = \frac{5}{4} \therefore$
$\frac{d\theta}{dt} = \frac{4}{5} \cdot \frac{1}{75000} \cdot 16500 = \frac{22}{125}$ (C)

(16) $2(1 - \cos^2 B) - \cos B = 1 \Rightarrow 2\cos^2 B + \cos B - 1 = 0 \Rightarrow (2\cos B - 1)(\cos B + 1) = 0 \therefore$
$\cos B = \frac{1}{2}$ or $\cos B = -1 \therefore B = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ (D)

(17) 

$x = b\cos A$
$y = a\cos B$ $\therefore b\cos A + a\cos B = x + y = AB = 50000$ (C)

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18. $y = \sin^3 x \frac{\cos^3 x}{\sin^3 x} \cos^2 x \frac{1}{\cos^5 x} = 1 \therefore y' = 0$ (B)

19. $(\sin\theta)(1+\tan\theta) = 1+\tan\theta \therefore \sin\theta = 1$ or $1+\tan\theta = 0 \therefore$
when $\sin\theta = 1$, $\tan\theta$ undef, so $\pi/2$ is not a sol'n. $\tan\theta = -1$ yields $3\pi/4, 7\pi/4$ (D)

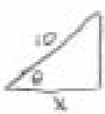
20. $\cos BCD = \cos CAB = \frac{5}{13}$, $\sec ACD = \sec CBA = \frac{13}{12}$ $\quad \frac{5}{13} + \frac{13}{12} = \frac{229}{156}$ (A)

21. $\frac{1473}{23} = \approx 64.04\pi$ $\quad \frac{773\pi}{12} \approx 64.416\pi$; x is always in 1st quadrant, so
$\sin x$ is strictly increasing (C)

22. $y' = \frac{10}{3}\cos\frac{5x}{3} - \frac{15}{4}\sin\frac{3x}{4}$, which has amplitude $\frac{10}{3} + \frac{15}{4} = \frac{85}{12}$ (C)
(Max attained @ $x = 18\pi$) (Find max by looking for x s.t. $\frac{5x}{3} = 2n\pi$ & $\frac{3x}{4} = 2m\pi + \frac{3\pi}{2}$

23. $\tan(\arctan\frac{22}{7}) = \tan(\arctan\frac{3}{x} + \arctan\frac{17}{x}) = \frac{\frac{20}{x}}{1 - \frac{51}{x^2}} = \frac{22}{7} \Rightarrow 11x^2 - 70x - 11(51) = 0 \therefore$
$x = 11, -\frac{51}{11} \therefore$ Clearly $x > 0$, so $x = 11$ (B)

24.



We seek $\frac{dx}{dt}$ (our answer is $|2\frac{dx}{dt}|$ since there are two spans equally moving away from each other) $\cos\theta = \frac{x}{10} \therefore -\sin\theta\frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt} \therefore$
$\frac{dx}{dt} = -10\sqrt{2}$ @ $\theta = 45° \therefore |2\frac{dx}{dt}| = 20\sqrt{2}$ (B)

25. $u = \cos t \therefore \frac{du}{dt} = -\sin t \therefore dt = \frac{du}{-\sin t} \therefore \int \sin^3 t \cos^4 t\, dt = -\int (1-u^2)u^4\, du =$
$\frac{u^7}{7} - \frac{u^5}{5} + C = \frac{\cos^7 t}{7} - \frac{\cos^5 t}{5} + C$ (C)

26. $\cos A = -\frac{45}{53}$, $\sin B = -\frac{20}{29} \therefore \cos(A+B) = \left(-\frac{45}{53}\right)\left(\frac{21}{29}\right) - \left(-\frac{28}{53}\right)\left(-\frac{20}{29}\right) = -\frac{1505}{1537}$ (B)

27. $V = \int_0^{\pi/4} \pi\left[(\sqrt{\cos x})^2 - (\sqrt{\sin x})^2\right] dx = \pi \int_0^{\pi/4} (\cos x - \sin x)\, dx = \pi\left[\sin x + \cos x\right]_0^{\pi/4} = \pi(\sqrt{2} - 1)$ (D)

28. Only $(\sin x, \cos x)$ of the $\binom{6}{2} = 15$ possibilities satisfy the criterion (B)

29. $y = 3\sin x - 4\sin^3 x = \sin x(3 - 4\sin^2 x) = 0$ at $x = 0$, $x = \pi/3$, and $y > 0$ for $0 < x < \pi/3$.
Hence, sought area is $A = \int_0^{\pi/3} (3 - 4\sin^2 x)\sin x\, dx$ let $u = \cos x$; $\frac{du}{dx} = -\sin x \therefore$
$A = \int_0^{\pi/3} (3 - 4(1 - \cos^2 x))\sin x\, dx = \int_1^{1/2} (1 - 4u^2)\, du = \left(u - \frac{4u^3}{3}\right)\Big|_1^{1/2} = 2/3$ (B)

30. $\sin^2 5\varphi = 1 \Rightarrow 5\varphi = 2n\pi + \pi/2 \Rightarrow \varphi = \frac{2n\pi}{5} + \pi/10$
$5\varphi = 2m\pi + 3\pi/2 \Rightarrow \varphi = \frac{2m\pi}{5} + 3\pi/10$
The first satisfies $\frac{47\pi}{50} < \varphi < \frac{8\pi}{5}$ for $n = 3$, the second for $m = 2, 3$.
The sum of the three sol'ns is $39\pi/10$ (B)

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(31)  $y = x + 25°$ $\frac{\sin x}{150} = \frac{\sin 25°}{200}$ ∴ $x = 18.47°$ ∴ $y \approx 43°$ (D)

(32) $x = \sin u$; $\frac{dx}{du} = \cos u \Rightarrow dx = du \cos u \Rightarrow \int \frac{dx}{x^2\sqrt{1-x^2}} = \int \frac{du \cos u}{\sin^2 u \cos u} = \int \frac{du}{\sin^2 u} = -\cot u + C$

$\sqrt{1-x^2} = \cos u$ ∴ $-\cot u + C = -\frac{\sqrt{1-x^2}}{x} + C$ (C)

(33) $\cos x - \sin 2x - \cos 3x = \cos x - 2\sin x\cos x - \cos 2x \cos x + \sin 2x \sin x =$

$\cos x\,(1 - 2\sin x - \cos 2x + 2\sin^2 x) = \cos x\,(1 - 2\sin x - 1 + 2\sin^2 x + 2\sin^2 x) =$

$\cos x\,(4\sin^2 x - 2\sin x) = 2\cos x\,(\sin x)(2\sin x - 1) = 0$ where $\cos x = 0$, $\sin x = 0$, $\sin x = 1/2$

∴ solutions are $0, \pi/2, \pi, 3\pi/2, 2\pi, \pi/6, 5\pi/6$ (B)

(34) Period of $2\sin 3x$ is $2\pi/3$, of $\tan \frac{5x}{4}$ is $4\pi/5$. Thus the period is the 'LCM' of these, or 4π, (B)

(35) $\sin\beta = \frac{2}{5}$ $\sin 3\beta = \sin 2\beta\cos\beta + \cos 2\beta \sin\beta = 2\sin\beta\cos^2\beta + (2\cos^2\beta - 1)\sin\beta$

$= 2\left(\frac{2}{5}\right)\left(\frac{21}{25}\right) + \frac{2}{5}\left(\frac{42-25}{25}\right) = \frac{118}{125}$ (B)

(36) $\ln y = x \ln \sin x \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln \sin x + x\frac{\cos x}{\sin x} \Rightarrow \frac{dy}{dx} = (\sin x)^x\left(\ln \sin x + x\frac{\cos x}{\sin x}\right)$ (A)

(37) $K = \frac{1}{2}(14)(12)\sin 18° = 7 \cdot 12\left(\frac{\sqrt{5}-1}{4}\right) = 21\sqrt{5} - 21$ ∴ $a + b = 26$ (A)

(38) Only $\sin x$ & $\cos x$ satisfy it (C)

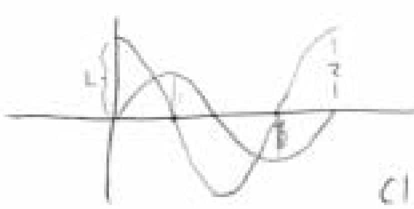
(39) $\cos(8\alpha) = \cos(9\alpha)$ when $8\alpha = 2\pi - \gamma$ & $9\alpha = 2\pi + \gamma$, giving solution $\alpha = 2\pi/17$

$8\alpha = 4\pi - \gamma$, $9\alpha = 4\pi + \gamma$, gives $\alpha = 4\pi/17$. Their sum is $6\pi/17$ (B)

(40) $1 + \sin x = 2(1 + \cos x) \Rightarrow \sin x - 2\cos x = 1$

Graphing $\sin x$ & $2\cos x$ gives:

$x = \pi/2$ is one solution



Also, combining $\sin x - 2\cos x = 1$ w/ $\sin^2 x + \cos^2 x = 1$ gives the solutions $x = \pi/2$ & $x = \operatorname{acos}(-\frac{4}{5}), \operatorname{asin}(-\frac{3}{5})$.

$\sin x - 2\cos x > 1$ for $\pi/2 < x < \pi$.

Clearly there is 1 more solution $\pi < x < 3\pi/2$.

There are no others

2 solutions (B)