

ALPHA TRIGONOMETRY TOPIC TEST ZODIAC CONVENTION

① THE INESTIMABLE BUDDHA ONCE NOTED THAT THERE ARE 3 INVARIABLE, INCONTROVERTIBLE TRUTHS: "A KINT'S A POUND THE WORLD AROUND", "GEORGE W. BUSH WILL NEVER SENSEBLY COMPLETE THREE SENTENCES IN A ROW" AND " $\sin^2 x + \cos^2 x = 1$ FOR ALL x ". GIVEN HIS EXEMPLARY ACCURACY ON THE FIRST TWO COUNTS, WE'LL GIVE HIM THE BENEFIT OF THE DOUBT ON THE 3RD. (A)

② $78^\circ \cdot \frac{\pi}{180^\circ} = \frac{13\pi}{30}$ (D)

③ ~~$0 - \frac{\sqrt{3}}{2} + 1 - 2 + 1 + \frac{\sqrt{3}}{2}$~~ $0 - \frac{\sqrt{3}}{2} + 1 - 2 + 1 + \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$ (D)


④ $1 + \cot^2 42^\circ = \csc^2 42^\circ = \sec^2 48^\circ$ (C)

⑤ (A)

⑥ $\frac{577}{36} \cdot \frac{180}{\pi} = 285^\circ$ (B)

⑦ $\arctan(\sin \frac{\pi}{4}) = \arctan(1) = \frac{\pi}{4}$ (A)

⑧ $90^\circ - 25^\circ 43' 37'' = 64^\circ 16' 23''$ (B)

⑨  $\sin \angle BCD + \csc \angle ACD = \frac{BD}{BC} + \frac{AC}{AD} = \frac{BC}{AB} + \frac{AB}{AC} = \frac{BC \cdot AC^2 + AB^2}{AC \cdot AB}$
 $= \frac{168 + 625}{7 \cdot 25} = \frac{793}{175}$ (B)

⑩ $C = 8\pi$; $\frac{\theta}{2\pi} = \frac{0.5}{8\pi}$; $\theta = \frac{1}{8}$ (C)

⑪ $5x - b = 9(\sec \theta - \tan^2 \theta) = 9 \therefore x = 3$ (C)

⑫ By Law of Sines: $\frac{AB}{\sin \angle ACB} = \frac{BC}{\sin \angle BAC} \therefore BC = \frac{\sin 68^\circ}{\sin 25^\circ}$ (C)

⑬ Periods of $\cos \frac{x}{4}$ & $\sin \frac{x}{2}$ are 8π , 4π resp. \therefore The period of their sum is 8π (B)

⑭ $\sin(\theta - \frac{3\pi}{2}) + \cos(\pi + \theta) = \sin \theta (\cos \frac{3\pi}{2}) - \sin \frac{3\pi}{2} \cos \theta + \cos \pi \cos \theta - \sin \pi \sin \theta = 0$ (A)

⑮ $\frac{\tan x \csc x - \cos^2 x}{1 - \cos^2 x} = \frac{\frac{1}{\cos x} - \cos x}{1 - \cos^2 x} = \frac{1 - \cos^2 x}{\cos x (1 - \cos^2 x)} = \sec x$ (C)

$$(16) z^4 = [2(e^{i\pi/7})]^{14} = 2^{14} e^{2\pi i} = 2^{14} \quad (D)$$

$$(17) \text{Expression} = \ln |\sec^2 u - \tan^2 u| + e^{\ln 1} = \ln 1 + e^0 = 1 \quad (C)$$

(18) $3600 \cdot \frac{\pi}{2} = 1800\pi \therefore$ The graph of $A(t)$ goes through 900 periods in the hour. Brett takes 900 breaths in the hour. (E)

$$(19) \cos(-u) = \cos(u); \sin(-u) = -\sin(u) \therefore \cos(-u) + \sin(-u)\cos(-u) + \sin^2(-u) = \cos u - \sin u \cos u + \sin^2 u = \frac{1}{\sqrt{3}} - \frac{2}{3} \left(\frac{1}{\sqrt{3}}\right) + \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{6 + \sqrt{3}}{9} \quad (D)$$

$$(20) \cos 2925^\circ - \tan 2400^\circ + \csc 2970^\circ = \cos 45^\circ - \tan 240^\circ + \csc 90^\circ = \frac{\sqrt{2}}{2} - \sqrt{3} + 1 = (A)$$

(21) $\frac{43\pi}{3} = 14\pi + \frac{\pi}{3}$ $\frac{31\pi}{2} = 15\pi + \frac{\pi}{2} \rightarrow 14\pi + \frac{\pi}{3} < x < 15\pi + \frac{\pi}{2}$, (D)
 $\sin x$ increases from $14\pi + \frac{\pi}{3}$ to $14\pi + \frac{\pi}{2}$, then decreases the rest of the way

$$(22) 4\cos 4t - 4\sqrt{3}\sin 4t = 8 \left[\frac{1}{2} \cos 4t - \frac{\sqrt{3}}{2} \sin 4t \right] = 8 [\cos 60^\circ \cos 4t - \sin 60^\circ \sin 4t] = 8 \cos(4t + \frac{\pi}{3}) = 8 \cos(4t - \frac{5\pi}{3}) \quad (C)$$

$$(23) = \frac{27}{27 \cdot 18} \cdot \frac{e^{i(4^\circ)} e^{i(64^\circ)}}{e^{i(27^\circ)} e^{i(18^\circ)}} = \frac{1}{18} \left(e^{i(4^\circ + 64^\circ - 27^\circ - 18^\circ)} \right) = \frac{1}{18} e^{i60^\circ} = \frac{1}{18} (\cos 60^\circ + i \sin 60^\circ) = \frac{1}{18} \left[\frac{1}{2} + \frac{i\sqrt{3}}{2} \right] \quad (B)$$

$$(24) \frac{\tan x}{1 + \sec x} + \frac{1 + \sec x}{\tan x} = \frac{\tan^2 x + (1 + \sec x)^2}{(\tan x)(1 + \sec x)} = \frac{(\tan^2 x + 1) + \sec^2 x + 2\sec x}{(\tan x)(1 + \sec x)} = \frac{2\sec^2 x + 2\sec x}{(\tan x)(1 + \sec x)} = \frac{2\sec x(1 + \sec x)}{\tan x(1 + \sec x)} = 2\csc x. \quad (A)$$

A YOU'VE FREQUENT W/ THE DANGER OF A FRODOGAN SLEEP.

$$(25) \text{I: } \cos 2x + \sin^2 x = \cos^2 x \geq 0 \text{ for all } x.$$

$$\text{II: } \sec^2 x + 2\tan x = \frac{1 + 2\sin x \cos x}{\cos^2 x} = \frac{1 + \sin 2x}{\cos^2 x} \geq 0 \text{ for all } x \text{ (since } \sin 2x \geq -1)$$

$$\text{III: For } x = \arctan(-5), \tan^2 x + 5 = -120.$$

$$\text{IV: } 4\cos^2 x - \sin^2 2x = 4\cos^2 x - 4\sin^2 x \cos^2 x = 4(\cos^2 x)(1 - \sin^2 x) \geq 0 \text{ for all } x.$$

I, II, IV

(C)

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26 $\cos^2 x \sin^2 x = \cos x (1 - \sin^2 x) \sin^2 x = \cos x (\sin^4 x - \sin^2 x) = \cos x (u^2 - u)$ (C)

27 $\sin^2 \frac{\pi}{12} + \cos \frac{\pi}{6} + \tan \frac{\pi}{4} = \frac{1 - \cos \frac{\pi}{6}}{2} + \frac{\sqrt{3}}{2} + 1 = \frac{3}{2} + \frac{\sqrt{3}}{2} = \frac{6 + \sqrt{3}}{2}$ (D)

28 $\sin y = \frac{\sqrt{2}}{2} \Rightarrow y = 45^\circ$ $\frac{\tan x + \tan y}{1 - \tan x \tan y} = \sqrt{3} = \tan(\arg) \Rightarrow x + y = 60^\circ \Rightarrow x = 15^\circ$ (B)

29 $\sin^2 5\psi = 1 \therefore 5\psi = \frac{\pi}{2} + 2n\pi$ & $5\psi = \frac{3\pi}{2} + 2m\pi$, or
 ~~$\psi = \frac{\pi}{10} + \frac{2n\pi}{5}$~~ & $\psi = \frac{3\pi}{10} + \frac{2m\pi}{5}$.

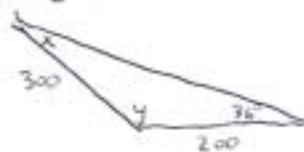
Under the restriction $\frac{3\pi}{5} < \psi < \frac{9\pi}{5}$, we find sol'ns $m=1, 2, 3$, & $n=2, 3, 4$.

The sum of these solutions is $3\left(\frac{\pi}{10} + \frac{3\pi}{10}\right) + \frac{2\pi}{10}(1+2+3+2+3+4) = \frac{36\pi}{5}$ (B)

30 From Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$. $a = 4 \sin 45^\circ$, $b = 4 \sin 30^\circ$
 $c = 4 \sin 105^\circ = 4 \sqrt{\frac{1 - \cos 210^\circ}{2}} = 4 \sqrt{\frac{1 + \sqrt{3}/2}{2}} = 2\sqrt{2 + \sqrt{3}} = 2\left(\frac{\sqrt{6} + \sqrt{2}}{2}\right) = \sqrt{6} + \sqrt{2}$.
 $a = 2\sqrt{2}$, $b = 2$. $P = 2 + 3\sqrt{2} + \sqrt{6}$ (B)

31 Let $x = \sin 2\theta = 2 \sin \theta \cos \theta \therefore$ given eqn is $4x^2 - 6x + 11 = 7 + 5x - 2x^2$:
 $6x^2 - 11x + 4 = (2x - 1)(3x - 4) = 0$: $x = \frac{1}{2}$ or $x = \frac{4}{3}$. latter produces
 no sol'ns for θ . Former gives $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ in
 $[\frac{\pi}{12}, \frac{25\pi}{12})$ (D)

32 Law of Sines: $\frac{\sin x}{200} = \frac{\sin 36^\circ}{300}$



33 $y \cos x - \sin 2x - \cos 3x = \cos x - 2 \sin x \cos x - [\cos^2 x \cos x - 2 \sin x \cos x] =$
 $\cos x - 2 \sin x \cos x - (1 - 2 \sin^2 x) \cos x + 2 \sin^2 x \cos x = \cos x [4 \sin^2 x - 2 \sin x] =$
 $2 \cos x \sin x [2 \sin x - 1] \therefore y = 0$ when $\cos x = 0$, $\sin x = 0$, $\sin x = \frac{1}{2}$.
 The first has 4 sol'ns on the interval, the 2nd has 5, and the last 4.
 13 total (B)

34 Law of sines: $\frac{8}{\sin 76^\circ} = 2R$: $R = 8$, Area = 64π (D)

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$$(35) \tan A = \frac{28}{45}, \tan B = -\frac{20}{21} \quad \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{28}{45} - \frac{20}{21}}{1 + \frac{28}{45} \cdot \frac{20}{21}} = -\frac{312}{1505} \quad (B)$$

(36) There are $\binom{6}{2} = 15$ ways to choose a pair; only one satisfies the given criterion $\frac{1}{15} \rightarrow (A)$

$$(37) (-r, \theta + 5\pi) = (-r, \theta + \pi) = (r, \theta + \pi - \pi) = (r, \theta) \quad (A)$$

$$(38) K = \frac{1}{2} ab \sin \theta, \text{ so the sought ratio is } \frac{\sin 2\theta}{\sin \theta} = \frac{2 \cos \theta \sin \theta}{\sin \theta} = 2 \cos \theta = 2 \left(\frac{95}{93} \right) = \frac{190}{93} \quad (C)$$

$$(39) \text{ Period is } \frac{2\pi}{\left(\frac{2}{3}\right)} = \frac{3\pi}{1} \quad (D)$$

$$(40) K = \frac{1}{2} ab \sin \theta = \frac{1}{2} (\sqrt{6})(2\sqrt{3}) \sin \theta = 3\sqrt{2} \sin \theta.$$

Max value of $\sin \theta$ is 1, so max area is $3\sqrt{2} \quad (B)$