Solutions – Equations & Inequalities
Alpha Page 1

1. $\text{Bart} \geq \text{Arthur} + \frac{100}{100}\text{Arthur} \geq 2\cdot\text{Arthur}$

2. $\text{Chris} \geq \text{Bart} + \frac{300}{100}\text{Bart} = 4\cdot\text{Bart} = 8\cdot\text{Arthur}$

$\text{Donald} \geq \text{Chris} + \frac{50}{100}\text{Chris} = 1.5\text{Chris} = 12\cdot\text{Arthur}$

$\text{Arthur} \leq \frac{\text{Donald}}{12} \leq 350{,}000$ A

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2. $x^2 - 7x + 17 > 5$

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

Case I: $x = 5$, $x^2 - 7x + 17 = 7$, $7 > 5$

Case II: $x = 3.5$ $x^2 - 7x + 17 = 4.75$, $4.75 < 5$

Case III: $x = 2$ $x^2 - 7x + 17 = 7$, $7 > 5$

$x < 3$ or $x > 4$, D

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3. $x^3 = -27$

$x = -3, -3\angle 120, -3\angle -120$

$x = -3,\ -3(\cos 120 + i\sin 120),\ -3(\cos 120 + i\sin -120)$

$x = -3,\ \frac{3}{2} - \frac{3\sqrt{3}}{2}i,\ \frac{3}{2} + \frac{3\sqrt{3}}{2}i$ B

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4. $\cot x = 1$ when $\tan x = 1$

$\tan x = 1$ at $45°, 225°$ A

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5. $|\sec x| = 1$ when $|\cos x| = 1$ D $0°, 180°$

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6. $x = \frac{8}{3 + 5f^{-1}(x)}$ $\quad 3x + 5x f^{-1}(x) = 8$

$f^{-1}(x) = \frac{8 - 3x}{5x}$ A

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7. $f(14) = \frac{2(14)^2}{3} = \frac{392}{3}$

$g(d) = g\left(\frac{392}{3}\right) = 3\left(\frac{392}{3}\right) + 2 = 394$ C

Equations and Inequalities
Solutions – Alpha page 2

8. $2^{f-4}2^{4f+16} = 2^{9f-9}$

$5f + 12 = 9f - 9$

$-4f = -21 \quad f = \frac{21}{4}$ D

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9. $2^{x-2}3^{x-2}2^{3x+6}3^{y+2} = 2^6 3^{10}$

$x + 3y + 4 = 6$

$x + y = 10$

$-x - y = -10$

$2y = -8$

$y = -4 \quad x = 14$

B

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10. $3x^3 + 6x^2 + 4x + 1 = 0$

First solution, -1 can be found by trial and error or with a calculator.

$$x+1 \overline{)\,3x^3 + 6x^2 + 4x + 1}$$

$3x^2 + 3x + 1$

$-3x^3 - 3x^2$

$3x^2 + 4x$

$-3x^2 - 3x$

$x + 1$

$-x - 1$

0

Sum of imaginary roots $= \frac{-b}{a} = \frac{-3}{3} = -1$ D

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11. $x^5 - 2x^2 + 3x + 5 = f(x)$

$\frac{-p}{E} = \frac{-3}{5}$ A

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12. $f(x-3) = 3x^2 - 5x - 2$

Let $u = x - 3 \quad x = u + 3$

$f(u) = 3(u+3)^2 - 5(u+3) - 2$

$f(u) = 3u^2 + 18u + 27 - 5u - 15 - 2$

$f(u) = 3u^2 + 13u + 10$

$f(x) = 3x^2 + 13x + 10$

$\frac{f(x)}{5} = \frac{3x^2}{5} + \frac{13x}{5} + 2$

C

13. $\frac{3x+9}{x^2+7x+10} = \frac{A}{x+2} + \frac{B}{x+5}$

$3x+9 = A(x+5) + B(x+2)$

$3 = A+B$ C

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14. $\frac{25x-11}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$

$25x-11 = A(x-1) + B$

$A = 25$ $\quad -A+B = -11$

$B = 14$ B

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15. $x + y + z = 1$

$3x - 2y + 3z = 5$

$2x + 2y + 2z = 2$

$5x + 5z = 7$

$x = -2 + \frac{7}{5}$ B

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16. $x + 3y + 4z = -5$

$-x + y - 2z = 5$

$-5x - y - z = 1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & -5 \\ -1 & 1 & -2 & 5 \\ -5 & -1 & -1 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 3 & 4 & -5 \\ 0 & 4 & 2 & 0 \\ 0 & 14 & 19 & -24 \end{array}\right] \begin{array}{l} \\ R_1+R_2 \\ 5R_1+R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 4 & -5 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 14 & 19 & -24 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{2} & -5 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 12 & -24 \end{array}\right] \begin{array}{l} R_1-3R_2 \\ \\ R_3-14R_2 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{2} & -5 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} R_1-\frac{5}{2}R_3 \\ R_2-\frac{1}{2}R_3 \\ \\ \end{array}$$

$x = 0$ $\quad y = 1$ $\quad z = -2$ B

Solutions - Equations & Inequalities
Alpha page 4

17. $\log_2(2^{2/3}x^{1/3}) + \log_2(2x^4) = -47/3$

$\log_2(2^{5/3}x^{13/3}) = -47/3$

$2^{5/3}x^{13/3} = 2^{-47/3}$

$x^{13/3} = 2^{-52/3}$ $\quad x = 2^{-4} = \frac{1}{16}$ E

$\sqrt{x} - x - 10x^2 = \frac{1}{4} - \frac{1}{16} - 10\left(\frac{1}{256}\right) = \frac{38}{256} = \frac{19}{128}$ ~~A~~

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18. $\frac{\log x}{\log 10} = \frac{\log 27}{\log 9}$ $\quad \log x = \frac{3}{2}$ $\quad x = 10\sqrt{10}$

$x^3 - 50x = 10\,000\sqrt{10} - 500\sqrt{10} = 9500\sqrt{10}$ A

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19. I want $\lfloor \log_2 N \rfloor$ and $\log_2 N$ the greatest possible distance apart. This is when $N = 3^x - 1$ where x is any positive integer.
In this problem, x is constrained to be 6 or less. The lowest value is $5 - \log_3 728$ B

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20. $\frac{x-2}{-3x-2} \geq -3$

Critical point at $x = -2/3$

$x - 2 \geq 9x + 6$

$x \leq -1$ Critical point at $x = -1$

case I: $x = -2$ $\quad \frac{x-2}{-3x-2} = \frac{-4}{~~8~~4} = \frac{5}{2} - 1 \geq -3$

case II: $x = \frac{-3}{4}$ $\quad \frac{x-2}{-3x-2} = \frac{-11/4}{1/4} = -11 \leq -3$

$x \leq -1$ or $x \geq -2/3$ E

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21. $\text{Bill} = \text{Mary} + 5$

$\text{Bill} + 3 = 7(\text{Mary} - 10)$

$-\text{Bill} = -\text{Mary} - 5$

$3 = 6 \cdot \text{Mary} - 75$

Mary = 13 $\quad$ Bill = 18 D

22. $24 - 448q + 11 = 5$

$q = \frac{15}{224}$ **E**

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23. $4x^2 + 30x + c = 0$

case I: $2\left(\frac{-30 + \sqrt{900 - 16c}}{8}\right) = \frac{-30 - \sqrt{900-16c}}{8}$

$-30 = -3\sqrt{900-16c}$

$10 = \sqrt{900-16c}$

$c = 50$ **C**

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24. $A + B = 3$
$A + D = 2$
$D + E = 2$
$A + E = 2$

$A + D = 2$
$-E - D = -2$
$A - E = 0$
$A + E = 2$
$2A = 2$

$A = 1$ $B = 2$ $D = 1$ $E = 1$ $C = 0$

C

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25. $m = 4k^{-1}(m) - 1$

$\frac{m+1}{4} = k^{-1}(m)$ **B**

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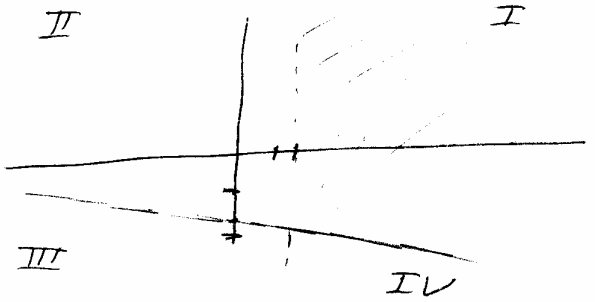
26. $-3y < x + 5$

$y \geq -\frac{1}{3}x - \frac{5}{3}$ $x \geq 4/3$



B

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27. **C** ellipse

28. $f(7) = \frac{35}{25} = \frac{7}{5}$ $\quad g(\frac{7}{5}) = \frac{63}{5} - \frac{10}{5} = \frac{53}{5}$ B

29. $9w - 39 < 94$
$9w < 133$
$w < 14\frac{7}{9}$ D

30. $t + 1 < -t - 4$ $\quad -t - 4 < 3t + 17$
$2t < -5$ $\quad -21 < 4t$
$t < -5/2$ $\quad t > -21/4$ D

31. $x = \frac{a}{1+r}$ $\quad y = \frac{a}{1-r^2}$ $\quad z = \frac{a}{1-r^3}$

$y = \frac{a}{(1+r)(1-r)}$ $\quad z = \frac{a}{(1-r)(1+r+r^2)}$

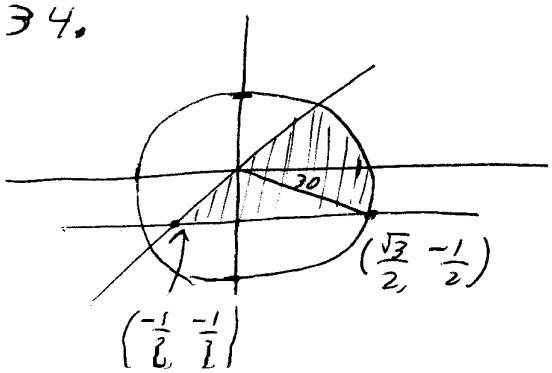
$\frac{a}{(1+r)(1+r+r^2)} = \frac{a(1-r)(1+r)}{(1-r)(1+r)(1+r+r^2)} = \frac{zx}{y}$ B

32. $\cot\theta \geq 1$, $\tan\theta \leq 1$ $\quad 0 \leq \theta \leq \frac{\pi}{4}$ D

33. period of $\sin(x)$ is 2π
period of $\sin(2x)$ is π
period of $\sin(3x)$ is $2\pi/3$
period of $\sin(4x)$ is $\pi/2$
period of $\sin(5x)$ is $2\pi/5$
period of function is $LCM(2\pi, \pi, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{5}) = 2\pi$

B

34.



Area of 75° wedge $= \frac{75}{360} \cdot \pi = \frac{5\pi}{24}$

Area of triangle $= \frac{1}{2}bh$
$= \frac{1}{2}(\frac{1}{2} + \frac{\sqrt{3}}{2})(\frac{1}{2}) = \frac{1}{8} + \frac{\sqrt{3}}{8}$

Total Area $= \frac{5\pi}{24} + \frac{1}{8} + \frac{\sqrt{3}}{8}$ C

Alpha Equations & Inequalities
Solutions page 7

35. $y(x) = \text{Arcsin}(\ln(x))$

domain of Arcsin is $-1 \le x \le 1$

$-1 \le \ln(x) \le 1 \qquad \frac{1}{e} \le x \le e$ **E**

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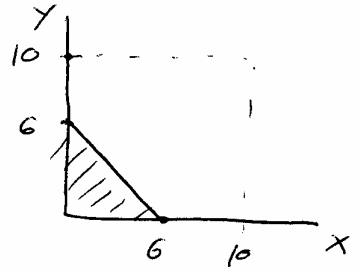
36. 

$\text{probability} = \frac{\frac{36}{2}}{100} = \frac{9}{50}$

A

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37. $|(4-3i)|\,|(5+12i)|\,|(8-15i)|$

$= (5)(13)(17) = 1105$ **A**

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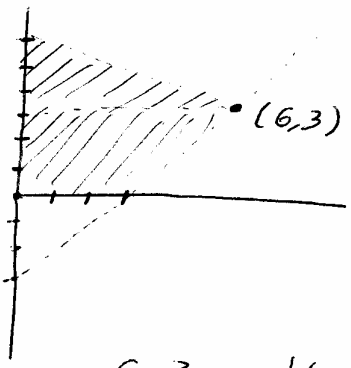
38. 

$y \ge x - 3 \qquad y \le \frac{-x}{2} + 6$

$\text{Area} = \frac{6 \cdot 3}{2} + \frac{1}{2}(6+3)(3) = 9 + \frac{27}{2} = \frac{45}{2}$ **C**

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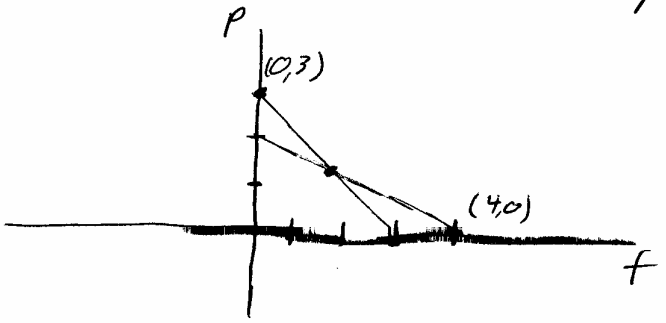
39. Let f = servings of french fries

Let p = servings of pizza

$600f + 600p \ge 1800$

$2f + 4p \ge 8$

minimize $f + 1.5p$



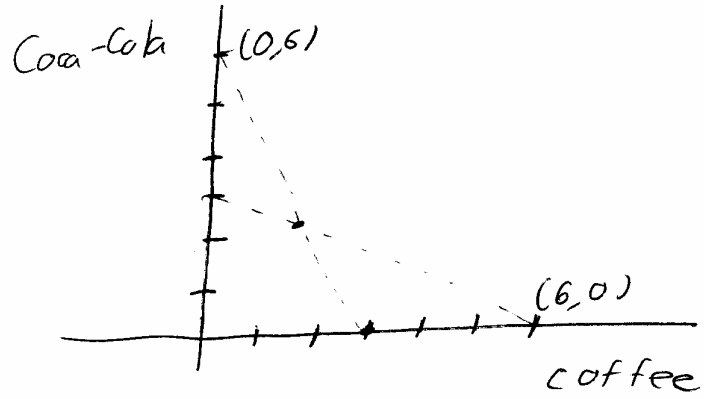
corner 1: $(f,p) = (4,0) = \$4.00$

corner 2: $(f,p) = (2,1) = \$3.50$

corner 3: $(f,p) = (0,3) = \$4.50$

D

$$25x + 50y \geq 150$$
$$10x + 5y \geq 30$$
minimize $x + 1.5y$



$x + 2y = 6$
$2x + y = 6$

$-3y = -6$
$y = 2$
$x = 2$

3 corners: $(6,0) = \$6.00$
$(0,6) = \$9.00$
$(2,2) = \$5.00$

A