

# THE MATHEMATICAL LOG

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## *A Bag of Tricks*

*Pete Pedersen  
Vinalhaven, Maine*

All competitors in mathematical competitions should have a bag of tricks...tricks that can crack problems quickly and most of all efficiently. Often, I have found competition writers utilize short cuts. Over the last ten years, I have gathered short cuts for problems in algebra, geometry and number theory. I do not claim to be the expert but a collector. The following will feature number theory problems which I have shared with students at regional and national conventions. Hopefully, these examples will be helpful in your competitive pursuits.

Often, we are presented with a problem that shocks us initially. An example of this was a ciphering problem:

123456789 123456789 - 123456798 123456780. First, substitute  $x$  for the fourth value. The second number is identical to the first. Again, substitute the second with  $x$ . The third number is nine greater than the first. Replace the third number with  $x + 9$ . The last number is nine less than the first. Use  $x - 9$ . We find that this problem is reduced to  $(x)(x) - (x + 9)(x - 9)$ . Simplify:  $x - (x - 81)$ . The  $x$ 's cancel leaving  $+81$ . Answer: 81.

Understanding how zeros are found can help you with several different problem types. The first example is "How many digits will occur in the product of  $(16)^8(125)^9$ ?" Rename these two numbers using 2's and 5's. This change in bases produces  $(2)^{32}(5)^{27}$ . For each two and five, we yield a zero. 27 such zeros can be found. This now leaves  $2^5=32$ . Write 32 followed by 27 zeros. This product has 29 digits.

Let's continue this by looking at factorial problems. "In how many zeros does  $30!$  end"? Nice question. It can be quickly found if we look for factors of two and five. We know that there are more two's than five's. Count the five's. Use the multiples of 5: 5, 10, 15, 20, 25, 30. 5, 10, 15 and 20 each have one factor of five, twenty-five has two factors of five and the 30 has only one factor of five. This gives a total of seven, fives. Answer,  $30!$  ends in seven zeros. At the *Mu Alpha Theta* National Convention in Denver, Colorado, there was an interesting number theory problem similar to the following: " $n!$  ends in exactly nine zeros. What is the largest possible value of  $n$  which is a multiple of three?" Again, we can use the solution from the preceding problem. We know that  $30!$  has seven zeros.  $35!$  - eight zeros,  $40!$  - nine zeros,  $45!$  - ten zeros. Thus,  $40!$ ,  $41!$ ,  $42!$ ,  $43!$ ,  $44!$  all have nine zeros. 42 is the only multiple of three. Answer:  $n = 42$ .

The number of positive, integral factors or divisors create a new set of problems. An example being, "Find the largest integer less than one hundred that has an odd number of factors". Odd factors are associated with perfect squares. 81 would be the answer. How many positive integer factors does 84 have? First, find the prime factorization of 84.  $84 = (2)(3)(7)$ . For each prime number base, use a set of parenthesis. ( ) ( ) ( ). In each set of parenthesis write the number one for each prime base plus the

## News

...continued on page 4

## • Fourth Annual High School Mathematical Contest in Modeling

COMAP, NSF, NCTM, and the MAA announce the Fourth Annual High School Mathematical Contest in Modeling. High school teams of up to four members each may choose a thirty-six hour period between 7 and 20 November, 2001, inclusive, to solve their choice of two real-world problems. Registration information, brochures, and contest rules can be found on the Web at

[www.comap.com/highschool/contests](http://www.comap.com/highschool/contests)

or contact Clarice Callahan at COMAP 217-862-7878 ext. 37 or [c.callahan@mail.comap.com](mailto:c.callahan@mail.comap.com) for information.

The required *pre-registration deadline* for the November 2001 contest is November 1, 2001.

## • National Convention

Come for a heapin' helpin' of  
Southern hospitality, charm, and

*Magnolia Math*

at the 32<sup>nd</sup> Annual  
Mu Alpha Theta National Convention  
to be held at  
Mississippi State University  
in  
Starkville, Mississippi

July 26- 31, 2002

Hosted by Mississippi School of Mathematics and Science  
The cost will be between \$250-\$300 per person.

For more information contact:

Claudia Carter ([ccarter@msms.k12.ms.us](mailto:ccarter@msms.k12.ms.us))

or

Aubrey Knight ([aknight@msms.k12.ms.us](mailto:aknight@msms.k12.ms.us))

## • MA $\Theta$ Student Delegate/Chapter President Listserve

Student Delegates and Chapter Presidents are invited to join a *listserv* by sending an e-mail to [subscribe-delegate@topica.com](mailto:subscribe-delegate@topica.com). Student Delegates and Chapter Presidents will all play a large role in the success of Mu Alpha Theta this year, through this mail list. The mail list throughout the year will have an open discussion of any topics related to Mu Alpha Theta. Sponsors are also

invited to join the list to provide some additional suggestions and comments to members.

## • Texas State Convention

If you have any information about the Mu Alpha Theta Texas Convention please contact Diane Rubin at the National Office either by phone (405-325-4489) or e-mail at [matheta@ou.edu](mailto:matheta@ou.edu).

## • Region IV MA $\Theta$ Convention

Carter High School,  
Strawberry Plains, TN,  
will host the Region IV Convention on  
December 7 & 8, 2001

If you have any interest in attending, please e-mail or write to us as soon as possible. Also, please choose a student delegate to join our e-mail list in order to keep up with the latest happenings for this convention. We can be reached at either [LeekJD@aol.com](mailto:LeekJD@aol.com) or [Briano32z@cs.com](mailto:Briano32z@cs.com).

## • MA $\Theta$ Website

Check out the National Office website at  
[www.mualphatheta.org](http://www.mualphatheta.org)  
for the latest information from  
the national office.

## • *The Art of Problem Solving*

*Paul Zeitz, former coach of the United States Olympiad team, referred to the texts as a **must** for any serious math library.*

The ART of PROBLEM SOLVING is a pair of texts for those students involved in extracurricular mathematics. The authors, Sandor Lehoczky and Richard Rusczyk are former Mu Alpha Theta students and participants in Math Olympiad Programs (students who pass the AHSME/AIME/USAMO series of tests produced by the Mathematical Association of America).

The texts have been used by successful students, teams, and clubs in grades 7-12 since 1994. Volume 1 covers mathematics commonly occurring in MATHCOUNTS and the AHSME, as well as many local and state math contests. Volume 2 covers more advanced topics found on the AHSME, as well as the AIME and USAMO. Included in the texts are hundreds of challenging problems from national tests such as ARML, MathCounts, the national Mu Alpha Theta conventions, and the MAA series of exams which determine the national math team. If you have further questions about the texts, contact the national office at [matheta@ou.edu](mailto:matheta@ou.edu) or 405-325-4489. Prices are shown below; make checks and purchase orders payable to Mu Alpha Theta. Discounts for bulk orders are available.

|                                    |      |
|------------------------------------|------|
| Volume 1 text                      | \$26 |
| Volume 1, text and solutions       | \$34 |
| Volume 2, text                     | \$28 |
| Volume 2, text and solutions       | \$37 |
| Volume 1 and 2, text               | \$50 |
| Volume 1 and 2, text and solutions | \$64 |

## *2001 Kalin Award Winner*

### David Smith

Catholic High School  
Baton Rouge, Louisiana

David Smith, the winner of the Kalin award is a most deserving recipient. He has had an extensive relationship with Mu Alpha Theta at the local, district, state and national levels. He served as club treasurer during his sophomore year, district governor in his junior year and state vice-president, as well as national president in his senior year. He has done an enormous amount to get others involved in the organization and has been tireless in his enthusiasm to get everyone involved. During all of his high school career, David encouraged a wide variety of students to become involved with Mu Alpha Theta. He always served as a mentor to younger students trying to find a place to belong in the school. He felt that anyone could benefit from involvement and always generated warmth and caring.

David has had an interest in mathematics from the time he was first challenged by a math problem. He loves to analyze a challenging problem and discuss it with others. He was a Mathcounts team member as an eighth grader and was part of a team that advanced to state competition. He was so energized by the atmosphere and the competition and the sheer joy of learning that it was obvious very early that his magnetic personality would take him to exciting places.

David has grown a lot as an individual through his involvement with Mu Alpha Theta. It has enhanced his leadership qualities and has taught him a lot about working with others. He networked with students from other schools in our city, state and the nation. He has formed lasting friendships through Mu Alpha Theta and has enjoyed the fruits of a lot of hard work.

David has begun his college career at Massachusetts Institute of Technology where he is anxious to embrace new learning experiences. He met several incoming freshmen at the

national convention and will continue to be involved with these outstanding young people as he begins college.

In conclusion there has been a symbiotic relationship between David and Mu Alpha Theta. He has gained an enormous amount through his work with the organization and the organization is better for having had him involved. I feel priveleged to have worked with David as a teacher and mentor and am a better person for having known him.

*Peggy Patout*  
Sponsor

Catholic High School, Baton Rouge, LA

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## *Congratulations to the 2001 Mu Alpha Theta Award Winners*

### **Kalin Award:**

David Smith  
Catholic High School, Baton Rouge, LA

### **Kalin Award Finalists:**

Esther Lee  
Thomas Jefferson High School, Auburn, WA  
David Held

Miami Sunset High School, Miami, FL

Eric Seymour

Mississippi School of Math and Science, Columbus, MS

### **Huneke Award:**

Susan Doker  
Lincoln High School, Tallahassee, FL

### **Sister Scholastica Award:**

Peggy Patout  
Catholic High School, Baton Rouge, LA

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*Once again The Mathematical Log takes on a  
new look. No longer will the column*

*$\sqrt{\text{At the Root of it All}}$*

*appear.*

*Since the early 1980s Deborah Patonai,  
Activities Editor, has written the column which  
features the Mu Alpha Theta Awardees. It is  
with great thanks that we must wish well to Ms.  
Patonia as she leaves a history of award winners  
behind.*

*Thank you.*

***National Convention Scholarship  
Winners***

Last Spring the Governing Board Council awarded scholarships to the National Convention held in Denver, Colorado. The winners each received travel expenses (up to \$500) and registration to the convention. The winners were:

Region I: Ed Pivarsky  
Sponsor: Malinda Shirley  
Tahoma High School  
Kent, Washington

Region II: Adriana Vasquez  
Sponsors: Consuelo Rubio  
Karl C. Parrish High School  
Barranquilla, Columbia, South America

Region III: David Lin  
Sponsor: Todd Taylor  
Auburn High School  
Auburn, Alabama

Kelly Ochs  
Sponsor: Christine Free  
Lassister High School  
Marietta, Georgia

Region IV: Ashley Wyrick  
Sponsor: Lina Demarotta  
Gibbs High School  
Corryton, Tennessee

Region IV: John Starks  
Sponsor: Susan Fritsch  
Providence High School  
Charlotte, North Carolina

*Congratulations to all the scholarship winners. We hope to see you at upcoming conventions.*

## *A Bag of Tricks* continued

power of the base..  $(1+2)(1+1)(1+1) = (3)(2)(2) = (12)$ . There are 12 factors. Let's continue. "What is the smallest, positive integer which has exactly 12 divisors?" Think products: (2 by 6), (3 by 4) and (3 by 2 by 2). We will work backwards.  $(2)(6) = (1+1)(1+5)$ . Take the two smallest, positive prime numbers. 2 and 3. Raise the two to the fifth power and the three to the first power.  $(32)(3)$  or 96. Do this for 3 by 4.  $(1+2)(1+3)$ . Take 2 to the third and three to the second.  $(8)(9) = 72$ . Now, the final one.  $(3)(2)(2) = (1+2)(1+1)(1+1) = 2$  to the second, 3 to the first and five to the first.  $(4)(3)(5) = 60$ . Find our three calculations, we find 96, 72 and 60. Since 60 is the smallest, positive answer; 60 is the answer.

What if we were asked "to find the smallest, positive integer with exactly 9 integral factors which is not divisible by 3?" Same method as before. Look at  $(3)(3) = (1+2)(1+2)$ . This time use the prime numbers 2 and 5, but not the three. 2 to the second and 5 to the second.  $(4)(25)$  or 100.

Finally, "What is the sum of the positive, integral factors of 60?" Find the prime factorization namely  $(2)^2(3)(5)$ . We will not name the factors. Take the largest power of 2 or 2 in this case. Add it to subsequent powers of two until you reach the zero power. Here, we have  $(2+2+2)$ . Do this for each of the remaining factors.  $(3+3)$  and  $(5+5)$ . Take the results and multiply them together:  $(4+2+1)(3+1)(5+1) = (7)(4)(6)$  or 168.

These are only a few of the many short cuts. Enjoy these, investigate new ones and share them with friends.

*Pete Pedersen*

## The Mathematical Log

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The *Mathematical Log* is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association of America (MAA), the National Council of Teachers of Mathematics (NCTM), and the Society of Industrial and Applied Mathematics (SIAM). Correspondence may be directed to Mu Alpha Theta National Office, 610 Elm Ave., Room 423, Norman, OK 73019, email: [matheta@ou.edu](mailto:matheta@ou.edu) or to Log editor Pat Bowler Johnson, New Trier High School, 385 Winnetka Avenue, Winnetka, IL 60093, email: [bowlerjp@newtrier.k12.il.us](mailto:bowlerjp@newtrier.k12.il.us) © 2001 Mu Alpha Theta

### National Officers:

|                      |  |
|----------------------|--|
| President:           | <b>Harold Reiter</b><br>University of North Carolina - Charlotte                       |
| Past-President:      | <b>Claudia Carter</b><br>Mississippi School for Math and Science                       |
| Secretary-Treasurer: | <b>Paul Goodey</b><br>University of Oklahoma - Norman                                  |
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### National Student Officers:

|                      |  |
|----------------------|--|
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| Sargent at Arms:     | <b>Nathaniel Watson</b><br>Farragut High School, Knoxville, TN<br><a href="mailto:Smimi999@aol.com">Smimi999@aol.com</a>                 |

## Letters from...

- **Harold Reiter,**  
*National MAΘ President*

Dear Mu Alpha Theta member,  
I am happy to be your new president. This note in *The Mathematical Log* traditionally enables new presidents to introduce themselves and to discuss

plans for the organization. I am a 32-year veteran of teaching at the college level. I've taught at universities in Hawaii, Maryland, South Carolina (Clemson), and London, England, besides the University of North Carolina Charlotte, where I've spend most of my career. My work with pre-college students includes working with math clubs and working on mathematics contests.

In 1987 I founded the Charlotte Mathematics Club for students in the Piedmont region of North Carolina. It is still going strong today with about 50 members a year from more than a dozen schools. In 1990 I founded the Mecklenburg Math Club for children in grades 4 to 6. This club also is still thriving. Although I do not run either club, I still enjoy volunteering at the meetings and going on trips with them. You can learn more about these clubs by going to their websites,

<http://www.davidson.edu/math/cmc/default.html> and  
<http://www.math.uncc.edu/~hbreiter/clubs/mmc.htm>

Since 1981 I have been involved with organizing, writing and promoting math contests for middle and high school students. I still work hard promoting the UNCC's local contest, the American Math Competitions and MathCounts. I believe in challenges, more so in students challenging themselves to find solutions than in winning competitions. I believe problem solving can be taught and learned. Problem solvers are people who can sometimes reach within themselves for new ideas. They have the confidence to change their new environment.

I want to be sure we provide these growth opportunities for all our members:

1. Problem solving.
2. Organizing activities for local chapters and regional conferences.
3. Speaking locally, at state meetings, and at nationals about mathematics.
4. Learning new mathematical ideas and applications.

I look forward to a great two years as your president.

Cheers, **Harold Reiter**

- **Susan Hiller,**  
*Governor of Region III*

The National Mu Alpha Theta Governing Board is in the process of refining the National Convention Guidelines. We would like for some of the things at

the convention to be standard so there are no surprises when attending from year to year. However, each convention host should also have some leeway to have some events that are unique to their state or region. As the Governor of Region III, I was asked to be the person in charge of this task. I am looking for suggestions from sponsors and students. Could you please take the time to respond to the questions.

1. How many questions should be on the Individual Tests? What should the time limit be?
2. How many questions should be on the Topic Tests? What should the time limit be?
3. What are some topics that you would like to see included in the topic tests? What level are those tests (Euclidean, Theta, Alpha, Mu)?
4. What other events would you like to see included at the convention that are not included now?
5. What events could be used for the first night of the convention for students to get to know each other?
6. What events could be used instead of or at the same time as the scheduled dances (for those who do not want to attend a dance)?
7. What different kind of items could be given as door prizes?
8. What would like to add that hasn't been asked above?

You can respond to these questions by email at [hillerse@yahoo.com](mailto:hillerse@yahoo.com). Also, these questions will be put on the Mu Alpha Theta *listserv* in a few days. Your input is needed. Thanks for taking the time to give your opinions.

**Susan Hiller**

### What would you like to see in the Mathematical Log?

Articles written by Mu Alpha Theta Members/Sponsors? Math Applications? Technology Articles? Interesting Math Concepts? Puzzles? Articles about chapters? Let me know what you'd like to see! Send your comments and any topics you would like to see in the Mathematical Log to:

Pat Bowler Johnson  
New Trier High School  
385 Winnetka Avenue  
Winnetka, IL 60093  
email: [bowlerjp@newtrier.k12.il.us](mailto:bowlerjp@newtrier.k12.il.us)

- **Alex Lee,**  
*National MA $\Theta$  Student Delegate President*

Dear Mu Alpha Theta Members,

My name is Alex Lee, I am a student at North Broward Preparatory School in Coconut Creek, FL and I am this year's MA $\Theta$  Student Delegate President. During the convention, it is my duty to make sure our delegate meetings run smoothly and stay on topic. Yet, before next year's convention, I have many more tasks. This year, it is

my intention, working with my fellow officers Elaine Hart (Vice President), Megan Repar (Secretary) and Nathaniel Watson (Sergeant at Arms), to deal with a number of issues which we as a delegation believe needs to be looked into with regard to the National Convention and our local chapters. Following is a short list of the topics we have on the table at the moment:

Online Test Database (Downloadable Site)  
 Business Sponsorship (Microsoft, HP, ...)  
 Fundraising for local chapters  
 Getting smaller chapters to the National Convention  
 Recruitment

I believe that with enough time and effort, we as a national assembly can get these items and more resolved before next year's convention, and truly make an impact.

If you have any other topics that you believe should be addressed, please feel free to e-mail me at [StormLX@aol.com](mailto:StormLX@aol.com).

If you are a student delegate and you have not yet joined out List Server, send a blank e-mail to [subscribe-delegate@topica.com](mailto:subscribe-delegate@topica.com).

See you all in Mississippi!

Yours Truly,  
**Alex J. Lee**  
 MAΘ Student Delegate President

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## 2001-2001 National Student Officers

President: Alex Lee  
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 Secretary/Treasurer: Megan Repar  
 Lafayette High School, Louisiana  
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 Farragut High School, Tennessee

## New Mu Alpha Theta National Listserve

Sam Koski, Miami Springs Senior High

At the recent national convention in Denver, the board approved the creation of an email *listserve* for Mu Alpha Theta. In an organization as large and as geographically vast, a *listserve* can provide a convenient way for members to keep in touch and up-to-date. Since every member sees each posted email, it provides an excellent way to quickly distribute information. The list also provides a forum for extended discussions. Members can read each post when convenient and if desired submit their comments or suggestions. One of the most helpful

uses of the list serve is for getting information. Since the most knowledgeable members of Mu Alpha Theta see each post, it is a great way to get answers to many questions or help with a variety of problems. Each sponsor has access to the accumulated wisdom of every other sponsor. All this is available by posting an email.

Since email messages are quickly posted to all members, it is important that you take care with any message that you post. Without the extra cues available in face-to-face communication, it is much easier to misinterpret written material. As a writer, chose your words with care. As a reader, give the poster the benefit of the doubt. There will be many &quot; hot &quot; ; topics discussed in the future. With such a diverse membership, it is natural that there will be many contested issues. It is acceptable to disagree on policy or topics, but it is not acceptable to vent your frustration with personal attacks. Remember, we ALL are working to improve our organization and each opinion should be read thoughtfully and discussed maturely. Each message should include the poster's name and school.

Some possible discussion topics are:

- Balancing & quot
- Convention & quot
- Competition & quot
- Effective Speakers
- Exceptional Chapter Activities.

To get the most from a *listserve*, it is necessary to include as many interested parties as possible. Encourage others to subscribe. They must subscribe in order to post or read.

There are two ways to subscribe

1. Send an email to [MuAlphaTheta-subscribe@topica.com](mailto:MuAlphaTheta-subscribe@topica.com). The *listserve* will automatically extract your email address and add you to the list.

2. A subscription form is also available at: <http://www.topica.com/lists/MuAlphaTheta/subscribe/?location=listinfo>

3. If you still can't get subscribed. Send me an email at [ASKoski@aol.com](mailto:ASKoski@aol.com) with the email addresses that you want to include. I will make sure that they get included.

**Sam Koski**