MU ALPHA THETA UNVEILS REDESIGNED WEBSITE!!!

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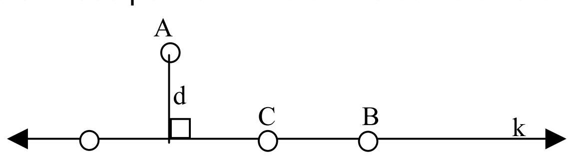
My Favorite Proof: Sylvester's Problem

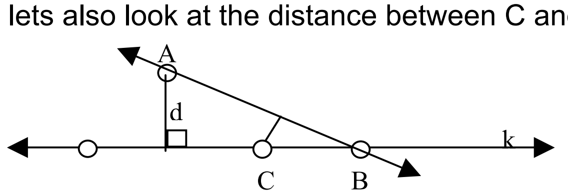
Nathaniel Watson
Mu Alpha Theta National Student Officer
Sergeant at Arms
Farragut High School, Knoxville, TN

Sylvester's Problem is to prove that "It is not possible to arrange a finite number of points so that a line through every two of them passes through a third unless they are all on a single line." If you drew a bunch of dots that would disprove this statement, I think you'll quickly be convinced that the statement. However, actually proving the statement isn't quite as easy. In fact, Sylvester himself, a very great mathematician, never proved it. In fact, this problem stumped the entire mathematical community for forty years, from 1893 when it was first proposed until 1933 when it was finally solved by T. Gallai. Gallai's proof was extremely complicated and accessible to only a fraction of mathematicians. However, in 1948 L. M. Kelly published the following proof of Sylvester's Problem, a proof that is extremely simple considering the amount of time that the problem went unsolved.

Assume the given statement is false. In other words, assume there is some way to draw a bunch of points on a piece of paper so that any line that passes through two of these points also passes through a third. Now think about the distances between the points in our diagram and the lines. For every point, write down its distance to every line that it's not on. Make a huge list with the distance between every point and every line, and take the smallest distance on this list. This represents the distance between the point and the line in our diagram which are closest together. Let's call the point A and the line k, and the shortest line between them d. It is very important to realize that no point can possibly be nearer than the length of d to any line, because d is the shortest distance between *any* point and *any* line in our diagram. Since we assumed for the sake of argument that every line between two points in our diagram passes through a third point, we know that line k has at least three of the points in our diagram on it. At least two of these points will be on the same side of the line segment d.



Call the point that's further from line segment d "B" and the one that's closer "C". Now let's draw line AB. While we're at it, lets also look at the distance between C and line AB. Let's call it e.



Oh no! It's shorter than d, the distance that we decided was the absolute minimum distance between a point and a line in our diagram! That's impossible! This is a contradiction of our assumption! This means what we were trying to prove in the first place was true after all, and there is no way to draw a bunch of points on a piece of paper so that any line that passes through two of these points also passes through a third, unless all of the points are on one straight line. ---- ***Nathaniel Watson***

Resources:

Engel, Arthur. *Problem Solving Strategies*. New York: Springer-Verlag, 1998.

Singh, Simon. *Fermat's Last Theorem*. London: Fourth Estate, 1997.

Suggested further reading on unsolved problems:

Clawson, Calvin. *Mathematical Mysteries.* Cambridge: Perseus Books, 1996.

Singh, Simon. *Fermat's Last Theorem*. London: Fourth Estate, 1997.

Mu Alpha Theta sends a huge "*thank you*" and a gift to **Nathaniel** for his article. Student contributions to the Log are welcomed and appreciated. Please send submissions to Richard Rusczyk at webmaster@mualphatheta.org.

NEW AND IMPROVED...
WWW.MUALPHATHETA.ORG

Richard Rusczyk, co-author of the *Art of Problem Solving* and director of the new **Vector Scholars Program**, has accepted the position of Mu Alpha Theta webmaster. The redesigned site was unveiled on April 1, 2002. Check out the new layout, information, and expanded site at www.mualphatheta.org! The Mu Alpha Theta national website is *the* place for information on events, membership requirements, Mu Alpha Theta merchandise, and scholarship opportunities.

So you can't go to this summer's national convention?

Then clear your schedule now for the **2003 convention**!

Emory Conference Center
Emory University
Atlanta, Georgia
July 20-25, 2003

Coming this fall. . .
The Mathematical Log
goes **electronic**!!!

Starting with the October 2002 issue, *The Math Log* will no longer be mailed out to chapters, but will be available online to everyone at no cost. Remember to keep checking the national website for the next issue of the national Mu Alpha Theta newsletter!

MATH IN THE NEWS

Spinning egg riddle isn't all it's cracked up to be anymore

Try this at home—take a hard-boiled egg (size, type, and color don't matter, but it must be hard-boiled) and spin it on a tabletop. One end rises until the egg is spinning vertically, like a top—amazing! But why?

Mathematicians from England and Japan spent the past six months figuring out the answer to that cosmically important question. Their explanation? Friction. As the egg spins, it touches the table at only point because of the curve of its shell. But this point of contact moves in a small circle around a vertical axis. As the spinning egg slides across the table, the movement creates friction, which slows the egg's rotation just enough to throw the contact point a little off-center. This causes the egg to twist (and shout?), and one end rises—till the egg is standing vertically!

You can read all about their findings (and check out the **sixteen** equations needed to explain this phenomenon) in the 4 April 2002 issue of the journal *Nature*.

Mu Alpha Theta Launches $36,000 Scholarship Program

In the spring of 2003 Mu Alpha Theta will award up to $36,000 in scholarships to some of its members as part of the Mu Alpha Theta Scholarship Program. The Program is designed to inspire students to contribute to their communities and to explore mathematics outside the classroom. It promotes and rewards students achieving excellence in the popularization, teaching, and development of mathematics through the media of mathematical modeling, writing, interactive web page design, and student-run educational programs.

The Program is currently funded by the *Mu Alpha Theta Foundation* and a grant from *The Actuarial Foundation.* As the Program attracts greater sponsorship, the number and amount of scholarships will be increased. The recipients are selected by panels of professors, teachers, and others active in the mathematics education community.

Awards will be offered in four Disciplines. The Mathematics in the Community Discipline rewards those students who aid the mathematics education of others. Rewarded efforts could be tutorial groups, organized math events for area schools, service to Mu Alpha Theta chapters, or even inventive programs the judges hadn't imagined. Students can submit entries in groups of up to three students, so long as the students collaborated on the programs they submit. Entrants will submit a scrapbook describing their efforts in photos and various printed materials including testimonials and event programs.

The Mathematics on the Web Discipline identifies student-designed web sites that promote aspects of mathematics and math education. Winning sites will be entertaining, easy to use, informative, and interactive. These sites will be the first entries in a library of educational web sites Mu Alpha Theta will build from entrants in this Discipline over the following years. Collaborative efforts will be accepted.

The Proponents of Mathematics Discipline selects those individuals who write about math-related topics in a thorough and engaging manner accessible to mathematics neophytes. The Scholarship Program will select an essay topic in early summer. As The Actuarial Foundation is sponsoring this Discipline in its first year, the subject will be statistical in nature. While this Discipline calls for essays either about or employing mathematics, this Discipline is not meant to produce mathematical papers. On the contrary, non-mathematicians will be able to understand and enjoy the winning papers of this Discipline, much as non-scientists can appreciate the writings of Carl Sagan.

The details regarding the fourth Discipline have not yet been determined. It will likely call for detailed mathematical modeling either of problems selected by the Program or issues students face in their communities.

The information presented in this article regarding the individual Disciplines is tentative. Finalized details, most notably the rules and judging criteria, will be available on the internet in early summer. Keep an eye on the Mu Alpha Theta website, www.mualphatheta.org, for the launch of the Program's site. If you have any questions or comments, write to scholarships@mualphatheta.org.

Richard Rusczyk, co-author of *The Art of Problem Solving*

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The Buffon Needle Problem

Paul Goodey
Mu Alpha Theta National Secretary-Treasurer
University of Oklahoma

The Buffon Needle Problems was described in 1777 by George-Louis Leclerc, Comte de Buffon [Essai d' arithmetique morale, in volume 44 of the *Supplement à l'Historie Naturelle*]. It leads to an experimental method of approximating π is irrational (indeed transcendental) means that we will never be able to give all the digits in the decimal expansion of π. The results of various Buffon needle experiments have appeared in the literature. In many ways the most incredible (that is, least credible) of these is due to M. Lazzarini. [Un' applicazione del calcolo della probabilità alla ricerca sperimentale di un valore approssimato di π, *Periodico di Matemtica* **4** (1901), 140-143].

This note is intended to provide readers with an analysis sufficient for them to draw their own conclusions as the veracity of Lazzarini's experiments. Much more detailed and interesting information can be found in the papers of N. T. Gridgeman [Geometric probability and the number π, *Scripta Math*., **25** (1960), 183-195] and L. Badger [Lazzarini's lucky approximation of π, *Math. Magazine* **67** (1994), 83-91], on which this article is based.

A good starting point for our story is in the works of Archimedes (possibly the most famous of all male streakers) who showed that 22/7 is a good approximation of π-- this is accurate to 2 decimal places. In increasing order of magnitude of their denominator, the next best rational approximations to π are

$\frac{333}{106}$	accurate to 3 decimal places	Athoniszoon, 1583
$\frac{355}{113}$	accurate to 6 decimal places	Tsu Chung-Chin, 480
$\frac{52163}{16604}$	accurate to 5 decimal places	Found by computer
$\frac{103993}{33102}$	accurate to 8 decimal places	Lambert, 1767

A very interesting (and opinionated) history of π can be found in the book *A History of π* (St. Martin's Press, New York, 1974) by Petr Beckmann.

The Buffon Needle Problem envisages the experimenter dropping a needle of length *l* onto a set of parallel lines distance *d* apart $(d > l)$. The problem asks what is the probability with which the needle will hit a line. In fact, Buffon himself gave the correct answer, $2l/\pi d$. Consequently, if one carries out this experiment the numbers *N* of experiments and *H* of hits can be observed. The quantities *l* and *d* can, of course, be measured. According to the experiment, the probability of a hit is H/N and according to the theory, it is $2l/\pi d$. Setting these two equal yields an equation of π, that is,

$$\pi = \frac{2lN}{dH}$$

Lazzarini reported that he carried out this experiment with $l/d = 0.83$. He dropped the needle 3408 times and observed 1808 hits. No doubt you are already asking yourself why would anyone set out to do 3408 experiments and then stop! Well, if you know that the number of experiments was 3408 then you know that the value of π is going to be

$$\pi = \frac{2l}{d}\frac{3408}{H}.$$

Had you read something of the history of π you would know that 355/113 is an excellent approximation and you might further note the advantage of carrying out a number of experiments that share a factor with 355:

$$\pi = \left(\frac{2l}{d}\right)\left(\frac{2^4 \times 3 \times 71}{H}\right) = \left(\frac{2l}{d}\right)\left(\frac{2^4 \times 3 \times 355}{5H}\right).$$

By now you see the opportunity to have 355 in the numerator. Unfortunately, you now have a 5 in the denominator and this is not a factor of the number 113, which you would be delights to see down there. However, all is not lost. After all, you have to report of the length *l* of the needle. If you're lucky it might turn out to be 5. Naturally the distance between the lines has to exceed this, why not 6?

$$\pi = \left(\frac{2 \times 5}{6}\right)\left(\frac{2^4 \times 3 \times 355}{5H}\right) = \left(\frac{2^4 \times 355}{H}\right).$$

If only you could observe $2^4 \times 113$ hits! The prosecution rests its case.

Still curious about B?

Search the web and find sites like "The **Pi** Pages" (http://www.cecm.sfu.ca/pi/pi.html) to continue learning about the fascinating history and intrigues surrounding this mysterious number.