Attend the 27th Annual Convention in Seattle 7/31 – 8/6

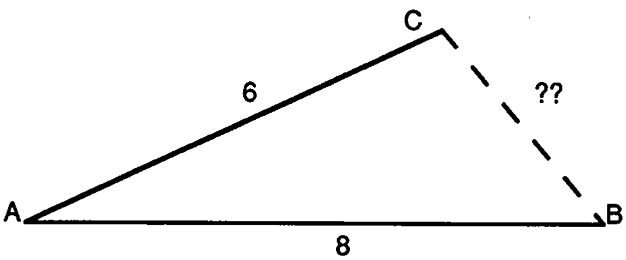
The Mathematical Log

Volume 41 Number 1 February 1997 ISSN 0025-5580

Tantalizing Investigations Involving the Triangle Inequality

When you consider the problem:

"Two sides of a triangle are 6 cm and 8 cm. Find the perimeter, P, and area, K, of the triangle."



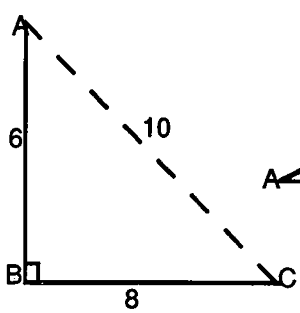
you will probably have one of two responses.

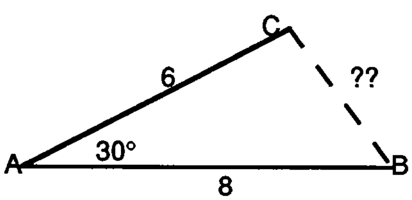
(1) "I need more information to solve the problem."

(2) "This problem has many solutions."

Let's consider both approaches. In (1), what additional piece of information is needed? Several answers are possible.

• Suppose we know side BC. If BC = 10, then ΔABC is a right triangle with P = 24 and K = 24. If BC = 9, then P = 23 and K can be determined using Heron's Formula: $K = \sqrt{11.5(11.5-9)(11.5-8)(11.5-6)} = 23.53$



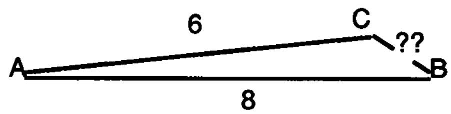


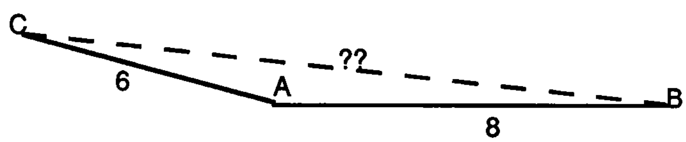
• Suppose we know $\angle A$, say $\angle A = 30°$. Then $K = \frac{1}{2} AC \cdot AB \sin 30° = 12$. To find P, we need to find BC. Using the Law of Cosines, $BC^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cos 30°$ or BC = 4.11 and P = 18.11.

<u>Problem 1</u> Suppose we know $\angle B$. Can we find P and K?

<u>Problem 2</u> Give an example of a piece of information that would yield unique values of P and K. Find the values of P and K for your choice. You might consider such things as an altitude, a median, ...

In (2), the key step is to determine the possible values of side BC.





Considering the extreme cases, we see that BC > 2 and BC < 14. Thus $16 < P < 28$. As side BC moves, we see that the area, K, increases, reaches a maximum when ΔABC is a right triangle, and then decreases. Thus $0 < K \leq 24$.

The visual examination of the way BC moves between the extreme cases essentially confirms the **Triangle Inequality** – the sum of any two sides of a triangle is greater than the third side. Formally, in ΔABC, $AB + BC > AC$; $AC + CB > AB$; $BA + AC > BC$.

We can prove this classic inequality using the Law of Cosines:
$c^2 = a^2 + b^2 - 2ab \cos C = (a+b)^2 - 2ab(1 + \cos C) < (a+b)^2$ --> $c < a + b$. [In a triangle, $1 < \cos C < -1$]. Similarly you can prove $a + c > b$ and $b + c > a$.
[continued on page 6]

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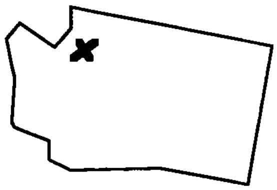
with Log Editor Tom Butts



We hope to see everyone at the 27th annual convention in Seattle.

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I am happy to share the following letter from Melinda Costello:

"I am currently a student at Eisenhower HS in Blue Island, IL. I am a junior in Algebra II Honors. My math teacher is Mrs. Cindy Charters. So many great teachers go unnoticed and not recognized for all of the hard work that they do. Mrs. Charters is such an example.

She's involved in Mu Alpha Theta, she's the freshman volleyball coach, and an all-around great person. She is a teacher who you can never forget. She makes learning fun. The way she teaches, mathematics is a puzzle. We another piece to it every day . Every day I look forward to her class because I feel that I am not just learning to solve an equation, but I am learning what it takes to succeed in the world. You really cannot get anywhere without mathematics.

Since I started high school, it has always been my dream to become a mathematics teacher. I know that I will pattern myself after Mrs. Charters; her manner and style make learning fun and that's how I want to teach some day.

I wrote this letter hoping that you could publish it in the near future. When Mrs. Charters reads it, she will realize that she has made a difference in one's person's life. I will never forget her. "

Sincerely,
Melinda Costello

Ed. Note: I'm sure that Ms. Costello speaks for many of us who studied with a mathematics teacher who made a real difference in our lives.

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SPEAK OUT !!!

This is your journal. Like Melinda, you are encouraged to send information about your teachers, your chapter, etc. Share a "mathematical" game you think others would like to play. Send your solutions to problems posed in the Log, a question about any area of mathematics - a person, a concept, a problem,.. or anything else on your mind. Send it to me at the address below or e-mail at tbutts@utdallas.edu.

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MATH LETTERS CHALLENGE

BY DON ALLEN

Ed. Note: Dr. Allen served as editor of the Log from 1980–1990 and currently works with prospective teachers in Canada's Eastern Arctic.

Have you ever played the game of taking three letters of a vehicle license plate and trying to find a word in which those three letters occur, not necessarily in sequence, but in the order they appeared. For example, if a license plate contains RND, then an appropriate word is "RANDOM"; LBA could be "ALGEBRA", etc. Here's your chance to play this game where each word you are trying to find is probably part of your mathematics vocabulary [e.g. random]. Listed below are 35 three-letter permutations, each of which might be part of a license plate or airport code, but is also part of a mathematical term. Try the activity "solitaire" or with a group. Send your solutions to me, Don Allen, P.O.Box 162, Coral Harbour, Northwest Territories, CANADA, X0C, 0C0 or to the Log editor.

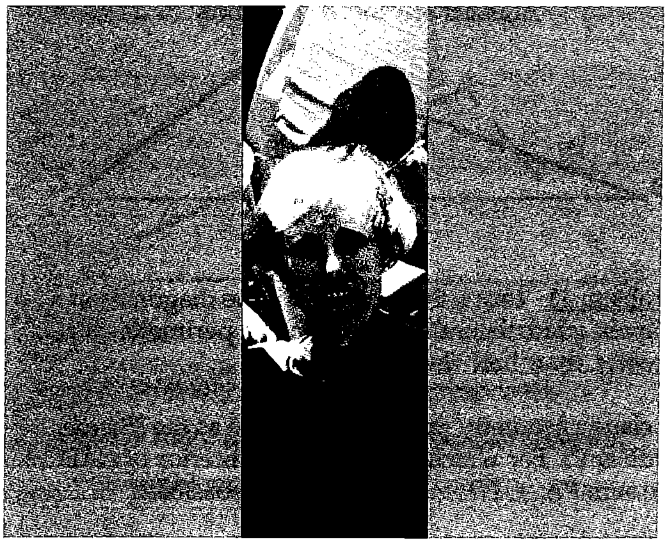
ACA	ADA	AEO	ASL	BAO
BUO	CUE	DOA	EEE	EIE
GOO	HMS	ICE	IGE	LMN
LPA	MOE	MRT	NGN	OAT
OCS	OCU	OYO	PRE	PZO
RMC	SPO	TTT	UCO	UTR
VII	YAD	YDE	YEA	YEU

The Mathematical Log
Volume 41 Number 1 February, 1997
The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu AlphaTheta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. e-mail: MATHETA@uoknor.edu or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson,TX 75083, e-mail: tbutts@utdallas.edu

1997 √At the Root of It All

Deborah Patonai Phillips, Activities Editor
St. Vincent-St. Mary HS, 15 North Maple Street, Akron, OH 44303

Each summer at the national convention, awards are presented to several outstanding students and committed sponsors. This past summer in Orlando, one special lady, who is neither a student nor a sponsor, was honored with a plaque for dedicated service to Mu Alpha Theta. She is Diane Rubin, the Executive Assistant in the National Office at the University of Oklahoma. √At the Root Of It All takes great pride in spotlighting this remarkable woman.



Originally from New Orleans, Diane received her bachelor's degree in history from Newcomb College of Tulane University. There she met her husband, Lenny. Prior to coming to Norman, she taught history and civics in New Orleans and Coral Gables, FL. When Lenny took a position as a topologist at the University of Oklahoma, Diane's tie to MAΘ began. Lenny was hired by Richard Andree, the chairman of the Mathematics Department as well as the founding father of Mu Alpha Theta.

"Perhaps all of it can be summarized by two words: efficiency and personableness."

When the Rubins arrived, they found that two other colleagues had also been hired that year – past Secretary-Treasurer Thomas J. Hill, a mathematics educator, and current Secretary-Treasurer Stanley B. Eliason, an analyst. Also part of the mathematics department was Harold Huneke, who served as MAΘ Secretary-Treasurer for 22 years.

In 1982 Diane was hired into her position by Harold Huneke. Recalling those early days, Harold reminisces, "It was a fortunate day for me and for MAΘ when I found out that Diane was interested in the position. " She was "very efficient" and "was always thinking of ways we might do things better." He was delighted with her enthusiasm and the fact that "she really cared about MAΘ."

When Harold retired, Tom Hill became Secretary-Treasurer. "From the outset, " Tom recognized that "Diane viewed MAΘ as an organization of which she was-- and is-- an essential part. She provided a 'human face' for the 'national office'. .... Working at MAΘ is more than a job to her. Perhaps all of it can be summarized by two words: efficiency and personableness."

In 1993, Stan Eliason took over as Secretary-Treasurer to reflect the continuing interest of the University of Oklahoma's Mathematics Department in Mu Alpha Theta. He portrays Diane as "the springboard for ideas... She enthusiastically and earnestly seeks ways to attract more students and schools to the organization."

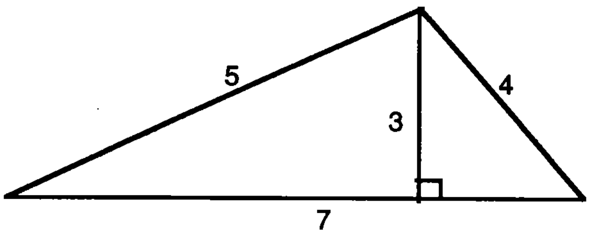
As the only full-time paid employee of this growing organization with over 48,000 members in more than 1300 chapters, Diane is the hub of the MAΘ wheel. For over 13 years she has been handling all MAΘ requests and correspondence. On any given day, she responds to inquiries from sponsors, fills orders for MAΘ products, mails certificates and charters, responds to letters, e-mail, faxes, and telephone calls. Four times a year, she oversees the printing and the distribution of the Log [and corrects the Editor's errors while tolerating his tardiness.]

"... thanks for being the glue that holds MAΘ together. Perhaps I'll start calling you Elmer."

Diane seems to take care of just about everything. She is a valuable link between the national office and the Governing Board, keeping them informed about the many developments within the organization. She monitors the progress of the national conventions. "Diane's work and presence at the conventions were [and are] marked factors in their success," recounts Tom Hill. In fact, Diane initiated and encouraged the current contracts now being signed with C/M Planners for their assistance in running future national meetings. In her spare time, she handles the MAΘ booth at regional and national NCTM conventions,
[continued on page 4]

"TAKE ME OUT TO THE OLD BALL GAME"

In attempting to create a test item that can be used to assess several concepts, every teacher has probably had the experience of designing an impossible situation. For example, to test the perimeter and area of a triangle using 'simple numbers', a teacher designed the following exercise:



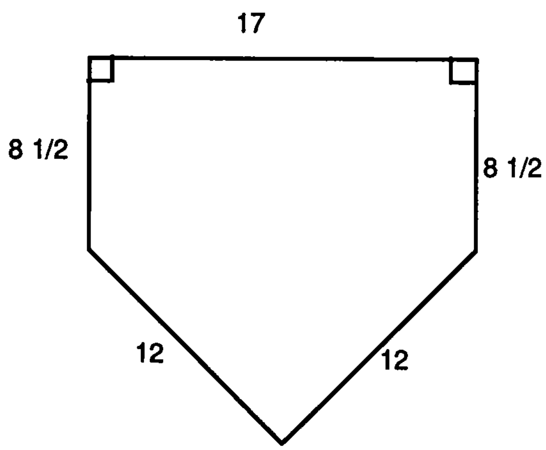
Find the perimeter and area of this triangle.

• Why is this situation impossible?

• Can you given another example of an "impossible situation"?

An interesting analogous situation concerns the specifications for the home plate in baseball.

"Home base shall be marked by a five-sided slab of whitened rubber. It shall be a 12-inch square with two of the corners filled in so that one edge is 17 inches long, two are 8 1/2 inches long and two are 12 inches long."



The question: Is it possible to build a home plate conforming to the specifications given in the rule book?

Reference: "Building Home Plate: Field of Dreams or Reality?", M. Bradley, Mathematics Magazine, Feb. 1996, pp. 44-45.

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[√At the Root of It All, continued from page 3]
oversees the sponsor get-together at the national NCTM convention [see Sponsors in Cones and Cuttings], and attends several state and regional MAΘ conventions.

Even after so many years Diane still enjoys working with MAΘ and finding ways to improve it. She gets to meet new people and to develop long-lasting friendships with students and sponsors. She was instrumental in the marketing of MAΘ T-shirts, putting some of the publications on microfiche, and is now involved in "computerizing" the national office.

Through her work, Diane has helped many individuals. One of them is Log editor, Tom Butts. He characterizes Diane as "the sort of manager that 'executives' dream about – someone with the ability to get everything done while maintaining a friendly, caring attitude, even at times when she must be exasperated."

Helen Dostal, long-time MAΘ sponsor and recent recipient of the Huneke Award, sums up Diane best. "Diane," says Helen, "much of the longevity and excellence of MAΘ is due to your tireless dedication. You have made so many memories possible for me ... Thanks for being my friend and thanks for being the glue that holds MAΘ together. Perhaps I'll start calling you Elmer."

Congratulations, Diane, on this long-overdue award. Thanks for your many years of faithful service to Mu Alpha Theta. We appreciate you much more than any plaque can ever say!

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[Nat Fredin - continued from page 5]
Just e-mail me at **natfredin@aol.com** and send me your name, hometown, school name, age, etc. and I will put the addresses together and send the directory to everyone who participates. Try to e-mail me by March 30 so I can get the directory out as soon as possible. If you get an e-mail address after March 30, send it anyway - I plan to periodically update the directory.

I'm looking forward to getting lots of mail so the directory will be filled with lots many interesting people. It will be a great opportunity to meet new people and share ideas.

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[Steiner Problem... , continued from page 6]

Problem 12 Solve the Steiner Problem for convex quadrilaterals.

Your initial guess is probably correct. You can justify it using several applications of the Triangle Inequality.

Reference

Mathematical Circles, Fomin et al, American Mathematical Society, 1996, Chapter 6

Cones & Cuttings

The Bulletin Board

• STATE AND REGIONAL MEETINGS in 1997•
March 23-24 1997: Mississippi State Meeting, Jackson MI. Contact William Payne, Jackson Academy, 6300 Old Carlton Road, Jackson, MI 39211.

March 1997: Louisiana State Meeting
March 1997: South Carolina State Meeting

April 1997: Florida State Meeting

• FUTURE NATIONAL CONVENTIONS
1998 Chicago area

Send information about your state or regional meetings to the national office.

Mu Alpha Theta on the Web
Mu Alpha Theta, which comes from the Greek letters for M-A-TH, currently has 48,000 members in over 1300 chapters. This is but a small part of the information available, and planned, at our web site through the University of Oklahoma:

http://www.math.uoknor.edu/~mualpha/.
Check it out.

Sponsors

Notice: As a result of action taken by the MAΘ Governing Board last year, the cooperative venture between MAΘ and the Who's Who organization is no longer functioning. That is to say, the national office is no longer forwarding names of MAΘ initiates to Who's Who.

Mu Alpha Theta is branching out. Come by our booth at the NCTM Meetings in Atlanta [Feb.][, San Jose [March], or Minneapolis {April] . We are trying to the word out that it's cool to start a Mu Alpha Theta club.

Speaking of the NCTM Annual Meeting in Minneapolis in April, we are planning a Friday dinner meeting for sponsors [instead of the traditional breakfast meeting]. Call the national office for more details.

Submit information for **Sponsor Swap**, things your have found useful that could be helpful for other sponsors; questions for other sponsors, etc. Setting up a section at the web site might be another way to exchange ideas.

Students:

• Take a few minutes and contact one of your student officers or join the e-mail directory. [see column below.]

• This is your last chance to nominate your sponsor for the Huneke Award or the Sister Scholastica Award. Contact the national office for nomination forms.

• **Student Delegate Officers for 1996 - 97**•

President: Daniel Neill, Hillsborough HS, 5000 N. Central Ave. Tampa FL 33603; 1903 Cape Bend Ave. Tampa, FL 33613

Vice-President: Adam Swensek, Riverdale HS, 240 Riverdale Blvd. Jefferson, LA 70121; 8702 Darby Lane, River Ridge, LA 70123; e-mail: aswensek@aol.com

Secretary-Treasurer: Nat Fredin, Lincoln Way HS, 1801 E. Lincoln Hwy., New Lenox, IL 60451; 330 Gina Dr. New Lenox IL 60451; e-mail: dfredin@aol.com

Sergeant-at-Arms: Allan Martin, Farragut HS, 11237 Kingston Pike, Knoxville, TN 37922; 11504 Gates Mill Dr. Knoxville, TN 37922

From Nat Fredin, Student Delegate Secretary-Treasurer

Greetings from the student delegate officers! My name is Nat Fredin, and I'm the Student Delegate Secretary-Treasurer. I live in New Lenox, Illinois, which is about an hour southwest of Chicago, and I am a senior at Lincoln-Way HS.

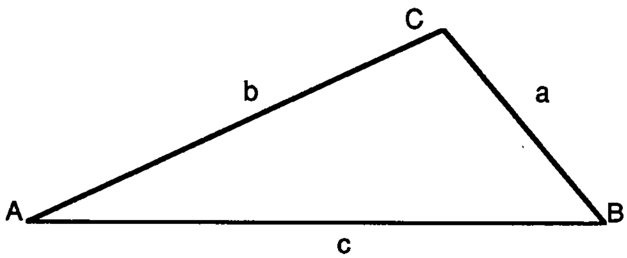
When discussing this article, our sponsor came up with a great idea. He suggested setting up a **student e-mail directory**, and that's what I'm doing. I think it will be a good chance for everybody to meet and keep in touch with other members around the country.
[continued on page 4]

[Tantalizing Investigations ... continued from page 1]
There are many interesting problems and investigations in which the Triangle Inequality plays a part. We discuss some of them.

Problem 3 Why is the Triangle Inequality simply a restatement of the familiar saying that "the shortest distance between two points is [along] a straight line."?

Problem 4 In ΔXYZ, $XZ = 3.8$, $XY = .6$ and YZ is an integer. Find YZ.

Problem 5 Prove that the sum of any three sides of a convex quadrilateral is greater than the fourth side.



If $b + c > a$, then $a + b + c > 2a$ or $a < \frac{a+b+c}{2}$. These calculations indicate how the Triangle Inequality shows that any side of a triangle is less than half its perimeter. We can use this observation to help solve the problem in the following example.

Example Find all triangles with integer sides whose perimeter is 13.

Solution
The longest side of such a triangle is 6. We make an organized list to consider all possibilities by writing the length of the sides in decreasing order.

6 6 1	5 5 3	~~4 4 5~~
6 5 2	5 4 4	
6 4 3	~~5 3 5~~	
~~6 3 4~~		

There are 5 such triangles.

Problem 6 For what values of the perimeter, P, is there only one triangle with integer sides whose perimeter is P? Justify your answer.

Problem 7 Is it true that as P increases, the number of triangles with integer sides whose perimeter is P also increases? Justify your answer.

Problem 8 For what value of P, $3 \le P \le 26$, is there the largest number of triangles with integer sides whose perimeter is P?

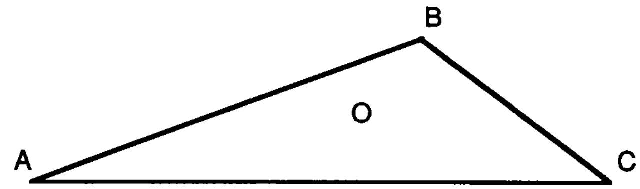
A few more problems involving the Triangle Inequality:

Problem 9 If O is a point in the interior of ΔABC, prove

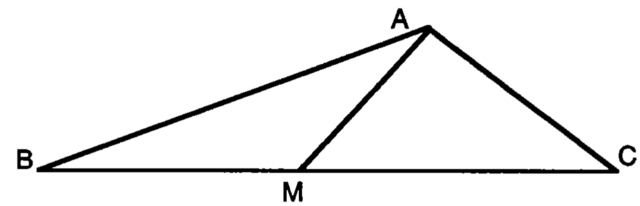
a. $AO + OC > AB + BC$

b. $OA + OB + OC <$ perimeter of ΔABC.

c. What happens in parts a. and b. if O is outside ΔABC?

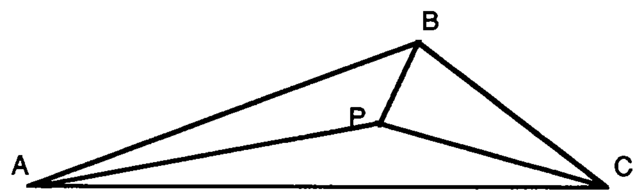


Problem 10 If AM is a median in ΔABC, then $AM < \frac{AB + AC}{2}$



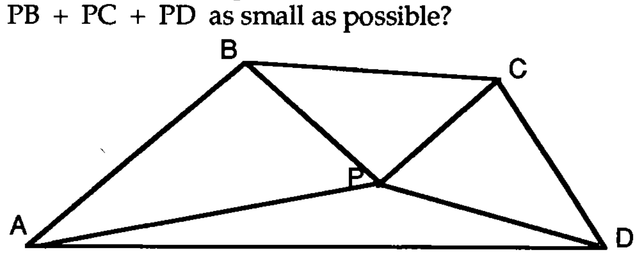
Problem 11 Prove that the sum of the diagonals of a convex quadrilateral is less than the perimeter but greater than half the perimeter.

A classic geometry problem in known as the "Steiner Problem": For which point P in the interior of ΔABC is the sum $PA + PB + PC$ as small as possible?



• Investigate this problem using the Geometer's Sketch Pad or the geometry application in the TI-92.

Unexpectedly, The "Steiner Problem" for convex quadrilaterals has a much easier solution than that for triangles. Most problems get harder when you consider the "next case". Specifically: For which point P in the interior of convex quadrilateral ABCD is the sum $PA + PB + PC + PD$ as small as possible?



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