

The

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Sensitivity Analysis - an Interesting "What If" Question

When we solve a problem, we often stop after we find one solution. In this article, we indicate one "looking back" strategy – using sensitivity analysis – that is especially appropriate for problems involved in mathematical modeling.

Example: The Pig Problem [from [2]]

A pig weighing 200 pounds gains 5 pounds per day and costs 45¢ a day to keep. The market price for pigs is 65¢ per pound, but is falling 1¢ per day. When should the pig be sold?

There are several key steps in solving such problems involving mathematical modeling:

1. Ask the question
2. Identify the problem variables.
3. Find assumptions about and relationships among these variables.
4. Formulate a model
5. Solve the model
6. Answer the question

1. When should the pig be sold so that we maximize profit?

2. Variables:

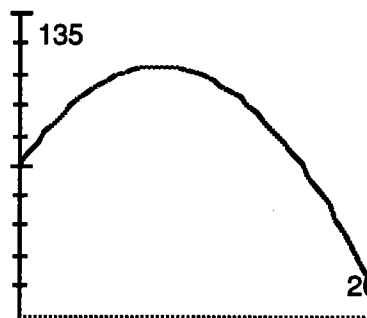
x = time in days
 w = weight of pig
 p = price for pigs in \$/lb
 R = revenue from selling pig in \$
 P = profit from sale of pig in \$

3. Assumptions/Relationships:

$w = 200 + 5x$
 $p = .65 - .01x$
 $C = .45x$
 $R = pw$
 $P = R - C$
 $x \geq 0$

4. $P = R - C = pw - .45x = (.65 - .01x)(200 + 5x) - .45x$

5. By using a graphing calculator, we find that P is maximized when $x = 8$ where $P = 133.20$



6. Sell the pig 8 days from now.

When we perform **sensitivity analysis**, we ask the question: "What happens when the inputs change?" "How does the solution change if some of the data or assumptions are changed?"

Example: Investigate the assumption that the price of the pig is falling 1¢ per day.

We look at several values for this price, r , [column 1 in the table] and then solve the problem as before, i.e. we maximize the profit, P , for the values of r in the table.

$$P = R - C = pw - .45t = (.65 - rx)(200 + 5x) - .45x$$

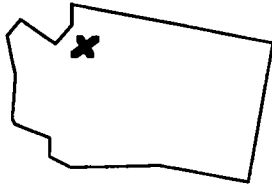
The amount per day, r , the price of the pig is falling	Optimal number of days, x , from now on which to sell
.008	15.0
.009	11.1
.010	8.0
.011	5.5
.012	3.3

[See graph on page 6]

We see that the optimal time to sell is quite sensitive to the assumption about how much the price of the pig is falling per day. This assumption needs to be carefully considered.

[continued on page 6]

dia Log ue
with Log Editor Tom Butts



We hope to see everyone at the 27th annual convention in Seattle.

EXTENDING SYNTHETIC DIVISION

I am happy to share the following discovery of Leonard Ford, a student in Mrs. Sheryl Gates's Pre-AP Precalculus class at White Station HS in Memphis, TN.

Synthetic division is a "shorthand" method for dividing a polynomial by a linear polynomial using only the coefficients.

To divide $P(x) = x^3 - 4x^2 - 3x + 2$ by $(x - 6)$ we write coefficients and the '6' as follows:

$$\begin{array}{r|rrrr} 6 & 1 & -4 & -3 & 2 \\ & & 6 & & \\ \hline & 1 & & & \end{array}$$

Bring down the '1' and then multiply 1×6 and put the result under the -4 above the line. Add -4 and 6 and write '2' below the line. Multiply 2×6 and repeat the process. The first three numbers below the line are the coefficients of the quotient, $x^2 + 2x + 9$, and the last number, 56, is the remainder and the value of $P(6)$.

$$\begin{array}{r|rrrr} 6 & 1 & -4 & -3 & 2 \\ & & 6 & 12 & 54 \\ \hline & 1 & 2 & 9 & 56 \end{array}$$

Leonard discovered that we can extend this method to cases where the divisor is a quadratic polynomial that can be written as the product of two linear factors $(x - k)(x - h)$. Let us illustrate Leonard's method with an example:

Divide $x^4 - 3x^3 + 2x^2 - 8x - 6$ by $(x - 2)(x - 3)$. First synthetically divide by $(x - k) = (x - 2)$.

$$\begin{array}{r|rrrrr} 2 & 1 & -3 & 2 & -8 & -6 \\ & & 2 & -2 & 0 & -16 \\ \hline & 1 & -1 & 0 & -8 & -22 \text{ [=R]} \end{array}$$

Then synthetically divide the quotient by $(x - h) = (x - 3)$. You can just continue as shown:

$$\begin{array}{r|rrrrr} 2 & 1 & -3 & 2 & -8 & -6 \\ & & 2 & -2 & 0 & -16 \\ \hline 3 & 1 & -1 & 0 & -8 & -22 \text{ [=R]} \\ & & 3 & 6 & 18 & \\ \hline & 1 & 2 & 6 & 10 & \text{[=P]} \end{array}$$

The quotient is $x^2 + 2x + 6$ and the linear remainder is $Px + (R - Pk) = 10x + [-22 - 10 \cdot 2] = 10x - 42$.

We can explain why this method works using the Division Algorithm for Polynomials:

Divide $P(x)$ by $(x - k)$: $P(x) = Q_1(x)(x - k) + R$

Divide $Q_1(x)$ by $(x - h)$: $Q_1(x) = Q_2(x)(x - h) + P$

Thus $P(x) = [Q_2(x)(x - h) + P](x - k) + R =$
 $= Q_2(x)(x - h)(x - k) + Px + (R - Pk)$.

Problem: Illustrate this idea using divisors of the form $x(x - k)$. Choose an example, find the quotient and remainder, and explain your answer using the Division Algorithm for Polynomials.

MATH LETTERS CHALLENGE- SOME ANSWERS BY DON ALLEN

- ACA ADA AEO ASL BAO
- BUO CUE DOA EEE EIE
- GOO HMS ICE IGE LMN
- LPA MOE MRT NGN OAT
- OCS OCU OYO PRE PZO
- RMC SPO TTT UCO UTR
- VII YAD YDE YEA YEU

radical, quadrant, parallelogram, transversal, subtraction, substitution, curve, diagonal, repetend, ellipse, trigonometry, rhombus, circle, triangle, element, multiplication, composite, numerator, ...
See if you can complete these answers and find others.

The Mathematical Log

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The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu Alpha Theta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. e-mail: MATHETA@uoknor.edu or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson, TX 75083, e-mail: tbutts@utdallas.edu © 1997 Mu Alpha Theta

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1997 $\sqrt{\text{At the Root of It All}}$

Deborah Patonai Phillips, Activities Editor

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When sponsors meet during the Mu Alpha Theta Convention each summer, they always recount how many previous conventions they have attended. Sister Scholastica of San Antonio always wins. She has attended every convention except for the first one – the San Antonio Convention! There is another special lady whose roots in Mu Alpha Theta run as deep as our dear Sister Scholastica – those of our current president, Barbara Stott. Involved in MA Θ since she was sixteen [Eisenhower was in office!], she is a charter member and served as the first president of the East Jefferson HS chapter in 1961. $\sqrt{\text{At the Root Of It All}}$ is enraptured and fascinated by all the contributions of this remarkable woman.



After receiving her B.S. degree from Louisiana State University in 1965, Barbara moved to Georgia to begin her first teaching assignment. Seven years later she returned to her native state to teach at Riverdale HS in Jefferson, LA. She has been at Riverdale ever since. She became a Tulane fanatic when she earned her master's degree in mathematics there in 1976. Over the years she has kept up with the many changes in mathematics education by earning 30+ graduate hours beyond her master's degree and attending countless conventions.

"no one ever graduates from my MA Θ chapter."

Serving as Riverdale's MA Θ advisor since 1973, Barbara has turned her chapter into one of the most active in Louisiana and, maybe, in the entire organization. Besides tutoring other high school students, her members also work with students in grade schools and junior high schools. As part of their fundraising activities, they probably sell more Twinkies than any other chapter in America. Each

year they have a wonderful time producing a slide show or video that promotes the organization. They also enjoy creating scrapbooks and especially banners that are renown for their uniqueness and humor. The chapter also boasts of an active and loyal group of alumni who continue to help the chapter as they make their marks on the world. As Barbara says, "no one ever graduates from my MA Θ chapter."

Under her watchful eye, the chapter annually hosts several math tournaments. They originated the Junior High Math Tournament for South Louisiana and have run it every year since. They also host the Riverdale Math Tournament every winter. This tournament is conducted solely by the members who write and check all the questions, grade the papers, and present the awards.

Following her example, her students have held many district, state, and national offices. She has sponsored state MA Θ officers every year [except one] since 1977. Attending every national convention since 1977, she has also sponsored five national officers.

One of these national officers is current vice-president, Adam Swensek. Involved with MA Θ since the seventh grade, Adam is Riverdale's vice-president. A National Merit Finalist, he also devotes time to serving as captain of the cheerleading squad, the Quiz Bowl team, and the speech team. At the national convention in Orlando, he placed first in the Alpha Chalk Talk. He believes such events "serve to combine mathematical ability with good communication skills - a formula for success in today's corporate climate." At the recent NCTM Regional Conference in New Orleans, Adam spoke at the MA Θ session with the charm and charisma of a veteran. A real tribute to MA Θ and to his advisor -- Barbara!

All of these honors that Barbara has received are just a by-product of her one true goal - to reach her students.

Barbara has been an instrumental force at all levels of MA Θ . She has served as a member of the Louisiana Executive Board every year since 1978 [except one], sponsoring four state presidents. She was co-chair of the 1984 national convention in New Orleans, Governor of Region II from 1986 - 1988 and NCTM representative from, 1988 - 1991. For many years she has promoted MA Θ by speaking at professional meetings where she can usually be found at the MA Θ booth.

[continued on page 6]

CRYSTAL BRANDON - SIX YEARS LATER

Crystal Brandon was the 1991 recipient of the Andree Award. Today she is a mathematics teacher at Pine Ridge HS in Deltona, FL. This year she chartered a new MAΘ chapter at her school. We recall a few excerpts from $\sqrt{\text{At The Root Of It All}}$ and then share Crystal's letter about her life since then.

Log 1991

A 1991 graduate of Plant City, Florida, High School, Crystal Brandon is majoring in mathematics education at the University of South Florida. After graduation, she plans to return to teach mathematics at her high school alma mater. Her former mathematics teachers are " ... looking forward to the day when she can become a member of our faculty and continue to be a positive influence in Mu Alpha Theta."

Moving several times while she was in junior high school, Crystal encountered a rocky start in mathematics. At Plant City, she was initially placed in a low math class. While taking Algebra I in tenth grade, she was discovered to have high ability in mathematics. She was subsequently placed in geometry while still taking Algebra I. She continued by taking Algebra II, Trigonometry, Analytic Geometry, and Calculus over the next two years! Because of her achievements, she received the Outstanding Math Student Award for 1991.

Soon after she was placed in advanced courses, Crystal was asked to join MAΘ. Although unfamiliar with the organization, she was willing to try anything involving mathematics. Working at least two extra hours a day on mathematics, she competed on the Algebra I team that won several state awards. In her junior year, she was on the Algebra II team that placed second at the national convention. As a senior, she was a member of the calculus team that won several awards. ...

Wanting to be a teacher since kindergarten, Crystal had the opportunity to teach a few math classes on special occasions. She felt 'right at home'. "Teaching is something I really want to do. I want to give my future students what my teachers have given to me: confidence in myself, a desire to achieve, and an undying interest in mathematics."

College

After graduation from Plant City HS, I entered the University of South Florida in Tampa. I was able to pay for college through scholarships such as the Andree Award and the "Chappie" James Most Promising Teacher Scholarship. I really believe I would not have received these scholarships without the strong academic/math skills I obtained from Mu Alpha Theta. They gave me confidence, enthusiasm for

academics, and math skills – everything I needed to succeed. I was invited to join the Honors Program and I became president of local chapters of Pi Mu Epsilon and the Mathematical Association of America. During my junior and senior years, I worked part time as a freshman advisor. I graduated with a BA in Mathematics and a minor in Music [clarinet] with a GPA of 3.6.



The first year of teaching

I never want to be a first year teacher again! Don't get me wrong, I love my job but the first year is trial by fire. At one point in college I was taking 24 hours and working 20 hours per week. But when I began teaching, I was busier than I had EVER been before.

I encountered students who HATED math and parents who thought teachers were not doing their jobs when they made students work in groups instead of lecturing. But I do love Pine Ridge HS. It has only been open for 2.5 years. The school is very "technologically oriented". Each teacher has a computer in her classroom. Add to this access to computer labs, LCD panels, graphing calculators, and an e-mail address and you have a school that is going places.

Chartering a new chapter

Most people thought I was crazy to try and start up a new club while enduring the tortures of a first year teacher. Looking back, I probably was. All I knew was that Mu Alpha Theta had given me so much that I didn't want even one student to miss out on the opportunities that I had.

Sponsor's Fun

I remember my first competition as a sponsor, Permission forms, transportation, registration, scantron forms, ... this list goes on and on. Students had no idea how much work goes into just getting them to one competition. I only managed to bring three students, but I got there. Going into the previously forbidden sponsor's room was neat.

[continued on page 6]



Cones & Cuttings



MS. PHILLIPS GOES TO WASHINGTON



Debbie Phillips, $\sqrt{\text{At The Root Of It All}}$ editor and MA Θ sponsor extraordinaire, is now in Washington, D.C. as an Albert Einstein Distinguished Educator Fellow. She is working in the Eisenhower Program Development Project in the Department of Education where she will help implement on the President's initiative to improve teaching and learning in mathematics and science.

In addition, she is one of one hundred recipients of this year's Tandy Awards honoring excellence in the teaching of mathematics, science, and computer science. [see ad in Time, Mar. 31, 1997] She is expected to return to St. Vincent-St. Mary HS in January, 1998.

Mu Alpha Theta on the Web

A Problem of the Week and pictures of the staff and governing council are now available at our web site through the University of Oklahoma:

<http://www.math.uoknor.edu/~mualpha/>.
Check it out.

Sponsors

Submit information for Sponsor Swap, things you have found useful that could be helpful for other sponsors; questions for other sponsors, etc. Setting up a section at the web site might be another way to exchange ideas.

Students:

- Take a few minutes and contact one of your student officers or join the e-mail directory. [see column below.]

• Student Delegate Officers for 1996 - 97•

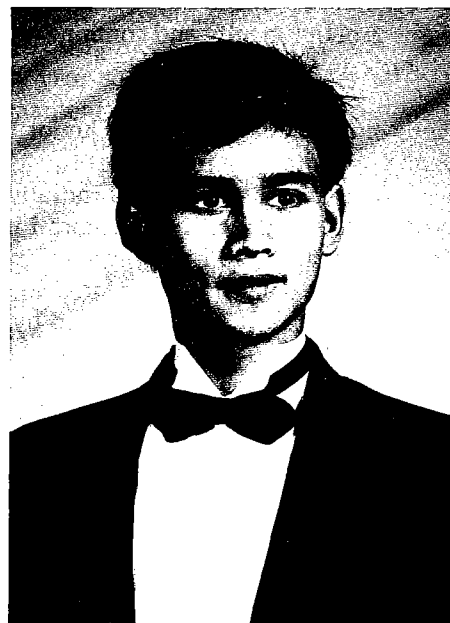
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From Allan Martin, Student Delegate Sergeant-at-Arms

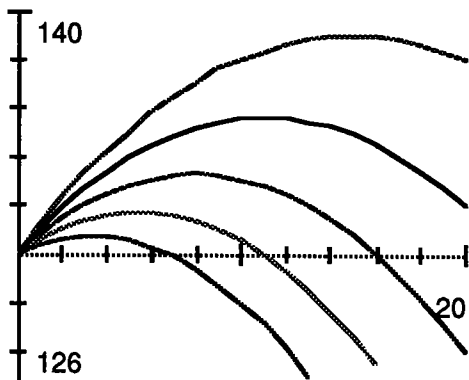


Hi! I'm Allan Martin, your Sergeant-At-Arms. I am a senior at Farragut HS, about 20 miles from Knoxville, TN. I'm also president of the Farragut MA Θ chapter - a very active school in the organization. We hosted the Region IV Convention and the Farragut Math Bowl and are planning to attend the Tennessee State Convention and the National Convention in Seattle.

Our organization offers some wonderful opportunities for students interested in mathematics and should be extended to as many students as possible. Try to convince other students in your school to join or students at a nearby school to form their own chapter.

Try to attend the national convention in Seattle. They are great fun and have something for everyone. You can meet new people from all over the country and become involved in great activities and competitions. This will be my third convention and I am really looking forward to it. I would like to thank Tom Norris and all his associates for their efforts in running the national convention. See you there!

[Sensitivity Analysis ... continued from page 1]



We often express this sensitivity using percents.

r	x	$\frac{\Delta x}{x}$	$\frac{\Delta r}{r}$	$\frac{\Delta x}{x} / \frac{\Delta r}{r}$
.008	15.0	.875	-.20	-4.375
.009	11.1	.3875	-.10	-3.875
.010	8.0			
.011	5.5	-.3125	.10	-3.125
.012	3.3	-.5875	.20	-2.938

This table shows that if r changes by 1%, then x changes by approximately 3.5%.

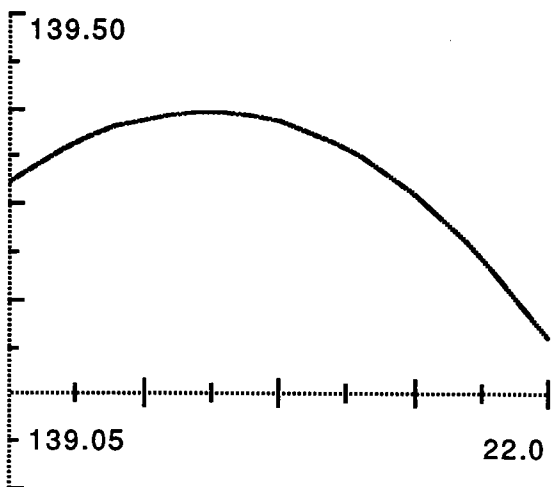
Type of Model

We may also investigate the question: "What happens when the model changes?"

- price of the pig, e.g. suppose price levels off so that $p = .65 - .01x + .00004x^2$ [Problem]
- increase in weight, suppose exponential $w = 200e^{.025x}$

If $w = 200e^{.025x}$ then $P = (.65 - .01x)(200e^{.025x}) - .45x$.

The maximum value of x is now about 19.5 compared to about 8 in other model.



Which model do you think is correct?

We can perform sensitivity analysis on almost any type of problem – those involving linear models, linear systems, linear programming, probability and expected value, etc.. Such analysis is routinely performed in mathematical modeling.

Reference:
Mathematical Modeling, Meerschaert, Academic Press, 1993

 [Crystal Brandon - continued from page 4]

First Awards

I had been taking students to competitions since last year and we hadn't won anything. I know it takes time to get a chapter started, but my students were getting discouraged. Then our largest group of 7 students went to the Lake Weir Regional. We brought home three trophies! I almost cried. During my time in MAΘ, I probably brought home over 20 trophies, but I have never been as proud as I was when my students walked onto that stage. The look in their faces will stay with me forever.

Ed. Note Thanks to Crystal for such an inspiring letter. It is good to see graduates succeed, especially as mathematics teachers.

[√At The Root Of It All - continued from page 3]

Being an active member of NCTM is one of many other interests Barbara has pursued over the years. She served on the Program Committee for the 1993 Annual Meeting and chairperson of Student Hosts for the 1991 Annual Meeting in New Orleans [Guess who served as hosts?]. A member of the Greater New Orleans Teachers of Mathematics, Barbara served as treasurer from 1986 - 1990. She was also treasurer of the Council of Presidential Awardees from 1985 - 1992 and a member of the Advisory Board for Quantum - a magazine of mathematics and science for secondary students and teachers.

An exceptional teacher, she was an early winner [1984] of the Presidential Award for Excellence in Mathematics Teaching. She was chosen Riverdale Teacher of the Year [three times], Outstanding New Orleans Mathematics Teacher, and Woodrow Wilson Master Teacher. All of these honors that Barbara has received are just a by-product of her one true goal - to reach her students. She teaches her students to love and enjoy both mathematics and life.

Throughout her term as President of MAΘ, Barbara has touched so many lives. As her terms ends this summer in Seattle, she leaves behind a wonderful legacy. Thank you, Barbara, for sharing so much of your life with Mu Alpha Theta!