

The

Mathematical Log

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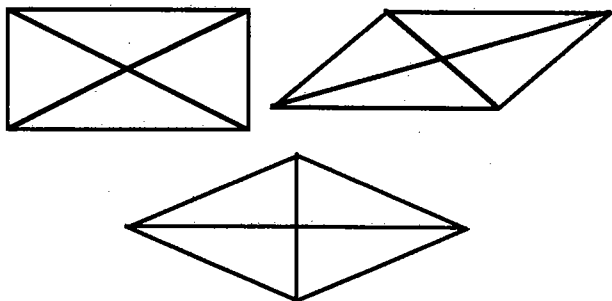
Challenging

If you exchange the hypothesis and the conclusion of a statement or theorem, you come up with another statement that is the **converse** of the original one. The false statement, "If I live in Texas, then I live in Dallas.", is the converse of the true statement, "If I live in Dallas, then I live in Texas." Let's warm up with several examples involving integers.

If $3 \mid a$ and $3 \mid b$, then $3 \mid (a + b)$, where $3 \mid a$ means 3 "divides" a or 3 is a factor of a , is a true statement that is an application of the distributive law. The converse, if $3 \mid (a + b)$, then $3 \mid a$ and $3 \mid b$, is false. [Find a counterexample.] In each of the following, a true statement and its converse are given. Decide which of the converse statements are true. If true, try to give a proof; if false, give a counterexample. [x, n, a, b are integers]

Statement	Converse
If $x > 0$, then $x^2 > 0$	If $x^2 > 0$, then $x > 0$.
If $6 \mid n$, then $6 \mid n^2$.	If $6 \mid n^2$, then $6 \mid n$
If $12 \mid n$, then $12 \mid n^2$.	If $12 \mid n^2$, then $12 \mid n$
If $a \mid b$, then $a^2 \mid b^2$.	If $a^2 \mid b^2$, then $a \mid b$.
If $a \mid b$, then $a^a \mid b^b$.	If $a^a \mid b^b$, then $a \mid b$.

Some of the most interesting examples come from geometry.



Three well-known properties of the diagonals of certain quadrilaterals are:

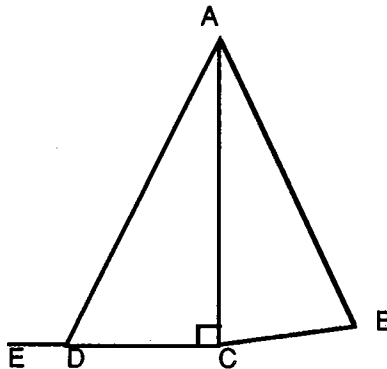
Converses

- The diagonals of a rectangle are equal.
- The diagonals of a parallelogram bisect each other.
- The diagonals of a rhombus are perpendicular.

Write the converse of each of these statements and decide whether the converse is true or false by giving a proof or a counterexample. The typical converse statement will be: "If the diagonals of a quadrilateral are ____, then the quadrilateral is a ____."

The Pythagorean Theorem is Proposition I.47 in the first book of Euclid's Elements. He closes this book with the converse of the Pythagorean Thm, whose proof is a logical "gem".

Proposition I.48 If a triangle with sides a, b , and c satisfies the condition $c^2 = a^2 + b^2$, then the triangle is a right triangle with hypotenuse c .



Proof 1: Given $\triangle ABC$ with $c^2 = a^2 + b^2$. We need to show $\angle ACB$ is a right angle. Draw CE perpendicular to AC at C . Then construct segment CD so that $CD = BC = b$. Let $AD = d$. Applying the Pythagorean Thm. to $\triangle ACD$, we have $d^2 = a^2 + b^2$. By hypothesis, $c^2 = a^2 + b^2$. Thus $c = d$ and $\triangle ABC \cong \triangle ADC$ by SSS so that $\triangle ABC$ is a right triangle.

Another version of this proof of the converse involves the properties of circles.

Proof 2: Given $\triangle ABC$ with $c^2 = a^2 + b^2$. With A as center and c as radius, draw circle A .

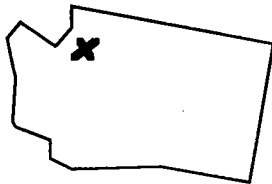
[continued. on page 6]

dia Log ue

with Log Editor Tom Butts

Many thanks to Sam Koski and all the other sponsors, students, and volunteers for making this year's national convention in Orlando such a success.

 Special congratulations to Debbie Phillips, the tireless editor of $\sqrt{\text{At the Root of It All}}$, who was this year's recipient of the Sister Scholastica Award as the Most Committed Sponsor, an award made by the Student Delegate Assembly.



Join us at the 27th Annual Convention in Seattle. More information in future issues of the Log.

SPEAK OUT !!!

This is your journal. You are encouraged to send your suggestions about ways to include more "visuals"; your thoughts on what type of articles and features to include; a question about any area of mathematics - a person, a concept, a problem,... ; news of your chapter; or anything else on your mind. Send it to me at the address below or e-mail at tbutts@utdallas.edu.

Transposition Errors

Rob Downes, a teacher at Mountain Lakes HS in Mountain Lakes, NJ, sends along this comment on an item from one of Marilyn Vos Savant's columns in Parade magazine.

Recently a reader asked Marilyn whether a discrepancy of \$63 in her checkbook indicated a transposition error since 63 is divisible by 9. Marilyn gave several examples and went on to say that "transposition errors always cause disagreements that are divisible by 9". How could you prove such a statement?

Let y and y' be the original and "permuted" numbers, respectively. We want to show $9 \mid (y - y')$.

Now $y = a_n 10^n + a_{n-1} 10^{n-1} + \dots + a_1 10 + a_0$ and $y' = b_n 10^n + b_{n-1} 10^{n-1} + \dots + b_1 10 + b_0$ where the a_i are digits and the $\{b_n, b_{n-1}, \dots, b_1, b_0\}$ is a permutation of $\{a_n, a_{n-1}, \dots, a_1, a_0\}$. In the difference $(y - y')$, group like coefficients together so that the difference is a sum of terms of the form $c_{ik}(10^i - 10^k)$. For $i > k$, $(10^i - 10^k) = 10^k(10^{i-k} - 1) = 10^k(99999 \dots 9)$. A similar result follows for $i < k$. Thus each of the terms is divisible by 9 and so is the sum $y - y'$.

Thanks to Rob for submitting this letter. You might also want to read the article on "What Do These Numbers Mean?", in the Log, Feb. 1994.

Perfect Parabolas

James Metz of the Mid-Pacific Institute in Honolulu, HI has contributed problems for each of the last two issue. In this issue we add another such problem. Recall the problem from the April 1995 issue for which there have been no solutions submitted.

- If a , b , and c are integers, under what condition(s) are the solutions to the equation $|ax + b| = c$ also integers? [Can the solutions be integers if a , b , and c are not integers?]
- Perfect Parabolas - A perfect parabola is one whose x -intercepts, y -intercept, coordinates of the vertex, and coefficients a , b , c [in $y = ax^2 + bx + c$] all have integer values. Can you find a procedure to determine perfect parabolas? Are all of the conditions necessary?

Send me your solutions to both of these problems.

The Mathematical Log

Volume 40 Number 3, October, 1996

The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu Alpha Theta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. e-mail: MATHETA@uoknor.edu or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson, TX 75083, e-mail: tbutts@utdallas.edu © 1996 Mu Alpha Theta

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1996 $\sqrt{\text{At the Root of It All}}$

Deborah Patonai Phillips, Activities Editor

St. Vincent-St. Mary HS, 15 North Maple Street, Akron, OH 44303

Two of the most prestigious MA Θ awards pay homage to three of its most important figures - founders Josephine and Richard Andree, and former governor and president, Robert Kalin. $\sqrt{\text{At The Root Of It All}}$ is proud to congratulate this year's Andree Award winner, Keith Bernicky, and Kalin Award winner, Dharmesh Mehta.

Striving to help students appreciate school for what it is, "an institution for learning", Keith Bernicky has already demonstrated his love of teaching. "Many high school students," he believes, "have the misconception that school is the awful building their parents make them go to in order to get good grades, or some see it as a place to socialize." To dispel this notion, Keith does everything he can to help as many students as possible by devoting his study halls and out-of-school time to tutoring in mathematics.

As Keith was growing up, his "heroes" were invariably teachers. "Teachers have always been the most intelligent people I have known," he proudly states. He hopes that he too can develop the work ethic, the patience, the ability to communicate, the insight, and the intelligence to become an effective teacher. "Keith enjoys seeing others learn almost more than his own learning," observes Keith's Math Team Coach Jim Aiu.

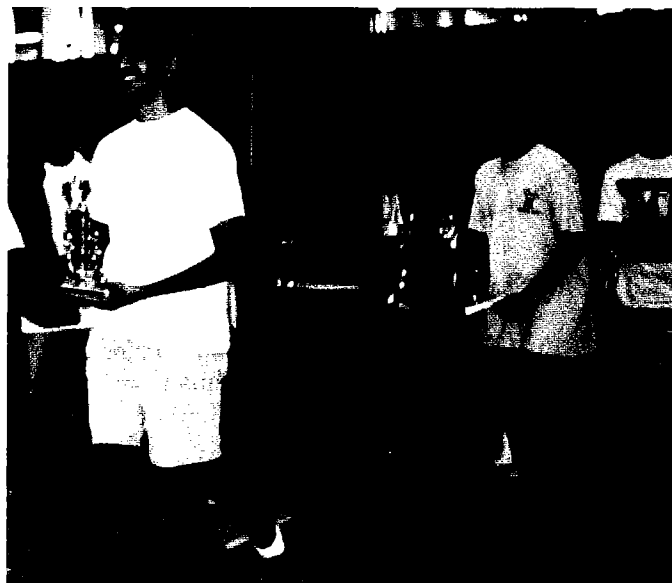
Joining the Math Team his freshman year, Keith was quiet, reserved, and respectful and did whatever was asked of him. "Keith's focus was on learning," according to Aiu. For the past two years, Keith spent much of the "normal" Math Team practice tutoring lower level students, but he still found time to make up the missed practices. Much to everyone's surprise, he performed extremely well in competitions this year.

In addition to MA Θ , Keith still found time to participate in many other activities. He was an active member of the history honor society, Future Teachers of America, the wrestling team, the track team, and he also contributed to his community's annual clean-up drive.

A well-rounded and versatile student with numerous skills, talents, and interests, Keith has the characteristics to become an effective and caring teacher. One cannot help but become excited about his future as a mathematics teacher.

Kalin Award winner Dharmesh Mehta from Marjory Stoneman Douglas HS in Parkland, FL represents the

epitome of a MA Θ member. He is a successful scholar, an erudite intellectual, and a service-oriented individual - a "Renaissance man".



Actively involved in MA Θ for the past six years, Dharmesh observed the positive impact MA Θ had on the students in his school and community. Serving as his chapter's vice-president last year and president this year, he put special effort into raising money for contest trips and organizing the mathematics awards ceremony recognizing successful students in competitions and classes. As Student Delegate president the past year, he encouraged many students and schools to attend the national convention. With his fellow student delegate officers, he discussed possible changes to benefit the organization.

"He is one of those rare geniuses who has it all."

"Dharmesh's accomplishments in mathematics are nothing short of incredible," according to his sponsor A. Singleton. All together, he has won well over 100 individual and team awards including a first place award in each of last five MA Θ national conventions. "As if all this were not enough", reports coach Singleton, "he scored a perfect 150 on both the 1994 and 1996 American High School Mathematics Examination - one of only five such scores in 1996 - and qualified for the USA Olympiad in three times." He also has written several tests for competitions hosted by his school. "Writing questions and solutions has taught me a great deal about teaching," Dharmesh reflected.
[continued on page 6]

[Cones and Cuttings, continued from page 5]

Sponsors:

Results of the past election: The new President-Elect is John Dossey, a former Mu Alpha Theta member. He is currently in the Mathematics Department at Illinois State University, Normal, IL. The new governor for Region I is Pat Bowler Johnson who teaches at New Trier HS in Winnetka, IL and is a familiar face at national conventions. Patricia Beck of Samuel Clemens HS, Sugarland, TX is the new governor of Region II. Pat sponsored the Texas State Meeting in last February.

Sponsor Swap:

Ed. Note: I received the following letter [edited] from Cindy Bunch, Fort Dorchester HS, 8500 Lincoln Blvd. North Charleston, SC 29420. It seemed that such letters could be the basis of this new section of Cones and Cuttings. Suggestions for another title are welcome.

"Regarding the Mathematicians Puzzle, my students made several large posters of the puzzle and placed them around the school. Interested students picked up a copy of the puzzle. Entries were collected and a prize was awarded to the student who was randomly selected from those with all correct answers. Last Saturday, eleven of our members attended our state convention and several have participated in various contests at local colleges, but for the most part the club has not been very active. I would be interested when other chapters hold their meetings. Our members also take part in many other activities, so attendance has not been good whether the meeting is after school or at night. I would love to hear from other sponsors about ways to get students interested and involved."

Students:

- Take a few minutes and write to one of your new student officers - see below.
- Huneke Award winners are nominated by their students. Think about nominating your sponsor for this award. Your sponsor must have been associated with Mu Alpha Theta for 5 years and have attended or will attend their second national convention. They cannot be a current member of the Governing Council. Write to the national office for nominating forms.
- **New Student Delegate Officers for 1996 - 97**
 - President: Daniel Neill, Hillsborough HS, 5000 N. Central Ave. Tampa FL 33603; 1903 Cape Bend Ave. Tampa, FL 33613

Vice-President: Adam Swensek, Riverdale HS, 240 Riverdale Blvd. Jefferson, LA 70121; 8702 Darby Lane, River Ridge, LA 70123; e-mail: aswensek@aol.com

Secretary-Treasurer: Nat Fredin, Lincoln Way HS, 1801 E. Lincoln Hwy., New Lenox, IL 60451; 330 Gina Dr. New Lenox IL 60451; e-mail: dfredin@aol.com

Sergeant-at-Arms: Allan Martin, Farragut HS, 11237 Kingston Pike, Knoxville, TN 37922; 11504 Green Mill Dr. Knoxville, TN 37922

From Daniel B. Neill Student Delegate President

Hi! My name is Daniel Neill and I am a senior at Hillsborough HS in Tampa FL. I have been involved in Mu Alpha Theta since the 7th grade. It has been a wonderful experience for me, not only as an opportunity to expand my knowledge of mathematics through practice and competition, but a chance to meet friendly and intelligent people from all over the country. This was my third national convention and, as always, I had an incredible time! We participated in many exciting activities including the Florida Challenge [a team scavenger hunt with many clever clues], entertaining speaker sessions, and trips to Disney World and the Kennedy Space Center. I appreciate the efforts of the students and sponsors from Miami Killian, Miami Springs, and Miami Sunset and everyone else who helped created a great convention.

This year the Student Delegate Assembly selected Deborah Phillips, sponsor of St. Vincent/St. Mary HS to receive the Sister Scholastica Award for the Most Committed Sponsor. The main topic of discussion concerned the redistricting of Alabama and Florida. Over half the schools attending the convention were from these two states, yet both states are part of Region III. Though no consensus was reached, recommendations from delegates included placing Florida and Alabama in different regions, making Florida its own region, and encouraging students from other states to become more active participants. Anyone is welcome to send me comments either through the national office or to the school address [above].

I feel that one of our main goals must be to encourage active nationwide participation in Mu Alpha Theta. I challenge every student and every sponsor to make a difference in any way they can – by encouraging other students and schools to participate, attending or hosting competitions and conventions, and serving in leadership roles. Most of all I encourage all of you [especially from schools from less active regions] to attend a national convention. It is a truly wonderful experience. I look forward to another year of Mu Alpha Theta and to the next national convention. I hope to see all of my old friends, and make many new ones, next year in Seattle.

1996 Overall School Results [Sweepstakes]

- | | |
|---------------------|---------------------|
| 1. Stoneman Douglas | 6. Berkeley Prep |
| 2. Vestavia Hills | 7. Grissom |
| 3. Lincoln | 8. J. P. Taravella |
| 4. Miami Springs | 9. Thomas Jefferson |
| 5. Farragut | 10. Gaither |



Cones & Cuttings



The Bulletin Board

• **STATE AND REGIONAL MEETINGS in 1996-97**
December 6-7, 1996: Sixth Annual Region IV Convention
 Contact Grace Mutz, Farragut HS, 11237 Kingston Pike,
 Knoxville, TN 37922

February 21-22 1997: Texas State Meeting. Contact
 Juan Saavedra, Edinburg HS, 801 E. Canton Rd.,
 Edinburg, TX 78539

March 1997: Louisiana State Meeting

March 1997: South Carolina State Meeting

April 1997: Mississippi State Meeting

April 1997: Florida State Meeting

• **FUTURE NATIONAL CONVENTIONS**

1997 Seattle

1998 Chicago area

Send information about your state or regional meetings
 to the national office.

Contests

Region I Contest: Contact Pat B. Johnson, New Trier
 HS, 385 Winnetka, IL 60093

Region III Contest: Contact Randy Long, Grissom HS,
 7901 Bailey Cove, Huntsville, AL 35802

Mu Alpha Theta on the Web

Our web site through the University of Oklahoma is
<http://www.math.uoknor.edu/~mualpha/>.

APPLAUSE !!!

Our laurels are showing

• The winners of the 1996 Presidential Awards for
 Excellence in Mathematics, who are also MA Θ sponsors:

- Melanie J. Nichols, Arkansas School of Science
 and Mathematics, Hot Springs, AR
- Patricia A. Daley, Fairfield HS, Fairfield, CT
- Jeffrey M. Choppin, Benjamin Banneker Academic
 HS, Washington, D.C.

• The winners of the 1996 Edith May Sliffe Award for
 Distinguished High School Mathematics Teaching
 who are also MA Θ sponsors:

- James Meier, Coral Springs, FL
- Patricia Zeran, Fort Lauderdale, FL

Congratulations to all!

Last year's winner of the Huneke Award, Helen Dostal, was the first recipient of the Dr. Betty Lichtenberg Award given by the Florida Association of Mu Alpha Theta [FAMAT]. Dr. Lichtenberg was president of Mu Alpha Theta from 1985 - 1987, editor of the Math Log from 1975 - 1980, and a founder of FAMAT.

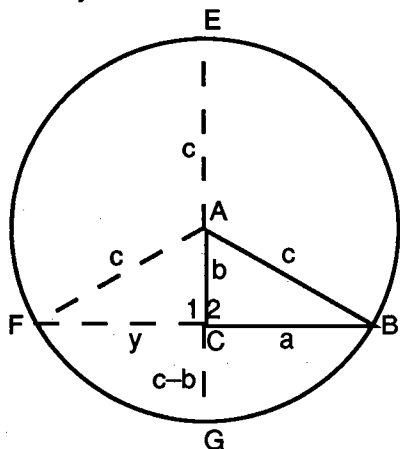
1995 At the Root of It All, Oct. 1995, contains a lengthy article on Helen. In a letter to FAMAT Board Members and Dr. Lichtenberg, she writes; "I was overwhelmed when I was named the first recipient of this award. Dr. Lichtenberg is one of many people involved with Mu Alpha Theta for whom I have the utmost admiration. ... To receive a plaque with her name engraved upon it touched my heart.

I have had a career not afforded to most. Many of the highlights of this rewarding segment of my life I credit solely to Mu Alpha Theta. I often wonder if Dr. and Dr. Andree every really understood the impact their organization would have on so many lives. Because of Mu Alpha Theta, I am a better mathematician, I have learned to relate to students in a way that is not possible in the regular classroom, and I have met students and sponsors whom I will ever treasure.

Some people think I have contributed to Mu Alpha Theta -- perhaps I have in some small ways. But, be assured, that I have received far more than I have given. I will simply say that Mu Alpha Theta has been my best investment!



[Challenging Converse, continued from page 1]
 Extend side BC through C to intersect the circle at F and let CF = y. Draw radius AF = c and extend side AC through A and C to intersect the circle at E and G, respectively.



By the Intersecting Chords Theorem, $(c - b)(c + b) = ay$ or $c^2 - b^2 = ay$. By hypothesis, $c^2 - b^2 = a^2$. Hence $a^2 = ay$, $y = a$ and $\triangle ABC \cong \triangle AFC$ by SSS. Thus $\angle 1 = \angle 2$ and both $\triangle ABC$ and $\triangle AFC$ are right triangles.

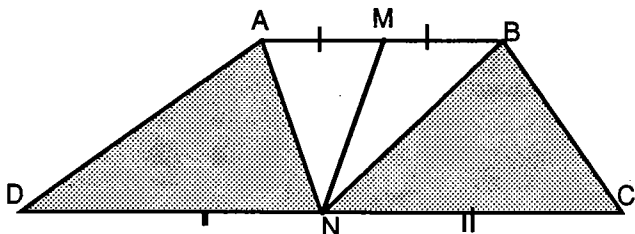
The converse of each of the next two theorems is true. One of the proofs is relatively easy and the other is very difficult. First state the converse of each theorem and then see if you can find the easy proof. [see Log, Oct. 1992].

- The medians to the equal sides of an isosceles triangle are equal.
- The angle bisectors of the base angles of an isosceles triangle are equal.

Two other provocative examples of theorems whose converses are true are given below. [see [2]].

Theorem: The segment joining the midpoints of the bases of a trapezoid [parallelogram] divide it into two quadrilateral of equal area.

Converse: If the line joining the midpoints M and N of sides AB and CD of a quadrilateral bisects its area, then these two sides are parallel.



Find a proof by showing $\triangle AND$ and $\triangle BNC$ have equal areas and that this implies that $AB \parallel CD$.

- In a parallelogram, the sum of the distances between the midpoints of the opposite sides is equal to the semiperimeter.

Converse: If the sum of the distances between the midpoints of the opposite sides of a quadrilateral is equal to the semiperimeter, then the quadrilateral is a parallelogram.

See if you can draw and label your own figure and then find a proof. **Hint:** Consider a diagonal of the quadrilateral and its midpoint.

Can find other examples of challenging converses? If you find one that you like, send it to the editor.

References

1. "The Converse of the Pythagorean Theorem", Rosenthal, J. Mathematics Teacher, Dec. 1994, pp. 692-693.
2. "Surprises of Conversion", Kushnir, I., Quantum, March/April 1996, pp. 38-39.

 [√At the Root of It All , continued from page 3]

Valedictorian of his class of over 600 students, he earned many other state and national academic awards including the Florida Young Scholars Program where one professor expounded, "Dharmesh's mathematical and academic abilities are phenomenal, yet his warm, caring, charming personality made him a favorite among his peers and staff. He is one of those rare geniuses who has it all."

Dharmesh was an active member of virtually every organization on campus - serving as an officer in most of them. "He is a sponsor's dream, ... never needed to be reminded of his responsibilities." He still found time for community work. He performs traditional Indian dances at cultural and religious programs, works as a hospital volunteer, spends time coaching and tutoring. He received the *Miami Herald* Silver Knight Award in Mathematics.

"He truly represents the best our nation has to offer in the educational domain," recalls one of his teachers. With all of these outstanding qualities and extraordinary talent, he is a most worthy recipient of the Kalin Award.

Mu Alpha Theta is fortunate to have two such mature, self-disciplined members. These exceptional young men will no doubt be successful in all the endeavors they may choose. Best of luck, Keith and Dharmesh!