Attend the 26th Annual Convention in Orlando 8/6 – 8/12

The Mathematical Log

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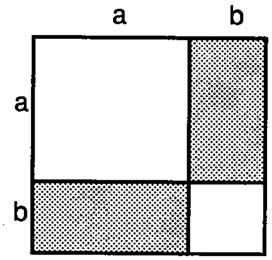
"Seeing is Believing"



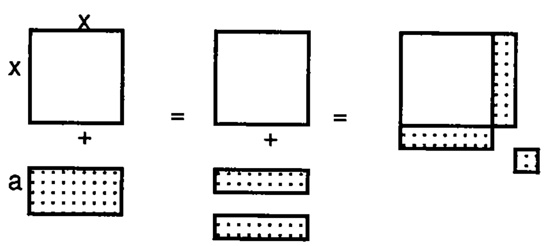
You have probably seen this diagram used to demonstrate the formula for squaring a binomial:

$(a + b)^2 = a^2 + 2ab + b^2$.

In this article we share several examples of such 'look-see' diagrams taken largely from Proofs Without Words by Roger Nelsen, MAA, 1993. Try to understand the examples and solve the problems. Then try to design a visual way to help you understand a concept or theorem that you find difficult.

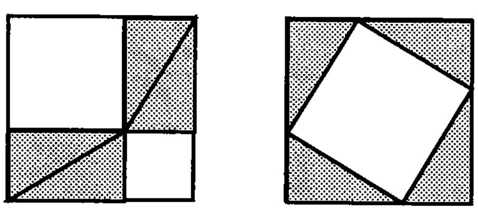
Example 1 Another way to use the diagram above is to demonstrate 'completing the square'.

$$x^2 + ax = (x + \frac{a}{2})^2 - (\frac{a}{2})^2$$



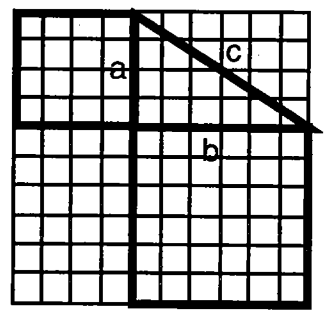
There are many visual diagrams illustrating the Pythagorean Theorem. Two of them are:

Example 2 [author unknown, circa 200 B.C.]



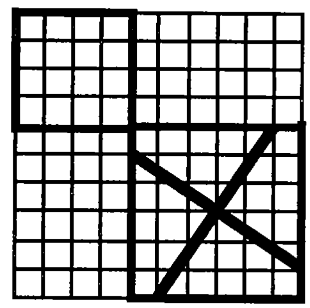
$a^2 + b^2 = c^2$

Problem 1 Explain how this sequence of diagrams, due to H. E. Dudeney [1917], illustrates the Pythagorean Theorem.



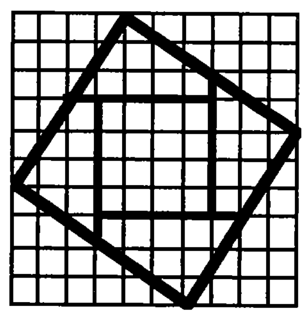
Example 3 Use the diagram below with the tilted rectangle and the labeled lengths and angles to explain why

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

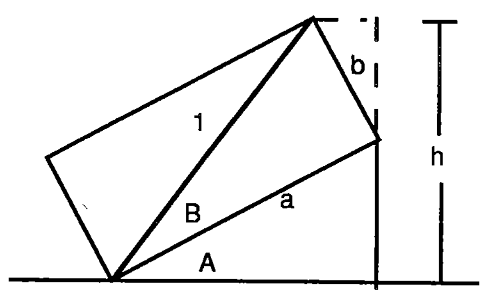


Solution

$\sin(A+B) = h = a \sin A + b \cos A = \sin A\cos B + \cos A\sin B$



Problem 2 Using the same diagram, explain why $\cos(A+B) = \cos A\cos B - \sin A\sin B$



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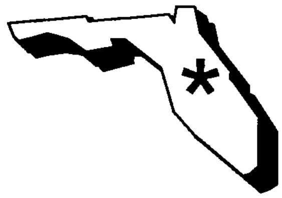
with Log Editor Tom Butts



A few questions and answers from Sam Koski about the 26th Annual Convention at the University of Central Florida in Orlando, FL. Miami Killian SR HS, Miami Springs SR HS, and Miami Sunset SR HS are the hosts.

Q: I am trying to plan my transportation to the convention. What are the exact dates and times?

A: The convention begins on August 6 with registration from 1:00 p.m. to 5:00 p.m. We serve dinner at 6:00 p.m.. Opening ceremonies and activities begin after dinner. The final banquet is on August 11 and we will serve breakfast early on August 12.

Q: How much does it cost to attend?

A: For schools that pre-register by April 1, the cost is $375 per person. All others must pay $425. See the convention package for more details.

Q: When will I get my package?

A: The packages are in the mail [Feb. 1]. If you have previously requested a package, you need do nothing further. See you in Orlando.

Highlights include a day at Disney World trip, a Cape Canaveral Tour, contests, and much more. For more information write to 1996 National MAΘ Convention, P.O.Box 161166, Miami FL, 33116-1166; send an e-mail message to Sam Koski at 76252.1660@compuserve.com; or contact the national office.
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SportsFigures

In the Oct. 1995 issue of the Log, Richard Rusczyk wrote a column about his involvement in **SportsFigures,** a series of seven 30-minute videotapes produced by ESPN. [" For example, we figured out why a curve ball curves and why golf balls have dimples. We used geometry to determine that 'flat' serves in tennis are impossible without gravity. ... It seems these TV people really do work hard – I always thought it looked so easy."] Each tape contains three segments featuring sports figures helping demonstrate various mathematics and physics concepts. These tapes are now available by calling the Customer Service Department of ESPN - 1-800-662-3776 [ESPN] between 9:00 a.m. and 5:00 p.m. EST. The national office and I have a copy of a flyer containing a complete description of the segments on the seven tapes and an order blank . Let one of us know and we will fax you a copy.
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The Unexpected Equation

I am pleased to share the first part of an article from James Metz of the Mid-Pacific Institute in Honolulu, HI. He writes:

"Find the equation of the circle with a diameter whose endpoints are (3,2) and (–5,6)."

When I posed this seemingly straightforward problem at the end of the period in my Algebra II class, I wanted to review the midpoint formula, the distance formula, and the equation of a circle. Much to my surprise and delight, one student, Cyrus Hanuna, found a formula that he said "somehow just came into my head". His answer to this problem was:

$(x-3)(x+5) + (y-2)(y-6) = 0$ and his theorem was:

If (x_1,y_1) and (x_2,y_2) are the endpoints of the diameter of a circle, then an equation of the circle is $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0.$

See if you can find a proof for Cyrus' theorem. We'll share Mr. Metz' comments on this experience in the next issue.

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The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association ofAmerica (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu AlphaTheta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. e-mail: MATHETA@uoknor.edu or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson,TX 75083, e-mail: tbutts@utdallas.edu

1996 √At the Root of It All

Deborah Patonai Phillips, Activities Editor
St. Vincent-St. Mary HS, 15 North Maple Street, Akron, OH 44303

On November 30, the 6-10 day forecast of the new weatherman for the Cincinnati area predicted mild temperatures in the high 60's or low 70's for December 8-9, the weekend of the Fifth Annual MAΘ Region IV Convention. As the weekend approached, the temperatures dropped and snow began falling. By the time the 273 convention participants arrived at Lakota HS in West Chester, OH on Friday evening, the temperature had dropped to near 0° with a wind chill factor of –25°. Chairperson Mary Rhein opened the convention by saying, "Welcome to the Ohio Valley where, if you wait a few minutes, the weather will change again."

Despite the frigid weather, eleven schools from Ohio, New York, and Tennessee experienced a great convention jammed full of contests, tournaments, and social activities. Mary, hosting her second regional, had five different contests including a Relay Competition and a Math Bowl. With the help of numerous teachers, parents, and students, there was also a chess tournament, a Euchre tournament, a dance, and delicious meals. Everyone was active and well-fed.

While the students were busy with contests, the sponsors had a chance to socialize with old and new friends. They detailed the joys and tribulations of their classes, discussed the merits of block scheduling, and shared fundraising ideas. A few even graded papers. One new face was Beatrice Tehi of Barberton HS in Barberton, OH. She has twenty active members in this chapter that is just two years old. Their main focus now is participating in contests, but they also find time to go bowling or ice-skating together. As a new chapter, they are faced with funding problems. While the school system does help with the registration fees, the chapter is selling poinsettias in conjunction with the local Kiwanis Club and is looking for other fundraisers.

In spite of their small numbers, the Barberton team was well-represented. Attending their first MAΘ convention, three seniors, six juniors, and one sophomore thoroughly enjoyed themselves as they competed in the various contests and made new friends. They are already looking forward to next year's convention.

While everyone attending the convention was a winner, some returned with trophies, rosettes, and ribbons. The top award of a paid registration fee at the national convention went to the Alpha student with the highest combined score on the Written Test and the Ciphering Test. Receiving the award this year was Halla Yang of Farragut HS. A new award this year, Math Champions, was presented to the school winning the most contests. This year's winner was Farragut HS. Other winners included:

WRITTEN COMPETITION - ALPHA

1. Halla Yang	Farragut
2. Dave Clark	Lakota
3. Chris Shen	Farragut
4. Andrew Schepler	Beavercreek

WRITTEN COMPETITION - THETA

1. Stephen Rochelle	Dobyns-Bennett
2. Charles Cushman	Dobyns-Bennett
3. Justus Kam	Farragut
4. Ann Rasmusser	Dobyns-Bennett

CIPHERING COMPETITION - ALPHA

1. Allen Martin	Farragut
2. Halla Yang	Faragut
3. Frank Petty	Beavercreek
4. Doug Lemme	Farragut

CIPHERING COMPETITION - THETA

1. Jerry Fu	Farragut
2. Shaline Shantharaju	Farragut
3. Eric Newland	Beavercreek
4. Brian Godsey	Lakota

INTERSCHOOL
1. Beavercreek
2. Farragut
3. Lakota

RELAY COMPETITION
1. Farragut A
2. Beavercreek A
3. Farragut B

MATH BOWL

ALPHA
1. Farragut
2. Beavercreek
3. Dobyns-Bennett

THETA
1. Farragut
2. Lakota
3. Kenmore West

For the past five years, Region IV has been the only region to host an annual convention. They require time and dedication on the part of the host, but they are a great motivator and allow participants to experience a convention atmosphere. They can be a stepping stone to the national convention. We do not yet have a host for the 1996 convention – contact the national office if interested. Thank you, Mary Rhein, and all of your co-workers, parents, students at Lakota HS for another great convention! We hope to see even more Region IV schools next year in ....

Mu Alpha Theta Contests

There are many types of contests given at Mu Alpha Theta conventions. At this year's national convention, all students will take a general test in their respective division: Calculus, Alpha [up to and including Precalculus], and Theta [up to and including Algebra II]. This year, those students in the Theta division who have not been enrolled in an Algebra II course may take several topic tests at the Mu level. There will also be a couple of new tests – one involving graphics calculators and graphs.

Here is a sampler of questions at various levels taken from several sources. In some cases, the multiple choice alternatives have been eliminated to allow space for more problems.

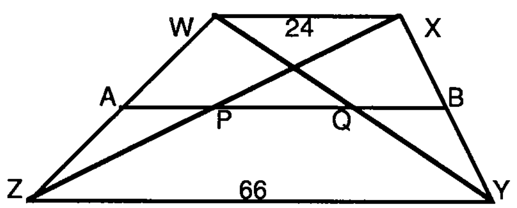
THETA CIPHERING - NATIONAL
Your score on these problems depends on the time needed solve the problem.

1. If $6x^2 + 29x + 35 = (ax + b)(cx + d)$, find $ac + bd$.

2. Find the sum of the four largest whole numbers less than 100 having an odd number of factors.

3. If the radius of a circle is increased by 100%, by what percent is the area of the circle increased?

4. WXYZ is a trapezoid, AB is the median. Find PQ.



5. If $N = \sqrt[4]{(12)(98)(21)(k)}$ is a natural number, find the smallest integral value of k.

6. Find the distance from the center of $x^2 + y^2 = 8y - 6x$ to the vertex of $8y = (x - 3)^2 + 16$.

ALPHA CIPHERING - NATIONAL
Your score on these problems depends on the time needed solve the problem.

7. If $x + y = 7$; $y + z = 20$; $w + u = 16$; $u + x = 8$, then $x + z =$ _____ .

8. If N is the smallest positive integer divisible by every two-digit square, then N = _____ .

9. Find the length of the longest umbrella that can fit in a box measuring 14 cm by 48 cm by 120 cm.

10. If $f(3t) = 18t^2 - 9t + 3$, then $f(-2) =$ ____ .

11. The distances from a point in the interior of an equilateral triangle to its sides are 5, 6, and 7 respectively. Find the area of the triangle.

MU TEST - FLORIDA

12. Find the volume of a cylinder with a lateral area of 192π and an altitude of 12.

13. Find the simplified ratio x:y for the solution to the linear system $4x + 3y = 1$; $6x - 2y = 21$.

THETA TEST - FLORIDA

14. In ΔABC, altitude $AD = 3$, $AB = AC$, $AB + BC + CA = 18$. Find AB.

15. Solve for x: $e^x - 4e^{-x} = 3$ [in terms of ln]

ALPHA TEST - FLORIDA

16. The graphs of the functions f and g are reflections of each other in the line $y = x$. If $f(x) = -x$, then $g(x) =$ ___ .

17. Given that $a + b = 1$ and $(ab)^2 = 9$, find $\frac{1}{a} + \frac{1}{b}$.

CALCULUS TEST - FLORIDA

18. For what values of x is $F(x) = x^3/3 - 5x^2/8 + 3x/8 + 1$ increasing?

19. Find the range of $f(x) = \frac{x^2 - 4x + 9}{x^2 + 4x + 9}$.

SCHOLARSHIP TEST - TEXAS

20. If $f(x) = 2x + 3$, find the degree of the polynomial $f(f(f(f(f(x)))))$.

21. In a 3-drawer chest, Hezy has 30% of his shirts in drawer A, 25% in drawer B, and the rest in drawer C. What is the fewest number of shirts he could have in drawer C?

23. A fifth number, N, is added to the set {4, 7, 10, 11} to make the mean of the set of five numbers equal to its median. Find all possible values of N.

24. All 120 permutations of the letters A, E, S, T, X are listed in alphabetical order. In what position in this list is TEXAS?

25. Find the number of solutions to the equation
$$(x-2)^{x^2 - x} = (x-2)^{12}.$$

26 Find the maximum value of the slope of a line that is tangent to the graph of $y = -x^3 + 3x^2 + 9x - 27$.

27. How many 8-digit natural numbers have exactly 8 different digits in descending order?

28. If $f(x) + 2f(6 - x) = x$ for all real x, find f(1).

Cones & Cuttings

The Bulletin Board

• STATE AND REGIONAL MEETINGS in 1996 •
March 9, 1996 South Carolina State Meeting, The Citadel, Charleston, SC. Contact Ann Long, First Baptist Church School, 48 Meeting St., Charleston, SC 29401. Phone: 803-722-6646.

March 1996: Louisiana State Meeting. Contact Barbara Stott, 240 Riverdale Dr., Jefferson, LA 70121

April 1996: Mississippi State Meeting. contact Rebecca Becnel, Long Beach HS, 300 East Old Pass Rd., Long Beach, MS 39560.

• FUTURE NATIONAL CONVENTIONS
1996 - Orlando Area - sponsored by Miami Sunset SR HS, Miami Killian SR HS, Miami Springs SR HS
1997 Seattle
1998 Chicago area

Send information about your state or regional meetings to the national office.

Our laurels are showing

Three sponsors were winners of the 1995 Presidential Award for Excellemce in Mathematics Teaching. They are:

• Patricia A. Daley, Fairfield HS, Fairfield, CT

• Jeffrey M. Choppin, Benjamin Banneker Academic HS, Washington, DC

• Linda H. Bowers, Alcorn Central HS, Glen, MS

Congratulations!

Students:
Did you know that you can purchase MAΘ jewelry, T-shirts, and books directly from the national office? Pins are $7.00; charms, $6.50; buttons or tie tacks, $6.00; patches, $1.50; medallions, $4.00, and T-shirts, $8.00. Write to the national office to place an order.

• Take a few minutes and write to one of your student officers.

• Student Delegate Officers for 1995 - 96 •
President: Dharmesh M. Mehta, Marjory Stoneman Douglas HS, 5901 Pine Island Rd., Parkland, FL 33067; e-mail: dharmesh@freenet.fsu.edu
Vice-President: Sonja Pruitt, Riverdale HS, 240 Riverdale Dr., Jefferson, LA 70121
Secretary-Treasurer: Nat Fredin, Lincoln Way HS, 1801 E. Lincoln Hwy., New Lenox, IL 60451
Sergeant-at-Arms: Chris Shen, Farragut HS, 11237 Kingston Pike, Knoxville, TN 37922

From Nat Fredin, Student Delegate Secretary-Treasurer

I am Nat Fredin, your student national secretary-treasurer for 1995-96. I am a junior at Lincoln Way HS in New Lenox, IL, which is about an hour southwest of Chicago. I thought I would share a few thoughts about attending a national convention. It's great fun, and we want you there! If you are interested, talk to your sponsor and convince him/her to take you [and some other members from your chapter].

This year's convention is in Orlando, FL from August 6 – 12. A typical convention day begins with breakfast around 8:00 a.m. followed by a couple of contests. Then lunch, followed by a few more contests and a break lasting until dinner. On several days, there is a guest speaker in the evening. Last summer we saw a comedian and a magician. Both were very funny and entertaining. The rest of the night is yours to enjoy with your friends.

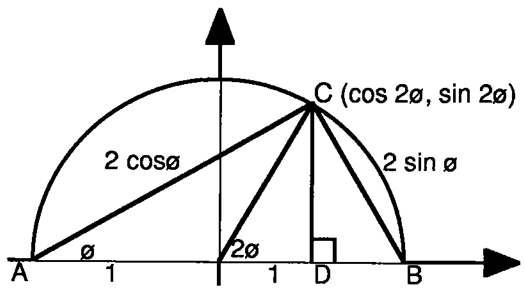
On one day, everyone has the opportunity to take part in a special fun activity. Last year in Maine, it was white water rafting. It was great adventure. This year a day at Walt Disney World is planned.

In addition to the fun, there are several types of competitions. Everyone takes a general test and up to three tests on specific topics. There are also several group competitions such as the State Bowl, the School Bowl, and relays. In the two bowls, individuals from both divisions who score well on the general test are grouped together with the top scorers from their school or state to take a group test. There are very nice awards to the top fifteen individuals or groups. Believe it or not, even though it's a math convention, it's really fun! [See page 4 for more information on Mu Alpha Theta contests.]

I hope I was able to give your a sense of what a convention is like and I hope I have convinced you to want to come! Talk to your sponsor or the national office. It's too great an experience to miss.

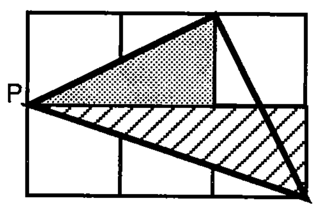
['Seeing is Believing' - continued from page 1]

Problem 3 Verify the labeling in the diagram and then use the fact that $\Delta ACD \sim \Delta ABC$ to explain the double-angle formulas: $\sin 2ø = 2\sin ø \cos ø$ and $\cos 2ø = 2\cos^2 ø - 1$.



Example 4 On your calculator,

[tan⁻¹] (1/2) + [tan⁻¹] (1/3) [ENTER] yields .785398. The exact value of this sum appears to be $\frac{\pi}{4}$. We can justify this conjecture with a diagram.



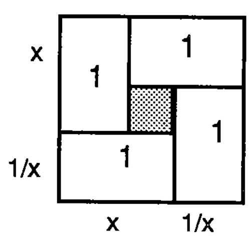
The angle at P measures $\frac{\pi}{4}$ radians since it is a base angle of an isosceles right triangle.

Example 5 Illustrate the validity of $x + \frac{1}{x} \geq 2$, for any $x \geq 1$, using a diagram.

Solution

Take four rectangles, each x by $\frac{1}{x}$ with an area of 1.

They can be rearranged to form a square with a square 'hole' in the middle.

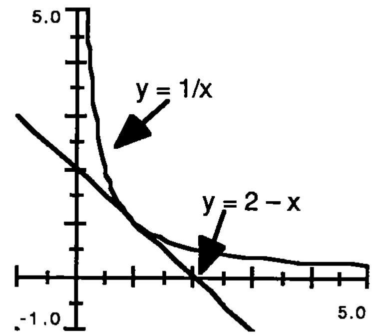


The area of the square is greater than or equal to 4, so the side of the side of the square, $x + \frac{1}{x}$, must be greater than or equal to 2. Equality occurs when $x = 1$; when each of the four rectangles 'becomes' a square and the hole vanishes.

Problem 4

Explain how the graph illustrates the validity of

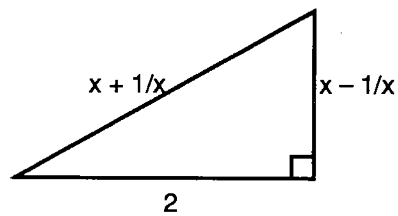
$x + \frac{1}{x} \geq 2$, for any $x \geq 1$.



Problem 5

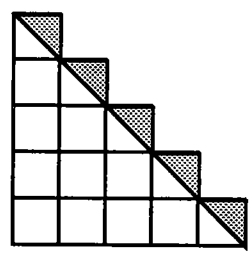
Explain how the right triangle illustrates the validity of

$x + \frac{1}{x} \geq 2$, for any $x \geq 1$.



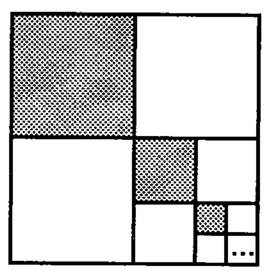
Two of my favorites are:

Example 6



$$1 + 2 + 3 + \ldots + n = \frac{n^2}{2} + \frac{n}{2}$$

Example 7



$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \ldots\ldots = \frac{1}{3}$$

Problem 6

Devise your own diagram to illustrate a favorite concept or theorem.