Attend the 26th Annual Convention in Orlando 8/6 – 8/12

The Mathematical Log

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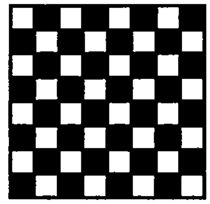
Checkerboard Challenges

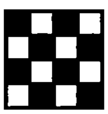


"How many squares on a checkerboard?"

There are, of course, 64 small ones, but when this question is asked as a mathematical puzzle, the desire is to count all squares whose sides are parallel to the sides of the board and whose vertices are points where several small squares come together. To determine this number, let's first consider a simpler problem - count the number of squares on smaller checkerboards.

4 [1x1 squares] + 1 [2x2 square] = 5

9 [1x1 squares] + 4 [2x2 squares] + 1 [3x3 square] = 14



16 [1x1 squares] + 9 [2x2 squares] + 4 [3x3 squares] + 1 [4x4 square] = 30

Continuing in this way, we find the table:

Size of Board	Number of Squares
1	1
2	5
3	14
4	30
5	55
6	91
7	140
8	204

and there are 204 squares on a standard checkerboard.

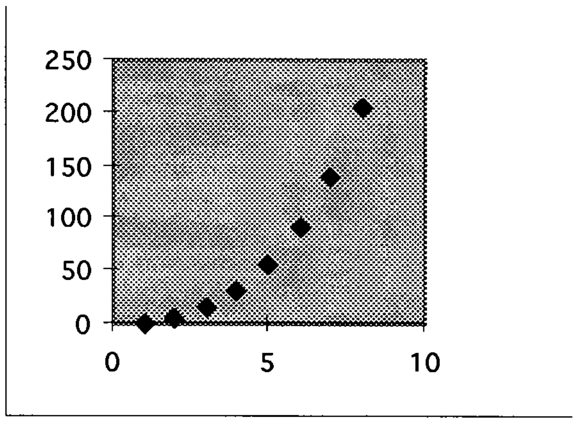
To generalize this problem, we ask: "How many squares are there on an nxn checkerboard? "

We see that this number is

$S(n) = 1^2 + 2^2 + ... + n^2$ or in recursive form,

$S(n+1) = S(n) + (n + 1)^2$, $S(1) = 1$.

To find a closed form for S(n), we examine a scatterplot of the data in the table and plot several polynomial functions to see if any of them fit the data. After some effort, we conjecture that S(n) might be a cubic polynomial since $Y1= .5x^3$ was "fairly close" to the data. Since the sum $1 + 2 + ... + n = \frac{1}{2} n(n+1)$, a quadratic polynomial, it seems plausible that a sum of squares might be a cubic polynomial.



Entering the table as two lists on a TI-82 graphic calculator, for example, and checking cubic regression, we find

CubicReg
$y = ax^3 + bx^2 + cx + d$
a =.33333333
b = .5
c = .16666666
d = 6.1E-11

We conjecture that $S(n) = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$.

To prove our conjecture, we can use mathematical induction. [see Log Feb. 1993, April 1993] .

[continued on page 6]

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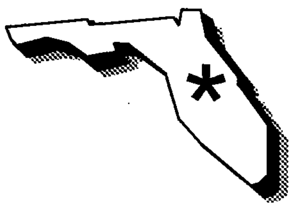
with Log Editor Tom Butts



Join us at the 26th Annual Convention at the University of Central Florida in Orlando, FL. Miami Killian SR HS, Miami Springs SR HS, and Miami Sunset SR HS are the hosts. Highlights include a Disney World trip, a Cape Canaveral Tour, contests, and much more. The estimated registration fee is $375 per person. For more information write to 1996 National MAΘ Convention, P.O.Box 161166, Miami FL, 33116-1166; send an e-mail message to Sam Koski at 76252.1660@compuserve.com; or contact the national office.

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The last issue was largely devoted to the annual convention and articles about sponsors and members. There was very little mathematics. Do you think we should have a "Convention Special" issue in the fall that is sent only to schools that attended the convention? I also continue to solicit your thoughts on what type of articles and features to include; a question about any area of mathematics - a person, a concept, a problem,... ; news of your chapter; or anything else on your mind. Send it to me at the address below or e-mail at tbutts@utdallas.edu.

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Famous Mathematicians

I have a good friend named "Schultz" and whenever I use my spell-checker to examine a message to him, it "suggests" the word "schmaltz" for "Schultz" since "Schultz" is not in its dictionary. Each of the following words occured as a suggestion from my spell-checker when I entered the last name of a mathematician, or person closely identified with mathematics. See how many of these mathematicians you can identify from their "spell-check names". Try to give at least one reason why the person is famous. Remember that some very famous mathematicians such as Gauss, Newton, and Pascal were recognized since, for example, their names are also scientific terms.

Send in your list of "spell-check names". I'll try to publish it.

Tom Butts, Log editor,
Moo Alpha Theta

Format	Pioneer
Ester	Bolero
Ruler	Apologies
Remain	Chronicler
German	Buffoon
Babble	Banker
Furor	Napa
Whales	Market

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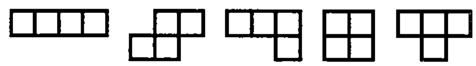
[Checkerboard Challenges - continued from page 6]

Problem 9 Is it possible to form a rectangle with these five tetrominoes?



Problem 10 Can a 10x10 checkerboard be covered with 25 straight tetrominoes like the one at the far left above?

Problem 11 Consider a 'mutilated' nxn checkerboard with the four corner squares removed. For what values of n can you cover such a board with L-tetrominoes like the middle one in the figure above.

In a future issues we will consider some problems that illustrate the use of various problem solving strategies on other situations involving a checkerboard.

The Mathematical Log
Volume 39 Number 4, December, 1995
The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association ofAmerica (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu AlphaTheta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. e-mail: MATHETA@uoknor.edu or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson,TX 75083, e-mail: tbutts@utdallas.edu

1995 √ At the Root of It All

Deborah Patonai Phillips, Activities Editor
St. Vincent-St. Mary HS, 15 North Maple Street, Akron, OH 44303

√At The Root Of It All is proud to showcase two outstanding members: Andree Award winner, Rory Gormley and Kalin Award recipient, Allen MacKenzie.

Named after MAΘ founders Richard and Josephine Andree, this award is given to someone who wishes to pursue a career in the teaching of mathematics. "Being a math teacher would give me a chance to do something that I do well and will enjoy as a career," says this year's winner, Rory Gormley, a graduate of Clarkstown HS in New York , NY who plans to attend Harvard University.

Introduced to MAΘ in his freshman year, he quickly demonstrated his mathematics ability, leadership skills, and teaching abilities. As vice-president in his junior year, he designed a presentation on the graphing calculator used at the induction ceremony. In his senior year as co-president, computer-generated fractals was the highlight of the induction ceremony presentation.

Because of taking a summer course at Johns Hopkins University in the summer following 7th grade, Rory was able to accelerate his high school program and take BC Calculus as a junior. He was encouraged by his teachers to participate in contests where he had many successes. He scored over 100 on the AHSME, participated in ARML, and did well in many state competitions.

While Rory sees mathematics as a "glorious challenge," he understands that not all students share his point of view. Helping students to a better understanding of a difficult concept has enabled him to share his love of mathematics. "Though it can show me many things," he says, "mathematics cannot show me what lies within the heart of my fellow man." By becoming a mathematics teacher, Rory will be able to work simultaneously with his two passions in life – "the symmetry and regularity of mathematics and the unpredictability and uniqueness of people."

The 1995 Kalin Award was presented to Allen MacKenzie of Farragut HS in Knoxville, TN. Extremely active in Mu Alpha Theta for the past four years, Allen has been chapter president for the past two years, Tennessee's State Vice-President, and National Secretary-Treasurer and Vice-President. In these positions, he has worked hard to increase every member's involvement in chapter activities and to promote the formation of new chapters.

Honing his leadership skills, Allen held executive positions in other organizations. He has served as captain of the Math Team, secretary-treasurer of the German Club, captain of the Science Bowl Team, and technical director for the school's production of "Fiddler on the Roof". In addition to being a member of several other school organizations, Allen is actively involved in his church as a camp counselor and a participant in a leadership training camp.

Earning the top AHSME score in Tennessee is one of many contest honors he has earned. He was the top Alpha winner at the Region IV convention, placed in the top ten for three consecutive years at the Tennessee Math Teachers Association Competition, and did well in several MAθ contests at the national convention. Awards in other areas include the Bausch and Lomb Science Award and a nomination for Georgia Tech's Scholar Award. He intends to pursue a career in electrical engineering at Vanderbilt University.

He credits Mu Alpha Theta with helping him "push my mathematical skills to the limit and developing my skills as a leader." His sponsor, Grace Mutz, takes great pride in recommending him as one of her finest students - "his potential is unlimited!"

Through involvement in Mu Alpha Theta, members are furnished with many opportunities to grow in mathematics and to develop as leaders. Allen and Rory are two examples who made the most of these opportunities. Congratulations and good luck to both.

Another Month of Mathematics [see Log, Feb. 1992]

1 •number of eyes of a Cyclops or humps of a dromedary •sum of .9 + .09 + .009 + ...	**2** •number of sides in an argument •an integer is the sum of consecutive integers if it is not a power of 2*	**3** •number is divisible by 3 if the sum of its digits is divisible by 3* •number of colors in the French flag	**4** •number of seasons •a regular polygon with lattice point vertices must be a square*	**5** •only prime that is the sum and difference of two primes* •Coco Chanel's lucky number – Chanel no. 5 was introduced on 5/5/21
6 • largest integer that is not a prime nor the sum two different primes* • number of wives of Henry VIII	**7** • number going to St. Ives ??? • number of ways of linking four congruent regular hexagons*	**8** •only power of a prime that differs from another prime power by 1* •number of legs on a scorpion	**9** •number of players on a baseball team • number is divisible by 9 if the sum of its digits is divisible by 9*	**10** •number of legs of a crab •only value of $n \neq k!$ for which $n! = a!b!$
11 • largest integer that is not the sum of two or more different primes* •number of players on a football or soccer team	**12** •number of "Days of Christmas" [how many gifts in all?]* •number of signs of the Zodiac	**13** •a "baker's dozen" •number of Archimedean polyhedra*	**14** •number of days for a waxing moon to become a full moon •one of four consecutive integers that are the sides and the area of a triangle*	**15** •number of crystal anniversary •one of the smallest pair of triangular numbers whose sum and difference are also triangular numbers
16 •number of pieces used by each player in a game of chess •only integer n for which $n = x^y = y^x$ has integral solutions*	**17** •parallel between North Vietnam and South Vietnam • $n^2 + n + 17$ is often prime*	**18** •age of adulthood •one of two numbers that can be both the area and perimeter of a rectangle*	**19** •number of amendment giving women the right to vote •third hexagonal number*	**20** •maximum number of questions in an identification game •second smallest pseudo-perfect number*
21 •winning score in a game of table tennis •number of guns in a presidential salute	**22** •one of three numbers n where n! has n digits* •a palindrome whose square is a palindrome*	**23** •number of people in a room where the probability of choosing two of them with the same birthday is .50. •number of one of most popular Psalms	**24** •smallest number for which the product of its proper factors is a cube* •number of furlongs in a league	**25** •number of pictures per second at which television is transmitted •only power that is 1 more than a factorial*
26 •smallest non-palindromic number whose square is a palindrome* •number of letters in the alphabet	**27** •equal to the sum of the digits of its cube* •number of books in the New Testament	**28** •number of digits [width of one finger] in a cubit •only perfect number of the form $x^n + y^n$*	**29** •maximum number of pieces into which a pizza can be cut with 7 cuts* •number of days in February in a leap year	**30** •minimum age to be a U.S. senator •one of two numbers that can be both the area and perimeter of a Pythagorean triangle*
31 •number of moves in the Tower of Hanoi puzzle with 5 disks* *number of letters is the Cyrillic alphabet		An asterisk (*) following a problem indicates that you may wish to solve the problem or find the reason. You are encouraged to send in your "events" for each day of the month to the editor.		

Cones & Cuttings

15 The Bulletin Board

• STATE AND REGIONAL MEETINGS in 1995-96 •
February 9-10 1996: Texas State Meeting. Contact Ms. Pat Beck, W.P. Clements HS, 4200 Elkins Rd., Sugarland TX 77479

March 9, 1996 South Carolina State Meeting, The Citadel, Charleston, SC. Contact Ann Long, First Baptist Church School, 48 Meeting St., Charleston, SC 29401. Phone: 803-722-6646.

March 1996: Louisiana State Meeting. Contact Barbara Stott, 240 Riverdale Dr., Jefferson, LA 70121

April 1996: Mississippi State Meeting. contact Rebecca Becnel, Long Beach HS, 300 East Old Pass Rd., Long Beach, MS 39560.

• FUTURE NATIONAL CONVENTIONS
1996- Orlando Area - sponsored by Miami Sunset SR HS, Miami Killian SR HS, Miami Springs SR HS
1997 Seattle
1998 Chicago area

Send information about your state or regional meetings to the national office.

Mu Alpha Theta on the Web
Our web site through the University of Oklahoma is now under construction. Look for us soon.

Students:
• Huneke Award winners are nominated by their students. Think about nominating your sponsor for this award. Your sponsor must have been associated with Mu Alpha Theta for 5 years and have attended or will attend their second national convention. They cannot be a current member of the Governing Council. Write to the national office for nominating forms.

Did you know that you can purchase MAΘ jewelry, T-shirts, and books directly from the national office? Pins are $7.00; charms, $6.50; buttons or tie tacks, $6.00; patches, $1.50; medallions, $4.00, and T-shirts, $8.00. Write to the national office for the list of available books or to place an order.

An idea for graduation --- become a "Friend of Mu Alpha Theta". Receive the Math Log regularly and any new publications for just $10 a year.

• Take a few minutes and write to one of your student officers .

• Student Delegate Officers for 1995 - 96•
President: Dharmesh M. Mehta, Marjory Stoneman Douglas HS, 5901 Pine Island Rd., Parkland, FL 33067; e-mail: dharmesh@freenet.fsu.edu
Vice-President: Sonja Pruitt, Riverdale HS, 240 Riverdale Dr., Jefferson, LA 70121
Secretary-Treasurer: Nat Fredin, Lincoln Way HS, 1801 E. Lincoln Hwy., New Lenox, IL 60451
Sergeant-at-Arms: Chris Shen, Farragut HS, 11237 Kingston Pike, Knoxville, TN 37922

From Sonja Pruitt. Student Delegate Vice-President

As a senior currently attending Riverdale HS in Jefferson, LA, I, like most of you, have been involved in mathematics competitions and Mu Alpha Theta since the 7th grade. This year my offices include Chapter Vice-President, District Secretary-Treasurer, and Student Delegate Vice-President.

Attending the National Convention in Maine was truly an experience that I will never forget. From the rafting trip to the student delegate meetings, I thoroughly enjoyed myself.

I would like to remind all the delegates that we pledged to write letters encouraging inactive chapters to participate in this year's national convention in Orlanda, FL. Before writing this column, I wrote a letter to Mount Vernon HS in Alexandria, VA. [Hey, guys, I would like to hear from you!] Delegates: get busy writing those letters.

I can't wait to meet delegates this summer from ALL the states.

[Checkerboard Challenges - continued from page 1]
Recall the Principle of Mathematical Induction:
Let P(n) be a statement that depends on the positive integer, n.

If 1. Anchor step: P(m) is true, and
2. Induction step: For all positive integers, $k \geq m$, $P(k + 1)$ is true if P(k) is true,
then P(n) is true for all integers $n \geq m$.

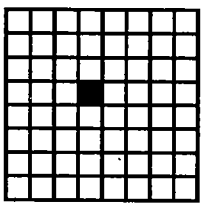
In this case, the statement P(n) is the formula S(n). For this conjecture m = 1 and S(1) is true since $S_1 = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$. The induction step becomes:

if $S_k = \frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k$,

then $S_{k+1} = \frac{1}{3}(k+1)^3 + \frac{1}{2}(k+1)^2 + \frac{1}{6}(k+1)$.

But $S_{k+1} = S_k + (k+1)^2 = \frac{1}{3}k^3 + \frac{1}{2}k^2 + \frac{1}{6}k + (k+1)^2 = \frac{1}{3}(k+1)^3 + \frac{1}{2}(k+1)^2 + \frac{1}{6}(k+1)$ [after some algebraic calisthenics]. Mathematical induction is especially easy to use when our conjecture was expressed as a recursive sequence.

Problem 1 How many of the 204 squares on a standard checkerboard do **not** contain the shaded one?



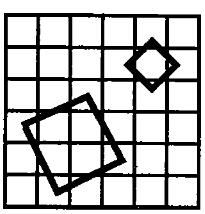
Generalize if you can.

Problem 2 How many mxn rectangles are there on an nxn checkerboard?

Problem 3 How many cubes are there in an nxnxn cube?

Problem 4 Pose, and solve if you can, a problem similar to Problem 1 on an nxn checkerboard.

An interesting extension involving even more squares on a checkerboard is the Game of Hip. [see Math Horizons, MAA, Nov. 1995, p. 20] Two players take turns placing counters on the small squares of a 6x6 checkerboard. Each player tries to avoid placing his counters so that four of them mark the corners of a square. The square may be of any size and tipped at any angle as in the figure below.



A player wins if his opponent forms a square.

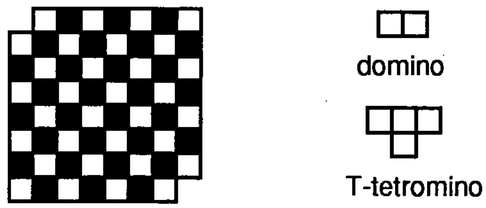
You might also try the game on other sized checkerboards.

Problem 5 Is it possible to have a draw in this game? Justify your answer.

Problem 6 Show there are 105 squares [including 'tipped' ones] on this 6x6 checkerboard.

Problem 7 Generalize Problem 6 by trying to find a formula for the number of squares [including 'tipped' ones] on an nxn checkerboard.

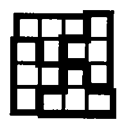
Another classic puzzle involves covering a 'mutilated' checkerboard with dominoes. The classic version of the puzzle asks whether it is possible to cover a checkerboard whose two opposite corner squares have been removed with 31 2x1 dominoes.



The impossibility of this task is nicely demonstrated by a coloring proof. Each domino covers one black square and one white square. But the 'mutilated' checkerboard has 30 black squares and 32 white squares.

Example A T-tetromino is shown above.
a. Can you cover a standard checkerboard with 16 T-tetrominoes?
b. Can you cover a 10x10 checkerboard with 25 T-tetrominoes?

Solution a. A T-tetromino covers 3 black squares and 1 white square or 3 white squares and 1 black square. We can cover a 4x4 block with 4 T-tetrominoes as shown and, hence, cover a standard checkerboard.



b. Since a 10x10 checkerboard has 50 black squares and 50 white squares, such a covering would require the same number of each type of T-tetromino. But since we need 25 T-tetrominoes, we have an obvious contradiction.

Problem 8 Suppose we remove one white square and one black square from a standard checkerboard. Can we cover the remaining 62 squares with 31 2x1 dominoes? Start with some special cases where the two squares are adjacent, the two squares are at opposite ends of a row, etc.

[continued on page 2]