

The

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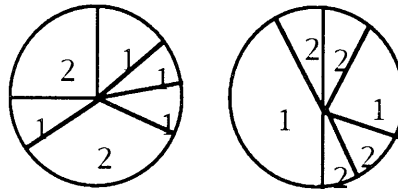
Some Probability Preconceptions

Answer each of the following multiple choice questions about situations involving probability. They are adapted from research studies on student's intuitions about some ideas from probability [see "Thinking About Uncertainty: Probability and Statistics", Shaughnessy and Bergman, Research Ideas for the Classroom, NCTM 1993] Many of the participants in these studies had little or no formal training in probability.

1. Two bags have black and white counters. Bag J: 3 black and 1 white Bag K: 6 black and 2 white. Which bag gives a better chance of picking a *black* counter?

- A) Bag J B) Bag K C) same chance for both bags

2. Two discs are marked with numbers. Each disc has a pointer that spins. If you want to get a 1, which disc would you choose?



- A) left one B) right one
C) doesn't matter - both discs have the same chance

3. The probability of a baby's being a boy is about $\frac{1}{2}$. Which of the following sequences is more likely to occur in having six children?

- A) BGGGB B) BBBGB C) BBBGG D) All sequences have about the same chance

4. The probability of a baby's being a boy is about $\frac{1}{2}$. Over the course of an entire year, where would there be more days when at least 60% of the babies born were boys?

- A) in a large hospital B) in a small hospital C) makes no difference

5. A large bag has the same number of black balls and white balls in it. Which is more likely?

- A) Choosing 7 whites in 10 draws B) Choosing 70 whites in 100 draws C) same likelihood

6. The probability of a baby's being a boy is about $\frac{1}{2}$. What is the probability that among six children, [exactly]

- three will be girls. A) less than $\frac{1}{4}$ B) between $\frac{1}{4}$ and $\frac{1}{2}$ C) $\frac{1}{2}$ D) more than $\frac{1}{2}$

7. A woman is bright, single, 31 years old, outspoken, and concerned with issues of social justice. Which of these statements is more probable?

- A) She is a bank loan officer. B) She is a bank loan officer and a feminist.
C) The two statements have about the same probability.

8. A cab was involved in a hit-and-run accident. There are two cab companies – Blue Cab [15% of all cabs] and Green Cab [85% of all cabs]. A witness identified the cab at the scene as being a Blue cab. Tested under similar visibility conditions, this witness was able to make the correct color identification in 80% of the trial instances. What is the probability that the cab involved in the accident was a Blue cab?

- A) less than $\frac{1}{4}$ B) between $\frac{1}{4}$ and $\frac{1}{2}$ C) between $\frac{1}{2}$ and $\frac{3}{4}$ D) more than $\frac{3}{4}$

[continued on page 4]

1994 $\sqrt{\text{At the Root of It All}}$

Deborah Patonai Phillips, Activities Editor

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For the first time in the thirty-five year history of the International Mathematical Olympiad, the United States team won. Each of the six team members [all boys] earned a perfect score. Held in Hong Kong for two days in July, the competition, usually dominated by China and Russia, attracted over 360 students from 68 countries. $\sqrt{\text{At the Root Of It All}}$ is proud to congratulate Jeremy Bern, Aleksandr Khazanov, Noam Shazeer, Jacob Lurie, Jonathon Weinstein, and, "one of our own" Stephen Wang from the Illinois Mathematics and Science Academy in Aurora, Illinois. Walter Mientka, Executive Director of the American Mathematical competitions and one of the coaches of the team remarked: "They're [the team members] a tremendous role model for how our country can achieve excellence."

At IMSA, Stephen was president of the chapter last year. He helped his club raise money by selling calculators [bought in bulk at reduced prices], and solution manuals to calculus books. Involved in MA Θ since his freshman year, he has competed in a wide variety of contests, including those at the national conventions in Huntsville, Honolulu, and New Orleans. At Huntsville, he was chosen for the Spirit Award by his space shuttle group. In Hawaii, he placed 5th in Geometry [Alpha Division], 4th on Logs and Exponents, and 1st in Theory of Equations. Winner of the MA Θ Region I Mathematics Contest, he earned a free registration to New Orleans. There he placed 1st in

Alpha Geometry and helped his team to 4th place in Sweepstakes.

To become a member of the Olympiad Team, he first achieved a perfect score of 150 on the AHSME Test taken, this year, by over 350,000 students. After performing well on the AIME and USAMO, he was chosen to attend the four-week training institute for the Olympiad [for the third time] held at the U.S. Naval Academy in June. He was subsequently chosen to be one of the six team members. During the training, the students attend lectures and take many practice tests.

After arriving in Hong Kong, the students played cards and sang Beatle songs to relieve the tension. The competition consisted of two sessions - each having three problems to be solved in 4.5 hours. One of the problems: "Determine all ordered pairs (m,n) of positive integers such that $\frac{n^3 + 1}{mn - 1}$ is an integer."

After their triumphant victory, Stephen and the rest of the team celebrated by playing another game of cards.

Besides excelling in mathematics, Stephen is an accomplished musician on the piano and several percussion instruments. He enjoys competing in basketball and Ultimate Frisbee. His favorite pastime, however, is playing cards - especially Hearts, Spades, and Bridge.

Mu Alpha Theta salutes Stephen Wang and wished him well in his study in International Relations at Harvard University.



CONTEST CORNER

This month's column contains the Power Question and some team and individual questions from the 1994 ARML Contest. They were composed by Gil Kessler and Larry Zimmerman. The solutions will appear in the next issue.

LATTICE POINTS

This question refers to figures in a two-dimensional coordinate system only.

Lattice points are points on a grid, both of whose coordinates are integers. A lattice segment is a line segment whose endpoints are lattice points, an oblique lattice segment is one that is not parallel to either axis.

The letter L will refer to the length of a lattice segment as well as the segment itself and $N(L)$ will represent the number of lattice points on segment of length L , NOT INCLUDING ITS ENDPOINTS. [For example: $N(\sqrt{2}) = 0$; $N(17) = 0$ if the segment is oblique and $N(17) = 16$ if the segment is not oblique.]

SEGMENTS

- Compute $N(L)$ for the lattice segment L whose endpoints are
 - $(0,0)$ and $(3,4)$
 - $(0,0)$ and $(12,16)$
 - $(0,0)$ and $(300,400)$
 - $(-3,11)$ and $(57,101)$
- Compute all possible values for $N(L)$ if L is oblique and
 - $L = 13$
 - $L = 4 \cdot 13$
 - $L = 5 \cdot 13$
- Compute the number of different possible values for $N(L)$ if L is oblique and $L = 5 \cdot 13 \cdot 17$

A lattice polygon is a polygon whose vertices are lattice points; an oblique lattice polygon is one whose sides are all oblique.

Pick's Theorem [1899] The area of any lattice polygon is given by $K = \frac{1}{2}B + I - 1$ [where K is the area, B is the number of lattice points on the boundary (sides and vertices) of the polygon, and I is the number of lattice points in the interior of the polygon.]

POLYGONS

- Compute the value of I for the triangle whose vertices are $(1,2)$, $(31,42)$, and $(81,-78)$.
- Compute the maximum value of I for a lattice square whose side is 25.
 - Compute the maximum value of I for a lattice square whose side is 26.
- Show that if two sides of an oblique lattice triangle are 5 and 13, then the third side cannot be an integer.

- Given any 5 lattice points, show there is a segment joining two of them that contains a lattice point (other than the endpoints of the segment).
 - Prove that every convex lattice pentagon must contain an interior lattice point.
 - A "trapezium" is a quadrilateral with no pair of parallel sides. For a convex lattice trapezium:
 - Show that if two successive sides each contain a lattice point (other than the vertices), then there is a lattice point in the interior of the trapezium.
 - Show that if there are two opposite sides neither of which contain a lattice point (other than the vertices), then there is a lattice point in the interior of the trapezium.
 [Together these show that every convex lattice trapezium contains a lattice point.]
- If the product of two sides of a lattice triangle is a prime, and the area is also a prime, prove that the area must be 2.

A team problem from 1994 ARML Competition:

- "An isosceles triangle has a median equal to 15 and an altitude equal to 24." This information determines exactly two triangles. Find the area of one of these triangles.

In the next issue, we'll have more problems from the ARML competition and a little more information about Pick's Theorem.

[Some Probability Preconceptions, continued from p.1]

Let's take a brief look at the answers now. We discuss the ideas behind these questions in the next issue of the Log.

- C Half the participants in the study chose bag K, many because it had more black counters than Bag J.
- B Nearly 25% chose the first spinner since it had more regions labeled '1'.
- C Even though both sequences have the same probability, far more participants chose A since they felt it was more representative of the 50-50 distribution of boys and girls.
- A 5. A 6. B Many people chose C for both #4 and #5 erroneously feeling that the size of the sample has no effect on the probability. All three questions are examples of Bernoulli trials, one of the topics of the first article in the next issue.
- A The probability of two events occurring simultaneously is less than the chance of either occurring separately.
- B Many participants neglected that fact that most of the cabs in the city are green, not blue.

From Joyce Becker, MA Θ President

continued from page 5

Miriam Goldstein was the \$1,000 Kalin Award winner. She attended Berkeley Prep School in Tampa, FL where the sponsor is Thomas Morris. He also accepted \$300 for the Berkeley Prep chapter. The \$500 winner of the Huneke Award for the outstanding sponsor was Pete Pedersen of Vinalhaven HS in Vinalhaven, ME. Several Governing Council members have completed their terms of office. They include NCTM Representative Mary Emma Bunch of Farragut HS in Knoxville, TN Region III Governor Frank Caballero of Miami Sunset HS in Miami, FL, Region IV Governor Debbie Phillips of St. Vincent-St. Mary HS in Akron, OH, and Past President Carol McGill of West Orange-Stark HS in Orange, TX. We thank these people for their many hours of dedicated service.

Newly-elected Governing Council members include NCTM Representative Gwen Snoddy of Austin HS in Decatur, AL, Region III Governor Randy Long of Grissom HS in Huntsville, AL, Region IV Governor Grace Mutz of Farragut HS in Knoxville, TN, and President-Elect Barbara Stott of Riverdale HS in Metairie, LA. Welcome aboard!

From Miriam Goldstein, Former Student Delegate President

In our three lengthy and productive delegate meetings, we discussed a variety of topics. I was able to discuss our list of suggestions for national conventions with the host of next year's convention in Maine, Pete Pedersen.

We also created a new award for sponsors to be selected and presented by the student delegates. It seemed to us that the one sponsor award, the Huneke Award, was not sufficient to honor the many distinguished sponsors in our organization. This award is unique in that it was initiated by students and that the selection process is fully student run. I hope you will take the opportunity to nominate your sponsor for this honor. The guidelines are:

- The sponsor must be in attendance at the national convention in the year nominated

- The nominee must have three letters of recommendation sent to the national office. Possible recommenders include any current or former MA Θ members, or colleagues of the sponsor. Topics to consider in the recommendation:

Students: Sponsor's influence on your participation in MA Θ . Sponsor's accomplishments and service to MA Θ .
 Colleagues: Sponsor's accomplishments and service to MA Θ .

- Nominations and recommendations should be sent to the national office no later than June 15 of the year in which the award is to be made.

At the national convention, nominees will be narrowed to three by the four student delegate officers. Copies of these nominations will be distributed to all delegates who will then vote and choose the winner after an appropriate discussion.

Another idea we would like to implement is a 'buddy system' between new and existing chapters. Each year new chapters can be encouraged to attend competitions and conventions if they are in contact with an active chapter. I suggest that members of the chapters exchange letters during the year. This correspondence should increase the number of active chapters and allow the sharing of information on fund-raisers and activities. If you would like your chapter to be involved in this program, please write to the national office. Please include the name and address of the contact person for your chapter.

Chapter Doings

The MA Θ chapter of George Washington Carver HS in Montgomery, AL had an exciting spring. Matthew Olson, Chris Evans, and De'andrea Barker entered kites in the **Go Fly a Kite Contest** while Christina Burdick entered a Hypercard program entitled "Math Kites". We also enjoyed taking Easter baskets to the children of Father Purcell's Hospital.





Cones

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Cuttings



MA Θ Bulletin Board

• STATE AND REGIONAL MEETINGS in 1994-95 •

December 2-3, 1994: Fourth Annual Region IV Convention Kingsport, TN; Sherry Cox, Dobyns-Bennett HS, 1800 Legion Drive, Kingsport, TN 37664

February 1995: Texas State Meeting. To be held in Austin, TX. Contact Mrs. Diane McGowan, Bowie HS, 4103 W. Slaughter Lane, Austin TX 78749

March 1995: Western Tennessee Regional Meeting.

March 1995: Louisiana State Meeting. Contact Ms. Barbara Stott, Riverdale HS, 240 Riverdale Ave., Jefferson. LA 70121.

March 1995: Tennessee State Meeting, McMinn County HS, 2215 Congress Parkway, Athens, TN 37303.
Contact: Luajean Bryan

April 1995: Mississippi State Meeting. Jackson, MS

May 1995: Florida State Meeting.

• FUTURE NATIONAL CONVENTIONS

1995 - Sugarloaf Resort in western Maine
1996 - Orlando Area - sponsored by Miami Sunset SR HS, Miami Killian SR HS, Miami Springs SR HS
1997 ???
1998 Chicago area ??

Send information about your state or regional meetings to the national office.

Other important dates:

Oct. 15, 1994 Deadline for entries to the Region I Contest

February 16, 1995 AHSME

Sponsors:

- Remember dues are now \$3 per student.
- Lincoln High School, Tallahassee, FL has organized a booster club to assist and support the extra-curricular and mathematical endeavors of their MA Θ members. If you want a copy of their by-laws and wish further information, please contact the national office.

Our laurels are showing

- The winners of the 1994 Tandy Technology Scholars Awards who are also MA Θ sponsors:
Annalee Rosenwie Henderson, State College Area HS, State College, PA

Duane C. Hinders, Henry M. Gunn HS,
Palo Alto, CA

Betty Pate Mayberry, Gallatin HS
Gallatin, TN

Diana Loewenstein Perkins, Las Vegas Academy for
the Performing Arts, Las Vegas, NV

Congratulations!

Students:

• Huneke Award winners are nominated by their students. Think about nominating your sponsor for this award. Your sponsor must have been associated with Mu Alpha Theta for 5 years and have attended or will attend their second national convention. They cannot be a current member of the Governing Council. Write to the national office for nominating forms.

See the information about a new sponsor award in the column by Miriam Goldstein.

Did you know that you can purchase MA Θ jewelry, T-shirts, and books directly from the national office? Pins are \$7.00; charms, \$6.50; buttons or tie tacks, \$6.00; patches, \$1.50; medallions, \$4.00, and T-shirts, \$8.00. Write to the national office for the list of available books or to place an order.

An idea for graduation --- become a "Friend of Mu Alpha Theta". Receive the Math Log regularly and any new publications for just \$10 a year.

• New Student Delegate Officers for 1994 - 95. •

President: Truong Tang, Thomas Jefferson High School, Auburn, WA

Vice-President: Allen McKenzie, Farragut High School, Knoxville, TN

Secretary: Harold Lubber, Coral Springs High School, Coral Springs, FL

Sgt.-at-Arms: Kevin Stancoven, West Orange-Stark High School, Orange, TX

From Joyce Becker, MA Θ President

Let me summarize some of the highlights of the past year. The winner of the \$1,000 Andree Scholarship for a future teacher of mathematics was Derinda Taylor of Austin HS in Decatur, AL. Her sponsor was Gwen Snoddy. The Andree Award was named in honor of Richard and Josephine Andree, the founders of MAQ. The two \$300 winners of the Kalin Awards for mathematics achievement and service to Mu Alpha Theta were Steven Parfitt of Lakota HS in West Chester, OH and Ronald DeBlanc of West Orange-Stark HS in Orange, TX. The Lakota sponsor is Mary Rhein and Don Griffin is the West Orange-Stark sponsor.

[continued on page 5]