Attend the 1994 Convention in New Orleans, Aug. 1–6

The

Mathematical Log

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Greedy Algorithms - Take the Best Choice

There are many interesting properties of the Fibonacci numbers {Fn}. Recall that $F_1 = 1$, $F_2 = 2$, and $F_{n+2} = F_{n+1} + F_n$. One of the most important is the special way they can be used to represent integers. Every positive integer, n, has a unique representation of the form $n = F_{k_1} + F_{k_2} + ... + F_{k_r}$ where $F_{k_1} > F_{k_2} > ... > F_{k_r}$ and no two terms in this representation are adjacent terms in the Fibonacci sequence. For example:

67 = 55 + 13 + 2 and

1,000,000 = 832040 + 121393 + 46368 + 144 + 55

We can find such a representation using a **greedy algorithm**. Choose F_{k_1} to be the largest Fibonacci number $\leq n$. Then choose F_{k_2} to be the largest one $\leq n - F_{k_1}$; and so on. More precisely, $F_{k_1} \leq n < F_{k_1+1}$ so that $n - F_{k_1} < F_{k_1+1} - F_{k_1} = F_{k_1-1}$ so that F_{k_1} and F_{k_2} are not adjacent terms in the Fibonacci sequence. In the same way, no two terms in this representation are adjacent terms in the Fibonacci sequence.

Exercise 1 a. Explain why you obtain the base 2 representation of a positive integer using a greedy algorithm. b. Can consecutive powers of 2 occur in such a representation? Either give several examples or explain why none exist.

Exercise 2 Using mathematical induction, prove this representation using Fibonacci numbers is unique.

This Fibonacci Number System is used in fast search algorithms. The golden ratio, ϕ, is the limit of the quotients of consecutive Fibonacci numbers, that is $\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \phi$ or $F_{n+1} \approx \phi F_n$. This property, combined with the curiosity that ϕ is approximately the number of kilometers in a mile, gives us a way to mentally convert mutually between kilometers and miles.

Example 1 Express 30 miles in kilometers.

Solution The number of kilometers is 30ϕ. But $30 = 21 + 8 + 1$ so that $30\phi = 21\phi + 8\phi + 1\phi = 34 + 13 + 2 \approx 49$ kilometers. For each summand, take the next Fibonacci number and add.

Exercise 3 a. Express 30 km in miles.
b. Express 65 mi/hr in km/hr.
c. How many sq. km are in 10 sq. mi.?

Another example of a greedy algorithm is illustrated Sylvester's algorithm for uniquely expressing a rational fraction, $\frac{m}{n}$, between 0 and 1 as the sum of unit fractions, those of the form $\frac{1}{q}$. Choose $\frac{1}{q_1}$ to be the largest unit fraction [the one with the smallest denominator] $< \frac{m}{n}$. Choose $\frac{1}{q_2}$ to be the largest one $< \frac{m}{n} - \frac{1}{q_1}$ and so on to obtain $\frac{m}{n} = \frac{1}{q_1} + \frac{1}{q_2} + \frac{1}{q_3} + ... + \frac{1}{q_r}$. To find q_1, for example, divide n by m and take the next integer greater that this quotient [the *ceiling* of the quotient] . thus $\frac{3}{7} = \frac{1}{3} + \frac{2}{21} = \frac{1}{3} + \frac{1}{11} + \frac{1}{231}$. [7/3 = 2.33; the ceiling of $\frac{7}{3}$ is 3 or $\left\lceil \frac{7}{3} \right\rceil = 3$; 21/2 = 11.5].

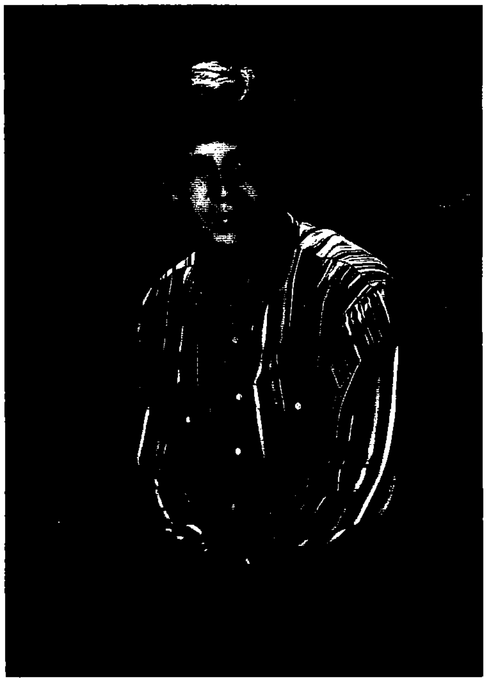
Exercise 4 Express several fractions between 0 and 1 as the sum of unit fractions using Sylvester's Algorithm.

Many of the most interesting uses of a greedy algorithm occur in finite graph theory. A **graph** is a finite set of **vertices** and **edges** connecting some of the pairs of vertices.
[continued on page 6]

A Quick Preview of the Annual Convention See p. 2.

[Cones & Cuttings continued from page 5]

From Russell Perry, Student Sergeant-At-Arms



At last year's national convention in Hawaii, there were only five schools from Region II which consists of Texas, Louisiana, Oklahoma, and Arkansas. Since the 1994 convention is being held in our region, I would like to encourage more schools to participate. I know you will enjoy the competitions, the many other activities, and, above all, the chance to meet members from other parts of the country. Share a bus with another chapter near you. See ya'll there.

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Gary Blackburn and all of his colleagues can't wait to have you join them at the annual convention August 1 – 6 at the Clarion Hotel, 1500 Canal Street, New Orleans, LA.

Planning is moving "full steam ahead" and a full slate of activities of all kinds are awaiting you. Included are:

Monday: Opening session with entertainment

Tuesday: A dance aboard the riverboat 'Natchez'.

Wednesday: General Session II with entertainment.

Thursday: A day at the Riverfront including a visit to the world-renowned aquarium, a cruise, and a visit to the zoo.

Friday: Closing Session and Banquet

There will be the customary competitions and speaker sessions.

Gary Blackburn: Home: 504–835–9793
School: 504–283–1561

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The third joint MAA/MAΘ joint lecture will be presented by Dr. Pam Drummond at the MAA annual summer meeting to be held in Minneapolis, Minnesota on August 15 – 17. Currently on the faculty of Kennesaw College, she is a past president of MAΘ, past regional governor, and former sponsor at Walton HS in Marietta, Georgia. She is now the MAA representative on the governing council.

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Governor James Aiu reports that Region I Contest winner this year is Steven Wang of IMSA (Illinois Mathematics and Science Academy). He achieved a perfect score year on the three contests and will receive a registration scholarship to the annual convention in New Orleans. Over 35 schools participated in the competition.

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[√At the Root of It All , continued from p. 3]

Stan is very impressed with the breadth, depth, and vigor of the active membership of MAΘ, especially in certain parts of the country. "It is important that the National Office continue to foster these active chapters." He hopes to expand the organization, especially in states and regions that currently have few or no chapters. In time, he hopes to meet and thank the many sponsors who have invested so much of themselves in MAΘ and the students who have profited from their efforts.

Room 423 , ~~608~~, ~~626~~, 625 is full of past memories of MAΘ. With the continuing efforts of Diane Rubin, with the new regime of Stan Eliason, and with the continuing techonolgical advances, there is room for many more of them.

The Mathematical Log

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The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association ofAmerica (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu AlphaTheta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson,TX 75083

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1994 √At the Root of It All

Deborah Patonai Phillips, Activities Editor
St. Vincent-St. Mary HS, 15 North Maple Street, Akron, OH 44303

Have you ever wondered about Room 423 on the return address Mu Alpha Theta envelopes? It's the mail room for the building on the campus of the University of Oklahoma where the national office is located. The office room number is ~~608~~, ~~626~~, 625 [It seems like it is moved every year.] Who works there and what do they do? I had always imagined it to be a very secretive place full of old math books, dusty Logs, and piles of membership lists. At the Thirteenth National Convention, hosted by David Drennan at the University of Oklahoma, I had the fantastic opportunity to see the room. It was much different that what I had imagined! Now, many years later, √At the Root of It All , takes a look at the hub of MAΘ, the national office.

The office ... is managed by Diane Rubin, who takes care of just about everything.

The office, under the auspices of the Mathematics Department, is managed by Diane Rubin, who takes care of just about everything. Since she began in the fall of 1983, she has been typing certificates, filling orders, answering the phone, and mailing the Log. All requests, as well as all correspondence passes through her capable hands. She is also the valuable link between the national office and the governing board and handles the MAΘ booth at the NCTM Annual Convention. She also finds time to attend regional conferences and to correct the mistakes of the copy for the Log.

Originally from New Orleans, Louisiana, Diane received her B.A. degree from Newcomb College of Tulane University where she met her husband, Lenny. She taught history in New Orleans and then in Coral Gables, Florida. When her husband accepted a position as a topologist at the University of Oklahoma, she moved to Norman, OK.

When the Rubins arrived in 1967, they met two other newly-hired colleagues: past Secretary-Treasurer Thomas J. Hill, in mathematics education, and current interim Secretary-Treasurer, Stanley B. Eliason, an analyst. Other key members of the mathematics department were Harold Huneke, who served as Secretary-Treasurer for 22 years and, of course, Richard and Josephine Andree, the founding "parents" of MAΘ.

Today Diane thoroughly enjoys her work with MAΘ. She "loves MAΘ and has a firm desire to see mathematics develop at levels." Her job allows her to meet many new people and to develop long-lasting friendships with students and sponsors. Former members who are enrolled in mathematics courses at the university often stop by to visit. Last year when Kristin Collett from Durant OK and David Miller from Natchitoches, LA came by the office, she hired them to help package the Log. Sponsors, on the other hand, maintain close contact with Diane throughout the year to keep abreast of other sponsors, upcoming meetings, and materials orders.

because of the installation of the Gateway 2000 computer .... she no longer has to type membership certificates and charters "by hand"

Lately Diane has been busy updating the office the routines because of the installation of the Gateway 2000 computer. She no longer has to type membership certificates and charters "by hand" and she is now able to communicate using e-mail. In the future, she hopes to add Quattro Pro and Paradox software.

The national office is also home for interim Secretary-Treasurer Stan Eliason. Stan grew up in a small town of 200 people in rural North Dakota where his father had settled after immigrating from Norway. He received his B.A degree from Concordia College in Moorhead, Minnesota and his PhD in Analysis from the University of Nebraska. During his 25+ years at the University of Oklahoma, he has worked vigorously to promote mathematics and mathematics education. He served as Associate Chair and Chairman of the department for several years during the mid 1980's.

Stan has presented numerous talks at schools, teacher workshops, colleges, and conferences. For the past two summers, he has been one of main presenters at the NSF-sponsored Mathematics Institute for Teachers at the University of Tulsa on the use of MATLAB software in mathematical modeling .

Along with his teaching responsibilities, Stan is active in various professional organizations. He is a member of the American Mathematical Society, NCTM, and Sigma Xi Society. He is currently chair of the publicity committee for CAMEO [Coalition for the Advancement of Mathematics in Oklahoma] and Secretary-Treasurer of the Oklahoma-Arkansas Section of the Mathematical Association of America.

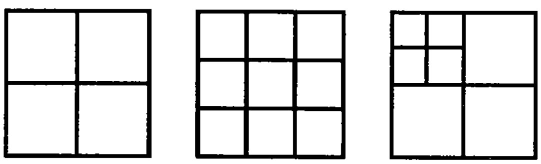
[continued on page 2]

dia Log ue

with Log Editor Tom Butts

The first item is another problem from longtime sponsor and former editor of the Log., Don Allen. Following Dr. Allen's custom, we will offer "fame, or at least, notoriety" to the first Loggers to submit a correct solution [with some indication of your method.]

A non-routine geometric question that students find challenging involves the partitioning of a square into smaller squares. A square is easily partitioned into four or nine smaller squares. Subdividing one of the four squares into four still smaller squares gives a partition of seven squares. Six squares takes some thought, and eleven is at least as great a challenge. The overall question: For what whole numbers n can a square be partitioned into n smaller squares?



The analog in three dimensions is also quite instructive. Partitioning a cube into 8, 27, 64, or, in general, n^3 smaller cubes is easily visualized. You can also visualize the removal of an outer shell containing $6n^2 - 12n + 8$ smaller cubes [The $6n^2$ counts the cubes in six faces; $-12n$ eliminates the doubly-counted cubes on each of the 12 edges, $+8$ restores the eight cubes at the vertices.] This leaves a central cubical region for further consideration. More pertinent, though, is a half-shell containing $3n^2 - 3n + 1$ smaller cubes which also leaves another cube for consideration. The overall question: For what whole numbers n can a cube be partitioned into n smaller cubes?

The fact that the number of instances where such a partitioning is not possible is finite, and relatively small, may be surprising. You also wish to pursue the problem into the fourth dimension, or even further.

Send in your solutions by September 1, 1994. The solution will be published in one of the first two issues next fall.

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Since the time is drawing near for Advanced Placement Examinations, I am happy to share this poem written by Julie Ciamporcero of Rutgers Preparatory School of Somerset, NJ and submitted by her sponsor, Beth Edmondson. Now a freshman at Yale University, Julie wrote this poem the night before she took the AP Calculus Test on May 11, 1993.

Thoughts on Calculus

It is truly an art,
this calculus

Not a messy art,
like writing,
where distinctions are bleared and blurred
and the gray area dominates.

Not calculus.

Here everything is orderly,
crystalline, beautiful,
symmetric and plane
like pieces of a puzzle
that fit, perfectly,
two lovers,
the composition of two inverse functions,
that's a Fundamental Theorem, you know,
that everything fits perfectly.

But not for me.

I dream of throwing out an integral sign
like a fish hook,
and catching on its curled but finely pointed tip
this utter perfection,
reeling it in and lifting it, dripping,
from the crystalline water
to admire the light
reflecting off it.
My hook comes back empty.
Or worse,
I have hooked another old rubber boot,
reflecting the grisly sun
into my face,
laughing.
You're still not a math person, it laughs.
Go write a poem or something.

So I do.

But a poem can never be perfect,
O calculus, stained-glass fish
dancing in the depths, glinting at me
to remind me that you are still there,
that I have just neglected to catch you.
I continued my imperfect art.

But perfect calculus,
I am still watching you,
and someday my reach
will no longer exceed my grasp.

Note: Julie received a '5' on her exam.

Cones & Cuttings

Bulletin Board

•STATE AND REGIONAL MEETINGS in 1994•

May 1994: Florida State Meeting. Contact the national office for details.

December 2–3 1994 Region IV Convention. Contact Sherry Cox, Dobyns-Bennett HS, 1800 Legion Drive, Kingsport, Tennessee 37664.

- FUTURE NATIONAL CONVENTIONS
  1995 - Bowdoin College, Maine
  1996, 1997, 1998 ?????

Send in information about your state or regional meetings to the national office.

STUDENTS

• Huneke Award winners are nominated by their students. Think about nominating your sponsor for this award. Your sponsor must have been associated with Mu Alpha Theta for 5 years and have attended or will attend their second national convention. They cannot be a current member of the Governing Council. Write to the national office for more details.

• Did you know that you can purchase MAΘ jewelry, T-shirts, and books directly from the national office? Pins are $7.00; charms, $6.50; buttons or tie tacks, $6.00; patches, $1.50; medallions, $4.00, and T-shirts, $8.00. Write to the national office for the list of available books or to place an order.

• An idea for graduation --- become a "Friend of Mu Alpha Theta". Receive the Math Log regularly and any new publications for just $10 a year.

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Congratulations to Daniel Cohan [pictured] and all the other students, parents, and sponsors who helped run the Texas State Convention in February. Over 500 students from more than 35 schools attended.

From Miriam Goldstein, Student Delegate President

Here in Florida, we have MAΘ competitions every two weeks from January to April. They are always well-organized. These successful competitions are the result of the efforts of many people -- people who are not always recognized and properly thanked.

Different schools sponsor each competition and there is much work involved. The sponsors from each school must organize the preparation of the questions, mailing of invitations, reproduction of the tests, making of the schedules, and printing of maps. Many test writers contribute to both the individual and team rounds. Teachers, students, and parents serve as proctors. Sponsors run the scoring room and settle disputes about test questions. Sponsors from participating schools must accompany their students, often traveling long distances. Students often practice before and after school. Parents support the competitions in many ways. as do the national office and the regional governing board.

I owe much to MAΘ. It has offered me the opportunity to improve my skills at something I love -- and I've had fun doing it. The friendships I have formed on the basis of this shared interest will endure. The hours I've spent working on mathematics with my team and the good times I've had at competitions and conventions are some of my best memories from high school. And, as a result of the efforts of all those mentioned above, high school students will continue to benefit from MAΘ.

Information about the national convention was mailed recently. The delegates from your region would like to know your suggestions for such meetings, so please write to us c/o the national office. I'll see you in New Orleans.