Attend the 1993 Convention in sunny Hawaii, Aug. 4 – 10

The Mathematical Log

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Induction and Induction: Guessing and Proving

When we examine a few specific cases of a phenomenon and then make a guess or conjecture that we think is true in general, we are using inductive reasoning or, induction. Induction involves reasoning from evidence. The validity of our conjecture can depend on how carefully we gather the evidence as well as how much evidence there is. Even if we carefully compile a "mountain of evidence", there is no guarantee, however, that our conjecture is correct. Mathematical induction, also sometimes called induction, is a method we can use to prove certain types of conjectures. It is an example of deductive reasoning. George Polya, the father of modern mathematical problem solving, has said, "It is unfortunate that the names are connected because there is very little logical connection between the two processes. There is, however, some practical connection: we often use both methods together."

We first illustrate Polya's remarks in a familiar example.

What is the sum of the first n odd positve integers?

To gather evidence, we examine the first few cases:

1	1
1 + 3	4
1 + 3 + 5	9
1 + 3 + 5 + 7	16
1 + 3 + 5 + 7 + 9	25
1 + 3 + 5 + 7 + 9 + 11	36

Based on this evidence, we make the conjecture that the desired sum is the square of the number of terms, or

$$1 + 3 + 5 + 7 + \ \ldots + [2n - 1] = n^2$$

We first prove our conjecture using the principle of mathematical induction: Let the first person in a line be a MAΘ member and let a MAΘ member follow every MAΘ member in the line. Then everyone in the line is a MAΘ member. A more serious, and more general, version of this principle: Let P(n) be a statement that depends on the positive integer, n.

If 1. Anchor step: P(m) is true, and
2. Induction step: For all positive integers, $k \geq m$, $P(k + 1)$ is true if P(k) is true,

then P(n) is true for all integers $n \geq m$.

For this conjecture m = 1 and P(1) is true since 1 = 1. Assume P(k) is true: $1 + 3 + 5 + \ldots + (2k - 1) = k^2$. Then $P(k + 1) = 1 + 3 + 5 + \ldots + (2k - 1) + (2k + 1) = P(k) + (2k + 1) = k^2 + 2k + 1 = (k + 1)^2$. This means that if our conjecture is true for a certain integer k, then it remains true for the next integer k + 1. But we know the conjecture is true for n = 1, so "by induction" it is true for all integral values of $n \geq 1$. [true for 1, hence true for 2; true for 2, hence true for 3; true for 3, hence true for 4, etc.]

Mathematical induction is especially easy to use if our conjecture is expressed in recursive form. In this case we can express our conjecture as follows:

Let $S_1 = 1$ and $S_{n+1} = S_n + (2n + 1)$. [This is another way to write $S_n = 1 + 3 + 5 + \ldots (2n - 1)$]. As before, we conjecture that $S_n = n^2$. The anchor step is given in the definition. The induction step becomes: if $S_k = k^2$, then $S_{k+1} = (k + 1)^2$. But $S_{k+1} = S_k + (2k + 1) = k^2 + 2k + 1 = (k + 1)^2$ and the proof by induction is complete.

Although the discovery of mathematical induction is usually attributed to Pascal [1654], it appears that the first person to apply it in rigorous proofs was the Italian scientist, Francesco Maurolico in 1575. Further improvements were made by Pierre de Fermat later in the 17th century. The phrase, "mathematical induction" was apparently coined by DeMorgan, the discoverer of several laws about sets.

We can also justify our conjecture using other methods.

Gauss' Method

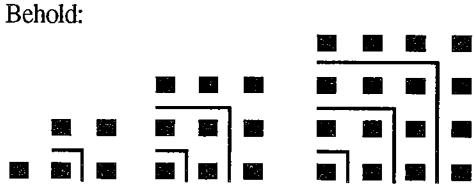
$$\begin{array}{lcccccc} \text{Let } S_n = & 1 & + & 3 & + 5 + \ldots + & (2n-3) & + (2n-1). \\ \text{Then } \underline{S_n =} & \underline{(2n-1)} & \underline{+} & \underline{(2n-3)} & \underline{+ \ldots \quad +} & \underline{3} & \underline{+ \quad 1} \\ 2S_n = & 2n & + & 2n & + \ldots + & 2n & + \quad 2n \end{array}$$

$$2S_n = n \cdot 2n = 2n^2. \text{ Thus } S_n = n^2.$$

Gauss' method can be applied to finding the sum of the terms in any arithmetic progression.

Visual Proof - "Proof Without Words"

Behold:



(continued on page 6)

MAθ Bulletin Board

• **HAWAII - Convention Update**
Paul Foerster will be one of the general session speakers and present a small group session [as will Jenny Thompson]. The beach picnic will include a magician and a juggler. Remember that registration begins at 1:00 p.m. on August 4 and the absolute deadline for being out of the dormitories is noon on August 10. If you have any questions or contributions, contact Jeanne Nelson, Kamehameha Schools, Kapalama Heights, Honolulu HI 96817; 808-842-8924 [school]. Aloha!

• **STATE AND REGIONAL MEETINGS in 1993**
March 1993: Tennessee State Meeting. Contact J. Michael Bradley, Hickam County HS, 1645 Bulldog Blvd., Centerville, TN 37033.
March 1993: Mississippi State Meeting. Contact Mrs. Claudia Carter, MSMS, P.O.Box W-1627, Columbus, MS 39701.
March 1993: Wisconsin State Meeting. Contact Joe Griesbach, Marquette University HS, 3401 W. Wisconsin Ave., Milwaukee, WI 53208-3842.
March 1993: Louisiana State Meeting. Contact Ms. Barbara Stott, Riverdale HS, 240 Riverdale Ave., Jefferson. LA 70121.
April 1993: Florida State Meeting. Greenleaf Resort. Contact Mrs. Merita Miller, A.C.Mosely HS, 501 Mosley Dr., Lynn Haven, FL 32444-5609

Attention Sponsors: There is a position open to represent MAΘ on a subcommittee of the Committee on American Mathematical Competitions. Duties include contributing possible questions, reviewing drafts, and proofreading one of the contest exams and attending one committee meeting each year. The normal term of office is 3 years. If you are interested, please send your resume to the national office by April 15.

The annual MAΘ sponsor's breakfast will be held at Friday, April 2 at 7:30 a.m. in the Seattle Hilton during the NCTM Annual Meeting. The cost is $7.95 per person and reservations [including payment] should be made with the national office.

$\sqrt[1993]{\text{At the Root of It All}}$ - continued from page 3)

Mark you calendars NOW for next year's Third Annual Region IV Convention to be held December 3-4 in Akron, OH. For more information, contact me at the address shown at the top of this column.

The Region IV Convention in Knoxville was full of mathematics, ideas, and fun. Everyone was a winner! Hope to see everyone next year in Akron.

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As an addendum to the column of October 1992, I want to acknowledge the contributions of the MAΘ members and sponsor Adolph Holbrook of the chapter at South Pike High School, Magnolia, Missiissippi in the running of the Mississippi State Convention there during the past several years.

Results: MAΘ Essay Contest

The judges had a difficult time selecting the seven winners from among the twenty entries submitted. The winners are given in random order.

• John Egan; Memphis, Tennessee
[Josephine Uttilla, teacher]
Determining Solutions of $y^x = x^y$ and Power Series Whose Coefficients are Arithmetic Progessions

•Andrew Chang and Sameer Shah
White Station High School, Memphis, TN
[Theresa Jennings, sponsor];
An Evening of MATH Radio

• Stephen D. Martin
The Mississippi School for Mathematics and Science
[Claudia Carter, sponsor]
The Development of Infinitesimal Analysis,

• Michael Nabavian
Clarkston North HS, New City, NY
[Dr. Mary Ann Gavioli, sponsor]
The Philosophy of Artificial Intelligence

• Pramod Achar
White Station High School, Memphis, TN
[Theresa Jennings, sponsor];
An Orthogonal Function Set in Two Variables

• Jeffrey Chuang
Bellaire HS, Bellaire, TX
[Eva E. Costa, sponsor]
Subword Combinatorics and Recursive Equations,

• Michael Golinko
Marjory Stoneman Douglas HS, Parkland, FL
[Barbara Nunn and Ann Singleton, sponsors]
Gravitational Deflection: Perturbation in the Fabric of Space-Time

The Mathematical Log

Volume 37 Number 1, February, 1993
The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association ofAmerica (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu AlphaTheta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson,TX 75083

1993 √At the Root of It All

Deborah Patonai Phillips, Activities Editor
St. Vincent-St. Mary HS, 15 North Maple Street, Akron, OH 44303

Region IV Convention

Mathematics conventions and competitions are places where MAΘ members can gather to test their abilities with some of the best students in the country and to meet new and exciting people. The Second Annual Region IV Mu Alpha Theta Convention was no exception. Held on December 4-5 at Farragut High School in Knoxville, Tennessee, the convention drew 178 student participants from fifteen schools in Ohio, Kentucky, and Tennessee. Sponsors running this convention were Mary Emma Bunch and Grace Mutz, both of whom hosted the 18th Annual Convention in 1988.

There were a variety of competitions including an interschool test, a written test, ciphering rounds, relays, and a Math Bowl to culminate the activities. To encourage participants of comparable ability, several of the contests were divided into two levels - Alpha for advanced students and Theta for less-experienced ones.

The awards ceremony was filled with excitement as students from different schools received their medals and trophies. Some of the school winners were:

Alpha Written Test	Theta Written Test
1. Beavercreek [OH]	1. Farragut
2. Centerville [OH]	2. Lakota [OH]
3. Farragut [TN]	3. St. Vincent-St. Mary [OH]

Alpha Ciphering Test	Theta Ciphering Test
1. Beavercreek	1. Lakota
2. Paul Blazer [KY]	2. Farragut
3. Centerville	

For the relay, the top schools were Beavercreek and Oak Ridge [TN].

... "Math Fair" where members ran various mathematical versions of BINGO, Jeopardy, Twister, and horse racing.

Individual prizes included:

Alpha Written Test	
1. H. Yalamanchi	Paul Blazer
2. D. Schepler	Beavercreek
3. M. Roh	Beavercreek

Theta Written Test	
1. T. Yam	Farragut
2. N. Chen	Farragut
3. C. Hutzelman	Lakota

On Alpha Ciphering, the top students were D. Schepler and H. Yalamanchi [tie]; and in Theta Ciphering the top student was K. Mendenhall [Lakota].

The results of the Team Bowl were used to determine the top school teams. The top four Alpha students from each school competed in team ciphering. The winners were: 1. Paul Blazer; 2. Beavercreek; 3. Oak Ridge. In addition to their trophies, individual winners in the Alpha and Theta competitions H. Yalamanchi and T. Yam received registration to the national convention to be held in Hawaii this summer. Congratulations to all the winners!

On Friday evening, the Farragut chapter planned a "Math Fair" where members ran various mathematical versions of BINGO, Jeopardy, Twister, and horse racing. Participants could win Euclid dollars that could be used to purchase prizes and candy. The Fair was a super way to meet people, to have fun, and, of course, to do more mathematics!

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Although an Ohio powerhouse in mathematics competitions, Beavercreek High School is a relatively new member of MAΘ. Bill Bagwell, the head of its math team, shared some of his secrets of success. First, his team does not have formal practices, they just take tests ... all kinds of tests. From a collection of over 15 contest books, his classes constantly work on all types of problems. When an unfamiliar topic arises, they take time to study it. These classes experience problem solving to the fullest!

The Math Team, composed of students from all grade levels, participates in over 40 contests every year. All the competitions are given before school, never in class. To participate, the students must arrive at school by 6:30 a.m.! Usually between 60 and 75 students do so, but on some test over 90 students may take part. With so many competitions, some students come early almost every day.

All this hard work really pays off since Beavercreek is always at or near the top most of the time. At the Denison University Competition, 17 of the top 51 finishers, and 7 of the first 8 female finishers, were from Beavercreek. They were first at the Rose Hulman Contest for the fifth year in a row.

Fundraisers are a necessity to pay for travel expenses and to offer small monetary prizes in some of the competitions. They raise over $5,000 annually by selling pretzels in math classes. The other major fundraiser is a Sadie Hawkins Dance in which the girls invite the guys.

(continued on page 2)

[MATH Radio, continued from page 5]

Lynn E. Orr: I've been a faithful listener for ten years, but this is the first time I've ever called ... can you hear me, Ed?

Ed J. Cent: Yes, go right ahead, Lynn.

Lynn E. Orr: OK, Mr. Euler, could you specify exactly how the Russians solved the bridge problem?

Lenny: Well, they merely added another bridge to make a total of eight. Keep listening, Lynn, and I'll explain my results and the Russian solution through a couple of very interesting theorems.

Lynn E. Orr: That would be nice. Thank you kindly, Lenny.

Lenny: Why don't I discuss the five theorems I've discovered through my study of networks. Before I start, I need to define the term *traverse*, which shows up in four of the theorems. A path in a network is said to traverse a network if and only if every arc of the network is included in the path. Now on to my theorems ...

Ed J. Cent: Sorry to interrupt, Lenny, but we need to go to a commercial.

Ed J. Cent: We're back. Call us, toll-free, at 1-800-LUV-MATH., to talk to Leonhard Euler. We now return to Mr. Euler [YOO LUR] and his theorems concerning networks.

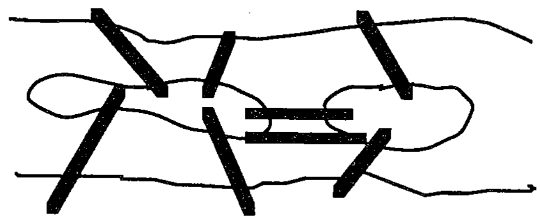
Lenny: For crying out loud, Ed, it's Euler [OILER]. Anyway, my first theorem: IN ANY NETWORK, THE NUMBER OF ODD VERTICES IS EVEN. This theorem is self-explanatory except for the fact that a network does not necessarily have to be traverseable.

The second theorem is the key to the bridge problem:

IF A NETWORK HAS MORE THAN TWO ODD VERTICES, IT CANNOT BE TRAVERSED. As you may remember, the network of the bridges of Koenigsberg had four odd vertices.

The third theorem is essential to the Russian solution to the bridge problem, which, as I said earlier, was to add another bridge. The theorem states:

IF A CONNECTED NETWORK HAS EXACTLY TWO ODD VERTICES, IT CAN BE TRAVERSED BY A SINGLE PATH WHOSE INITIAL AND TERMINAL VERTICES ARE THE TWO ODD VERTICES OF THE NETWORK. When the eighth bridge was built, two of the four odd vertices became even as you can see in the diagram.



Ed J. Cent: Excuse me, um, Lenny, you'll have to stick to verbal diagrams.

Lenny: Sorry. What they did was to add another bridge between the two islands. This way, there were two paths to each island and the traffic problem as well as the bridge problem were solved. On to the last two theorems. They are not the most important, but they are interesting. Number four:

IF A CONNECTED NETWORK HAS NO ODD VERTICES, THEN IT CAN BE TRAVERSED BY A SINGLE PATH.

And finally, number five:

IF A CONNECTED NETWORK HAS EXACTLY 2n ODD VERTICES, THEN IT CAN BE TRAVERSED BY A COLLECTION OF n PATHS AND CANNOT BE TRAVERSED BY ANY COLLECTION CONTAINING FEWER THAN n PATHS.

Ed J. Cent: Well, thanks for all that interesting information. Now, listeners, you don't have to call the 900-number. We go to Cosine, Kentucky, Welcome to our show.

Caller #3: Hi. This is Max E. Mumm. I'd like ...

Ed J. Cent: Hey, your name sounds familiar. Have you called before?

Max E. Mumm: Oh, no. That must have been my wife, Minnie. We're both math teachers and this show always provides interesting topics for discussion. Anyway, I was wondering if networks have any useful applications in today's world?

Lenny: They certainly do. Many engineers use my theorems to design transportation routes, to locate gas-water-electricity lines, and to plan parks. You know, landscape architects do not wish to face the same problem they did in Koenigsberg.

Ed J. Cent: Well, Lenny, we're running out of time. Thanks for being on the show. Are you working on anything new?

[continued on page 6]

An Evening of MATH Radio

Andrew Chang and Sameer Shah
White Station High School, Memphis, TN
Theresa Jennings, sponsor

Ed. note. This article is a slightly edited version of one of the winning entries in the MAΘ Essay Contest. According to its authors, it "can best be appreciated through actual dramatization of the dialogue." Complete results of the competition are on page 2. These two seniors also submitted the winning entry in last year's GEOBOGGLE Contest.

Announcer [Ed J. Cent]: The following broadcast is a presentation of MATH radio. I'm Ed J. Cent and our special guest tonight is the world renowned mathematician, Leonhard Euler. Here to brief us on the basics of his current field of study, topology, is MATH radio's resident mathematician, Polly Nomial.

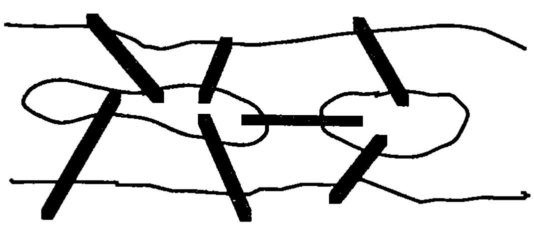
Polly Nomial: : Topology, as our listeners may know, is also called the science of wiggly lines. Formally, however, it can be defined as the branch of mathematics that decides what is and is not possible. Although topology is divided into two areas, rubber-sheet dynamics and network topology, Mr. Euler specializes in network topology. Therefore we request that our callers focus their comments on this area. Specifics will be discussed by Mr. Euler. Now back to Ed.

Ed J. Cent : Thank you, Polly. I remind our listeners of our telephone number: 1-800-LUV-MATH. And now please welcome Mr. Euler [YOO LUR] to the show. How's life been treating you?

Euler: Please, Mr. Cent, it's Euler [OILER], but if that's too much trouble, just call me Lenny.

Ed J. Cent : OK, Lenny. Last time you were on the show, you claimed you were on the brink of a discovery. Have you made any progress?

Lenny: I have exciting news, Ed. Let me start from the beginning. I was serving in the court of Catherine the Great, in 1736, when I was faced with a frightful quandary. I received word from the mayor of Koenigsberg [CONE iks berg] that his citizens were depressed. What could be the cause of this great anxiety? The bridges along the Preger River. They collapsed, you may ask? Actually not. The Sunday strollers were fond of walking along the banks of the river which meanders through town. Here's a diagram.



Ed J. Cent: Excuse me, Lenny, this is radio. You'll have to compose a verbal diagram.

Lenny: Ah! Sorry. You must realize how involved we mathematicians get in our work. Let me think. Between two banks of the river are located two islands, one a little larger than the other. From the larger island emanate five bridges. One goes to the smaller island, and four, two on each side, go to the banks. From the smaller island, two bridges, one on each side, go to the banks, and the same bridge that the large island sent also radiates from the smaller island ...

Ed J. Cent: I'm not sure if all of our listeners understand you, Lenny. Is there an easier description?

Lenny: Yes! I've developed a system of points and lines representing the islands and the bridges. It is formally called a network, a figure consisting of a positive number of arcs, no two of which intersect except at the vertices.

Ed J. Cent: What on earth are these arcs and vertices that you are talking about?

Lenny: Well, Ed, an arc is a line segment or a curve that can be obtained by an elastic motion of the segment. The vertices are the endpoints of these arcs. To further elaborate, a vertex is odd if it is an endpoint of an odd number of arcs and, similarly, an even vertex has an even number of arc ends.

Ed J. Cent: That's clear. Now what was that Koenigsberg problem?

Lenny: Sunday strollers were fond of walking along the banks of the Preger. The question was raised as to whether it was possible to take a stroll that crosses each bridge exactly once.

Ed J. Cent: Well, is it possible?

Lenny: Calm down, Eddie, I'm getting to that. I'm sorry to disappoint the strolling citizens, but it's not possible. However I want to thank the citizen who first raised the question. Based on his curiosity, I have discovered several theorems in network topology. By the way, if any of you mathaholics want a copy of my work, please send a self-addressed envelope to Leonhard Euler, Kalingrad, Lithuania or call my publisher at 1-900-NET-WORK.

Ed J. Cent: I'm glad you got in that telephone number, but one things still bothers me. Why Kalingrad and not Koenigsberg?

Lenny: Well, after World War II, when Russia took over the part of Germany containing Koenigsberg, they renamed the city.

Ed J. Cent: Sorry to interrupt, Lenny, but we have our first caller. Boise, Idaho, you're on MATH Radio.

Caller #1: MATH Radio ?! You've got to be kidding! No one listens to that stuff! I was scanning the stations and I heard the name "Leonard" mentioned with a phone number. Isn't this Leonard Bernstein?

Ed J. Cent: No, I'm afraid it's not. This is Leonhard Euler, the famous mathematician.

[DIAL TONE]

Hello? Hello? Oh, well, we go now to Arctangent, Oklahoma and Lynn E. Orr. What's your question, Lynn?

[continued on page 4]

(Induction - continued from page 1)

Telescopic Cancellation

Rewriting the key step in an induction proof offers another method:

$(k+1)^2 - k^2 = 2k + 1$

Then

$1^2 - 0^2 = 1$
$2^2 - 1^2 = 3$
$3^2 - 2^2 = 5$
$4^2 - 3^2 = 7$

$n^2 - (n-1)^2 = 2n - 1$

Adding the terms on both sides yields

$n^2 = 1 + 3 + 5 + 7 + \ldots + (2n-1)$

Exercises 1 - 2: Find and prove a formula for each of the following:

1. $(1 - \frac{1}{4})(1 - \frac{1}{9})(1 - \frac{1}{16}) \ldots (1 - \frac{1}{n^2})$, $n \geq 2$
2. $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2$, $n \geq 1$.

Example 1

For what positive integers n is $2^n > 4n^2 + 1$?

Solutions:

Checking cases, we conjecture the inequality is true for $n \geq 9$.

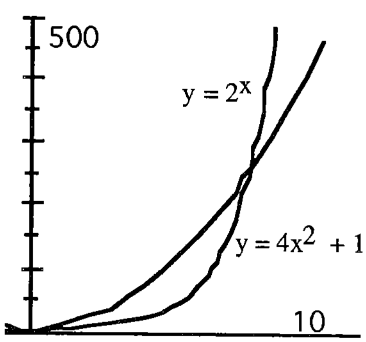
Method 1 Use mathematical induction.

1. Anchor step [m = 9]: $2^9 > 4 \cdot 9^2 + 1$ $[2^8 < 4 \cdot 8^2 + 1]$
2. Induction step: If $2^k > 4k^2 + 1$, then $2^{k+1} > 4(k+1)^2 + 1$.

Now $2^{k+1} = 2 \cdot 2^k > 2(4k^2 + 1)$ by the induction hypothesis and $2(4k^2 + 1) > 4(k+1)^2 + 1 = 4k^2 + 8k + 5 \Leftrightarrow 8k^2 + 2 > 4k^2 + 8k + 5 \Leftrightarrow 4k^2 - 8k - 3 > 0 \Leftrightarrow 4(k-1)^2 - 7 > 0 \Leftrightarrow (k-1)^2 > \frac{7}{4}$ which is clearly true for $k \geq 9$.

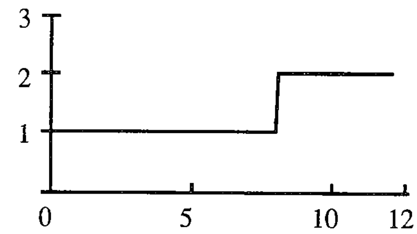
Method 2 Visual: Use a graphing utility.

2a.



Plot $y = 2^x$ and $y = 4x^2 + 1$ on the same coordinate system. After using ZOOM and TRACE, you can see that the inequality is false for $x \leq 8$ and true for $x \geq 9$.

2b. If $Y1 = (2^x > 4x^2 + 1)$ on the TI-81, for example, then Y1 = 1 when the statement is true and 0 when it is false. Thus if $Y2 = (2^x > 4x^2 + 1) + 1$, then Y2 = 2 when the statement is true and 1 when it is false. This graph is easy to see in the window $[0 \leq X \leq 12;\ 0 \leq Y \leq 3]$. It shows the inequality is false for $x \leq 8$ and true for $x \geq 9$.



Exercise 3: For what integers n is $2^n > n^2 + 4n + 5$? Justify.

Example 2: What is the largest positive integer that is a factor of $n^3 - n$ for all integers $n \geq 1$?

Solutions:

Checking n = 1, 2, 3, 4, 5, and 6 we find $n^3 - n = 0, 6, 24, 60, 120,$ and 210. We conjecture that the largest such integer is 6.

Method 1 Use mathematical induction.

1. Anchor step: 6 is a factor of $1^3 - 1 = 0$
2. Induction step: If 6 is a factor of $k^3 - k$, then 6 is a factor of $(k+1)^3 - (k+1)$. Now $(k+1)^3 - (k+1) = k^3 - k + 3k^2 + 3k = (k^3 - k) + 3k(k+1)$. Now, by assumption, 6 is a factor of $(k^3 - k)$ and 6 is a factor of $3k(k+1)$ since either k or k + 1 is even.

Method 2

$n^3 - n = n(n^2 - 1) = n(n+1)(n-1)$ - a product of three consecutive integers. One of them is even and one is a multiple of 3, so the product is a multiple of 6.

Exercise 4:

a. What is the largest integer that is a factor of $n^5 - n$ for all $n \geq 1$? Justify your answer.
b. What is the largest integer that is a factor of $n^6 - n$ for all $n \geq 1$? Justify your answer.

To be continued in the next issue

(MATH Radio - continued from page 4)

Lenny: I'm working on some problems involving coloring maps. I think you can draw any map using only four colors, but so far I can only prove that at most five colors are needed. Send any ideas you have to me in Kalingrad, Lithuania or call my publisher at 1-900-NET-WORK.

Ed J. Cent: You just had to get that phone number in, didn't you, Lenny? Our time is almost up for tonight. Join us next week when our guest will be Pierre de Fermat, who will share a problem that he thinks he has solved. For now, keep those fingers counting!

A Voice: This has been MATH radio. We close with our theme song: "I've Got Your Number" by Frank *Sine* atra