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Operation Equilateral

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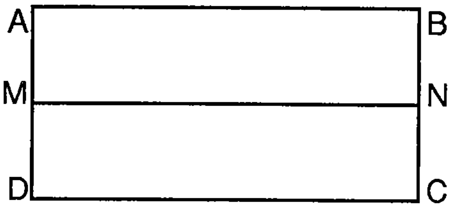
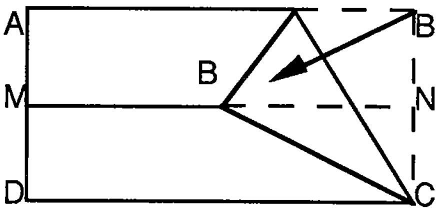
By folding and cutting paper, we can demonstrate many geometric concepts. Here we examine a few simple ideas related to equilateral triangles and regular tetrahedrons. [Some material has been added by the editor.]

1. **Operation Rectangle:** Take a rectangular sheet of paper; call it ABCD [Fig. 1]. Fold the paper along its length by putting A on D and B on C thereby obtaining M and N, the midpoints of segments AD and BC respectively.



(Fig. 1)

Open the paper and draw segment MN to make it clearer. Now put B on MN and fold the paper in such a way that the crease goes through C and B remains on MN [Fig. 2].



(Fig. 2)

Call the other end of the crease K [Fig. 3]. Extend segment KB until it cuts DC at H. ΔKCH is equilateral.

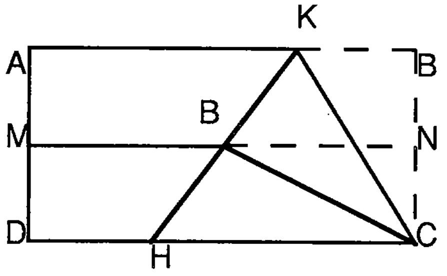


Fig. 3

- Why is ΔKCH equilateral? Answer the question several ways.

2. **Operation on a Circle:** Draw a circle using a compass or trace around a circular plate. By using the compass, you already have the center. If you trace around a plate, find the center by first carefully cutting out the circle and then folding the circle twice to obtain two diameters. Their point of intersection is the center.

Put the rim (circumference) of the circle at the center O and fold the paper. Call the crease AB. Put the rim of the circle at O and fold the paper in such a way that the crease contains A. Call the new crease AC. [Fig. 4]

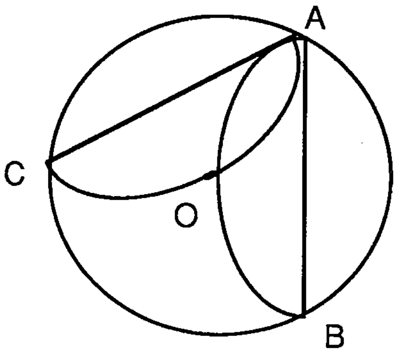


Fig. 4

- Why is ΔABC equilateral? Answer the question several ways.

3. **Operation Tetrahedron I:** A tetrahedron is a triangular pyramid. If the faces are equilateral triangles, the tetrahedron is regular. Choose equilateral ΔABC. Fold all sides carefully to obtain the midpoints L, M, and N of sides AB, BC, AC respectively. Draw LM, MN, and NL. [Fig. 5]

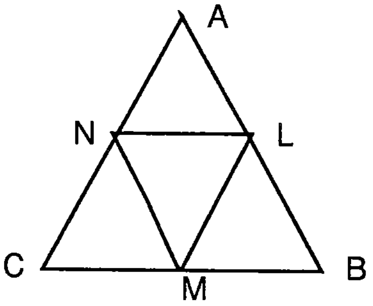


Fig. 5

Fold the triangle along these segments. Put AN and CN together and secure with tape. Similary for CM and BM; BL and AL. You obtain regular tetrahedron.

- What type of triangles can be folded along three midsegments to form a tetrahedron [not necessarily regular]?

You can also make a tetrahedron from a paper [circular] cylinder. Pinch together the top and tape it to form a straight edge [Fig. 6].

(continued on page 6)

MAθ Bulletin Board

• HAWAII Join the hundreds of members at Brigham Young University for the 23rd national convention in August, 1993. The schedule is almost set. Registration begins at 1:00 p.m. on August 4 and the absolute deadline for being out of the dormitories is noon on August 10. There will be three speaker sessions. We would like to have as many student presenters as possible. Students or sponsors - if you would like to present a session, please write: Jeanne Nelson, Kamehameha Schools, Kapalama Heights, Honolulu HI 96817. Call if you have questions: 808-842-8924 [school]. Aloha!

• STATE AND REGIONAL MEETINGS in 1993

February 1993: Tennessee Regional Meeting. Hosted by Kirby HS, 4080 Kirby Parkway, Memphis, TN 38115. Contact Mrs. Patricia Brownlee.

February 12-13: Texas State Meeting. Held at Corpus Christi State University. Contact Lorraine Dominguez at Moody HS, 1818 Trojan, Corpus Christi, TX 78416.

March 1993: Tennessee State Meeting. Contact J. Michael Bradley, Hickam County HS, 1645 Bulldog Blvd., Centerville, TN 37033.

March 1993: Mississippi State Meeting. Contact Mrs. Claudia Carter, MSMS, P.O.Box W-1627, Columbus, MS 39701.

March 1993: Wisconsin State Meeting. Contact Joe Griesbach, Marquette University HS, 3401 W. Wisconsin Ave., Milwaukee, WI 53208-3842.

March 1993: Louisiana State Meeting. Contact Ms. Barbara Stott, Riverdale HS, 240 Riverdale Ave., Jefferson. LA 70121.

April 1993: South Carolina State Meeting. Contact Mrs. Gloria Allen, South Aiken HS, 232 Pine Log Rd., Aiken, SC 29803-6158.

April 1993: Florida State Meeting. Greenleaf Resort. Contact Mrs. Merita Miller, A.C.Mosely HS, 501 Mosley Dr., Lynn Haven, FL 32444-5609

• Ms. Joyce Becker is the President-elect; James Aiu of Lincoln Way HS, New Lenox, IL is Governor of Region I; Gary Blackburn of Brother Martin HS, New Orleans, LA is Governor of Region II.

• TEST FILES The national office now maintains a test bank consisting of those tests used in regional, state, and national conventions. At this time, there are tests in the following areas: Algebra, Analytic Geometry, Calculator, Circles, Complex Numbers, Conics, Differential Calculus, Equalities and Inequalities, Functions, Gemini, Geometry, Integral Calculus, Logarithms & Exponents, Matrices and Determinants, Number Theory, Polynomials, Probability and Statistics, Radicals, Trigonometry, Sequences and Series, and Word Problems. These were gathered from the 1987, 1990, 1991, and 1992 national conventions. If you wish to purchase any of these tests, please contact the national office. If you have tests you wish to contribute to the files, please do so.

• FRIENDS It is not too late to become a "Friend of Mu Alpha Theta". For just $10 per year, FRIENDS will receive copies of selected MAθ publications and their own copy of the Log delivered to their front door. Write to the national office for details.

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Logniappe

Wednesday , January 20 marks the inauguration of the 42nd U. S. president, Bill Clinton. Two questions concerning Inauguration Day that you might enjoy researching are:

1. The second inauguration of Ronald Reagan in 1985 fell on a Sunday. What happens when Inauguration Day is on a Sunday ? The inauguration of Zachary Taylor is an especially interesting example. [Before 1937, Inauguration Day was March 4.]

2. The thirteenth of the month is more likely to be on a Friday than any other day of the week. In a 400-year cycle, on what day of the week is January 20 most likely to fall?

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The Mathematical Log

Volume 36 Number 4, December, 1992

The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association ofAmerica (MAA) and the National Council of Teachers of Mathematics (NCTM). Correspondence may be directed to Mu AlphaTheta National Office, 601 Elm Ave., Room 423, Norman, OK 73019. or to Log editor Thomas Butts, Univ. of Texas at Dallas, P.O. Box 830688 FN 32, Richardson,TX 75083

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1992 √At the Root of It All

Deborah Patonai Phillips, Activities Editor
St. Vincent-St. Mary HS, 15 North Maple Street, Akron, OH 44303

Kalin Award Winner

In a "normal" high school, there are "normal" students. Every once in a while there are exceptions to this rule. One such exception is Loren Looger, this year's recipient of the Kalin Award. Named after Robert Kalin, former governor and president of Mu Alpha Theta, this award is given annually at the national convention to an outstanding Mu Alpha Theta member. Representing all MAΘ members, this student demonstrates unusual ability in mathematics and dedicated service to the organization.

Loren, a recent graduate of Randolph High School in Huntsville, Alabama, has accumulated a remarkable list of accompishments during his high school career. Most noteworthy are his successes in contests and competitions. On the local level, he has won so many competitions that keeping track is almost impossible. He has captured first place in Geometry for the last four years at Grissom High, Decatur High, and Austin High; and in Advanced Math at Vestavia Hills, Grissom, Livingston University, University of Alabama, Wallace State, and Muscle Shoals. Loren has been the top-scoring student in the Alabama State Mathematics Contest for the last two years. He has also won first place three times in the Al-La-Miss Contest.

He has attended the last five MAΘ national conventions - Knoxville, Tampa, Chicago, Huntsville, Princeton - winnng a "ton" of awards. At his first convention, as an eighth grader, Loren placed second in the junior division and was a member of the winning Alabama team. In fact, he has been on the winning state team every year. In individual competitions, he has placed at or near the top being first place Individual, second place Number Theory, and fourth place Geometry at the '91 convention.

"He is the kind of student who makes ... Randolph School look very good."

Loren has also been quite successful in competitions sponsored by organizations other than MAΘ. He has qualified for the AIME for the past five years and the Olympiad for the past three years. This past year, he scored 139 [out of 150] on the AHSME, a 13 [out of 15] on the AIME, and did well enough on the Olympiad to earn a spot on the training program for the International Olympiad. He has also won first place in the National Mathematics League in Precalculus and Calculator Competitions, second in Geometry and fifth in Calculus. In addition, he has been a member of the ARML Team for three years and a book award winner in the U.S.A. Talent Search for three years. As the mathematics and computer science department chair, Mary Shepard Hughes, describes Loren's talent, "He is the kind of student who makes me, the Math Department, and Randolph School look very good."

His incredible perfromances are not limited to mathematics. Loren has displayed talent in the sciences as well. He has qualified for the Chemistry Olympiad for three year and was invited to the Chemistry Olympiad Training Camp for the last two. He has also won the Alabama Science Talent Search, was named a Westinghouse semi-finalist, and receivedd several other honors and awards.

On many occasions when math substitute teachers could not be found, Loren taught the class.

As president of his local MAΘ chapter for the past three years, Loren "has worked especially hard helping other with mathematics. " During this time the club has been involved in raising money for mathematics books for the library, bringing in special speakers, tutoring students, promoting math competitions, and training Math Counts participants. The club's most successful fundraiser has been the annual Halloween treat sale.

Developing a love for mathematics at an early age, Loren encourage others to enjoy the subject. Besides helping explain problems to fellow students, he ran practices for the Math Counts team and those wishing to prepare for the AHSME. On many occasions when math substitute teachers could not be found, Loren taught the class.

Through his associations with Mu Alpha Theta, Loren was given the chance to hear many different speakers and meet people who used mathematics in many fascinating ways. He "saw living proof that math can be fun, interesting, and rewarding." We certainly agree and offer our congratulations.

Be sure to send me news of your chapter, state and regional conventions, etc. Include a photo.

CONTEST CORNER

This month's column contains an outline of the solution to the Power Question from the 1992 ARML Contest. It was constructed by Gil Kessler and Larry Zimmerman, who have been creating the solutions for this contest since 1983. Their solution is intended to be instructive and is not necessarily the most elegant solution. You are encouraged to complete the solution on your own. Time spent examiningsolutions to problems and trying to devise your own solutions is time well spent.

Throughout this problem, the points A (a,a^2), B(b,b^2), C(c,c^2), and D(d,d^2) represent distinct lattice points on the parabola $y = x^2$. I. Let the area of ΔABC be K. It can be shown [using determinants, for example] that $K = \frac{1}{2}|(a-b)(b-c)(a-c)|$

1. Show that K must be an integer.
2. Show that K = 3 is the only possible prime value for K.
3. Show that K cannot be the square of a prime.
4. Show that the area of quadrilateral ABCD cannnot be 8.

Solution: 1. At least two of a,b,c must have the same parity, so at least one of the three differences must be even.

2. Suppose $a<b<c$. Set $b-a = e$, $c-b = f$ and hence $c-a = e+f$. Now $2p = ef(e+f)$ so $e=1$ and $2p = f(f+1)$. Then either $f=p$ and $f+1 = 2$ [impossible] or $f = 2$ and $f+1=p=3$.

3. $2p^2= ef(e+f)$. Consider cases as in 2. Either $e = 1$ or $e = 2$. ... How many cases are there? Why are they all impossible?
4.A diagonal divides ABCD into two triangles, each with integer area. Any of the four possible cases $8 = 1 + 7$, $8 = 2 + 6$, $8 = 3 + 5$, and $8 = 4 + 4$ contradicts either (2) or (3) above.

II. It can be shown that the slope of line AB is $a + b$.
1. A line passes through the point (3,5) and through two lattice points on $y = x^2$. Compute the coordinates of these two points being sure to find all possible pairs of such points.
2. A line passes through the point (2,4) and through three other lattice points on the "double parabola" $y^2 = x^4$. Compute the coordinates of these three points being sure to find all possible triplets of such points.

1. Let the two points be (a,a^2) and (b,b^2). Then $\frac{5-a^2}{3-a} = a + b$ leads to $b = \frac{5-3a}{3-a} = 3 + \frac{4}{a-3}$. Trying $a - 3 = \pm 1$, ± 2, ± 4 gives only two pairs of values for a and b [order not important] with $a \neq b$, namely $a = -1$, $b = 2$ and $a = 4$, $b = 7$. The pairs of points are (–1,1), (2,4) AND (4,16) (7,49).

2. Let the two points be $(a,-a^2)$ and $(b,-b^2)$ on the "lower" parabola $[y = -x^2]$ and (c,c^2) on the "upper" parabola. Then $\frac{4+b^2}{2-b} = \frac{4+a^2}{2-a} = 2 + c$. The first equality eventually leads to $b = \frac{2a+4}{a-2}$. Continue the analysis as in 1. to obtain three triplets of points. In general, if a line crosses $y^2 = x^4$ in four points whose abscissas are a, b, c, and d with $a < b < c < d$, then $a + b + c + d = 0$ and $ab = -cd$.

III. Consider quadrilateral ABCD.

1. Let the vertices be labeled (alphabetically) in a counterclockwise direction. Show that $\tan A = \frac{d-b}{1+(a+b)(a+d)}$.
2. A quadrilateral is "cyclic" if all four of its vertices lie on the same circle. Show that: If quadrilateral ABCD is cyclic, then $a + b + c + d + 0$; AND If $a + b + c + d + 0$, then quadrilateral ABCD is cyclic.
3. Use the previous result to show that: If a circle intersects the graph of $y = x^2$ in four points, and three of them are lattice points, then the fourth point must also be a lattice point.

1. This result comes from using the formula for the tangent of the angle between two lines with the fact that the slopes of these lines are $(a + b)$ and $(a + d)$ respectively.

2. If ABCD is cyclic, then its opposite angles are supplementary. Then $\tan A = -\tan C$ implies $(a+b)(a+d) = (c+b)(c+d)$. Simplification yields $a + b + c + d = 0$. More elegantly: Setting $y = x^2$ in $(x-h)^2 + (y-k)^2 = r^2$ [without multiplying out] yields a quartic polynomial with no x^3 term. Thus the sum of its roots, which are the abscissas of the intersection points, is 0. It can be shown that for four points on $y = px^2 + qx + r$ to be concyclic, the sum of their abscissas must be four times the abscissa of the vertex of the parabola. Try to show no cyclic quadrilateral whose vertices lie on $y = x^2$ can contain a right angle. Try to prove the converse by "reversing" the previous argument.

3. Let the four abscissas be a, b, c, and k. Then $a + b + c + k = 0 \rightarrow k = -(a + b + c)$ which is clearly an integer. Thus the fourth point (k,k^2) is a lattice point. Does a circle through three lattice points on a parabola always intersect the parabola in a fourth lattice point? [Hint: Examine the case $a+b+2c=0$.]

(Tossing Dice - continued from page 6)

We use the Inclusion-Exclusion Principle to compute this number. Since A_1 contains solutions to (*) for which $x_1 > 6$, a solution to (*) corresponds to choosing 3 of the last 14 spaces above. Thus $|A_1| = \binom{14}{3}$. Similarly $|A_2| = |A_3| = |A_4| = \binom{14}{3}$. Similarly since $A_1 \cap A_2$ contains solutions to (*) for which $x_1 > 6$ and $x_2 > 6$, we must choose 3 of the last 8 spaces. Thus $|A_1 \cap A_2| = \binom{8}{3}$. The same is true for each of the other 5 intersections of two sets. In the same way $A_1 \cap A_2 \cap A_3$ contains solutions to (*) for which $x_1 > 6$, $x_2 > 6$, and $x_3 > 6$ which means we must choose 3 of the remaining 2 spaces - an impossibility. So, in this case, $A_1 \cap A_2 \cap A_3$ and the other intersections of three sets are empty. So is $A_1 \cap A_2 \cap A_3 \cap A_4$. Putting all this together, $|A_1 \cup A_2 \cup A_3 \cup A_4| = \binom{4}{1}\binom{14}{3} - \binom{4}{2}\binom{8}{3} + \binom{4}{3}\cdot 0 + \binom{4}{4}\cdot 0 = 1456 - 336 = 1120$. Hence the number of solutions to the dice problem is $1140 - 1120 = 20$. This is certainly a long-winded way of solving this problem, but it illustrates a method that can be generalized to any such dice problem [like those considered by Zubair].

dia Log ue

with Log Editor Tom Butts

We present two more excerpts from entries submitted to last's years contest. [A few changes have been made by the editor.]The judging has not been completed, but we hope to have the results in the next issue. You are encouraged to send your comments on either one of them as well as questions, comments, suggestions on what type of articles and features to include in the Log, a question about any area of mathematics - a person, a concept, a problem, ... a comment about any of the other articles , any contributions for the *Logniappe* colummn, or anything else on your mind. Send it to me at the address on page 2.

* * *

From John Egan of Memphis, Tennessee [Josephine Uttilla, teacher]

Determining Solutions of $y^x = x^y$

How do we graph the solutions to the equation $y^x = x^y$ in the coordinate plane? Clearly all points on the line $y = x$ are solutions, but there are more. One example is (2,4) since $4^2 = 2^4$. Furthermore, if (a,b) is a solution, then so is (b,a). If we manipulate the equation $y^x = x^y$ by raising both sides to the power $\frac{1}{x}$, we have $y = x^{\frac{y}{x}}$, i.e. $y = x^m$ where m is the slope of the line containing the origin and (x,y). This suggests that if we consider only solutions that lie on a fixed line that intersects the origin, we have the two equations $y = mx$ and $y = x^m$ or $mx = x^m$. If $m = 1$, this is true for all values of x. Otherwise, $m = \frac{x^m}{x} = x^{m-1}$ or $x = m^{\frac{1}{(m-1)}}$. Then $y = x^m = m^{\frac{m}{(m-1)}}$. If we regard m as a variable and let it take on all positive values, we now have two parametric equations that describe the set of points (x,y) for which $y^x = x^y$.

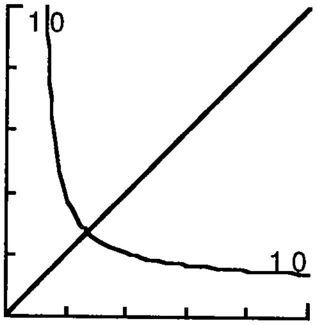
It is also possible to express this curve as a polar function if we convert the parametric equations to polar coordinates. Since the slope, m, is the tangent of the polar angle θ, we have $x = (\tan\theta)^{\frac{1}{(\tan\theta - 1)}}$ and $y = (\tan\theta)^{\frac{\tan\theta}{(\tan\theta - 1)}}$. Since the polar radius $r = \sqrt{x^2 + y^2}$, we can substitute and show that $r = (\sec\theta)(\tan\theta)^{\frac{1}{(\tan\theta - 1)}}$. We must also include the points on $y = x$ or $\theta = \frac{\pi}{4}$.

To determine the point of intersection of $y = x$ and the curve, we let $m \to 1$ in the parametric form and determine the resulting limit. Thus

$$\lim_{m\to 1} m^{\frac{1}{(m-1)}} = \lim_{s\to 0} (s+1)^{\frac{1}{s}} = \lim_{w\to\infty} \left(1 + \frac{1}{w}\right)^{w} = e$$

[setting $s = m - 1$ and $w = \frac{1}{s}$] and the point of intersection is (e,e).

The graph of the curve and $y = x$ is shown below. You can see it on the TI-81 graphing calculator using the parametric equations above and the window ($[0 \le t \le 10]$, $[0 \le x \le 10]$,$[0 \le y \le 10]$). You can $\boxed{\text{TRACE}}$ and $\boxed{\text{ZOOM}}$ to see the hole in the curve at (e,e).



- Show that $(\sqrt{3}, 3\sqrt{3})$ [and $(3\sqrt{3},\sqrt{3})$] is on the curve.
- Verify that the y-coordinate of the point of intersection is also e using a limit.
- Are there any lattice points other than (2,4) and (4,2) on the curve?

* * *

Zubair Talib of Stoneman High School, Parkland, Florida [Ann Singleton, sponsor]

Tossing Dice

While working through the questions we had compiled [for an upcoming regional contest], I came across a problem that turned out to have some most interesting results. The question read: What is the probability, when rolling four dice, of obtaining a sum greater than or equal to 17? My first instinct was to determine the number of ways of rolling the first eight sums [4,5,6,7,8,9,10,11] since I assumed the number of ways of rolling the eight desired sums [17,18,19,20,21,22,23,24] would be the same. I discovered the general pattern for the number of ways of rolling the nth sum was $\binom{n+2}{3}$. That is, the number ways of rolling the first sum, 4 [n = 1] , is $\binom{3}{3} = 1$; to roll the second sum [n = 2, sum = 5] is $\binom{4}{3} = 4$; to roll the third sum [n = 3, sum = 6] is $\binom{5}{3} = 10$, etc. Thus, in general, the number of ways the nth sum can be obtained by rolling x dice is $\binom{n + (x-2)}{x-1}$

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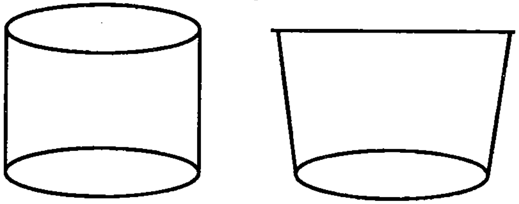

Fig. 6

Then do the same to the bottom, but at right angles to the edge formed at the top.

- For what values of the radius and the height of the cylinder will the resulting tetrahedron be regular?

A more sophisticated version of this construction always yields a regular tetrahedron.

4. **Operation Tetrahedron II**: Fold a large [e.g. 8.5 in x 11 in.] piece of paper in half and tape the three pairs of congruent edges together to form a "sealed envelope". Call the shorter edge of the envelope KH. Fold the paper along its length and obtain crease PQ in the middle. In a way similar to that used in Figure 2 above, put K on PQ, call it X, and fold the envelope so the crease contains H. Fold the envelope through X to get AB, a segment perpendicular to PQ [Fig. 7].

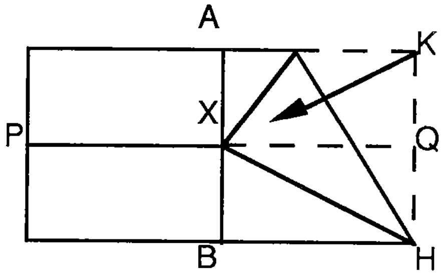

Fig. 7

Cut the envelope along AB (and unfold) and discard the piece containing P. Fold along XH and XK making sure the creases are well-formed [Fig. 8]. Then "pull apart" along side AB and "reverse" the creases formed above. Bring A and B together tape along the opening. You get a regular tetrahedron.

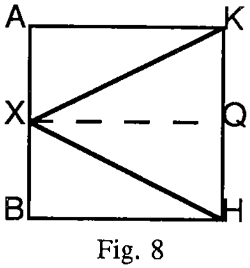

Fig. 8

- Why does this construction yield a regular tetrahedron?

Try to find some other paper folding experiments that will help you understand other geometric shapes and properties.

(**Tossing Dice** - continued from page 5)

However, the pattern breaks down for n > 6. For n = 7, the number of ways is not $\binom{9}{3} = 84$, but 80 - a difference of 4. For $n = 8, 9, 10, 11$ these differences are 16, 40, 80, 140 respectively. In fact, for n > 6, the difference between $\binom{n+2}{3}$ and the actual number of ways is $4\cdot\binom{n-4}{3}$. In general, the number of ways the nth sum can be rolled (n > 6) is

$$\binom{n+(x-2)}{x-1} - x\cdot\binom{n+(x-2)-6}{x-1}$$

Upon further investigation, I found the pattern breaks down again for n > 12. I noticed that the difference was again a pattern from Pascal's Triangle. The number of ways the nth sum can be rolled for n > 12 is:

$$\binom{n+(x-2)}{x-1} - x\cdot\binom{n+(x-2)-6}{x-1} - \left(\frac{x-1}{2}\right)\binom{n+(x-2)-12}{x-1}.$$

As anticipated, this pattern breaks down for n > 18. The number of ways the nth sum can be rolled for n > 18 is:

$$\binom{n+(x-2)}{x-1} - x\cdot\binom{n+(x-2)-6}{x-1} - \left(\frac{x-1}{2}\right)\binom{n+(x-2)-12}{x-1} - \left(\frac{x-2}{3}\right)\binom{n+(x-2)-18}{x-1}$$

Zubair goes on to notice a "general pattern for the general pattern". He ran a program to find the number of ways to roll a certain sum with 9 dice. "I found the computer took a little over 5 minutes for each sum." He included a copy of his output.

Zubair has, in essence, rediscovered a formula for the number of solutions to a linear diophantine equation of the form $x_1 + x_2 + x_3 + \ldots + x_n = k$ where $1 \le x_j \le 6$ for j = 1,2,3,..., n. The proof of the formula involves the Inclusion-Exclusion Principle [see the Log, April 1992]

Let's consider one example in detail. The number of ways of obtaining a sum of 21 when rolling four dice is the number of integer solutions to the equation (*) $x_1 + x_2 + x_3 + x_4 = 21$ where $1 \le x_j \le 6$ for j = 1,2,3,4.

First consider 21 1's separated by 20 spaces: 1s1s1s1s1s1s1s1s1s1s1s1 ... s1. To find a solution to $x_1 + x_2 + x_3 + x_4 = 21$ corresponds to choosing 3 of the 20 spaces. Suppose, for example, we choose the third, sixth and tenth spaces: 1s1s1$\boxed{s}$1s1s1$\boxed{s}$1s1s1s1$\boxed{s}$1s1...s1 . This choice corresponds to the solution: $x_1 = 3$, $x_2 = 3$, $x_3 = 4$, $x_4 = 11$ [not a solution to our dice problem]. There are $\binom{20}{3} = 1140$ such choices and hence there are 1140 positive-integer solutions to the equation. To solve the dice problem, we must subtract those solutions for which $x_j > 6$ for any j = 1,2,3,4. We define A_j = {solutions to (*) with $x_j > 6$} for j = 1,2,3,4. The number to be subtracted from 1140, then, is $|A_1 \cup A_2 \cup A_3 \cup A_4|$.

(Continued on page 4)