Attend the 1992 Convention in Princeton, New Jersey

The Mathematical Log

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Finding the Center of an Ellipse

Harry Ruderman

The material in this article comes from a talk given by Dr. Ruderman at the 1991 Convention in Huntsville AL. It has also been submitted for publication in the New York State Mathematics Teachers Association Journal.

"Can you find the center of a circle?"

"Sure. Construct the perpendicular bisector of a chord. This bisector will contain a diameter whose midpoint is the center of the circle." [Or construct the perpendicular bisectors of two non-parallel chords. Their point of intersection is the center.]

"Fine. How do you find the center of an ellipse?" [You might see if you can answer the question by yourself.]

Trying to find the center of an ellipse using the perpendicular bisectors of chords ends in failure. We must find another way to generalize the concepts used in the construction of the center of a circle. The key is the fact that the product of the slopes of a chord and the line containing the center of the ellipse and the midpoint of that chord is constant. For a circle, this constant is –1; for an ellipse of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ it is $-\frac{b^2}{a^2}$.

Theorem If $\overline{PQ}$ is a chord in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ that does not contain the center and is not parallel to an axis, and $\overleftrightarrow{OM}$ contains the center, O, of the ellipse and the midpoint, M, of $\overline{PQ}$, then the product of the slopes of $\overline{PQ}$ and $\overleftrightarrow{OM}$ is $-\frac{b^2}{a^2}$

Proof Let $P=(x_1, y_1)$ and $Q=(x_2, y_2)$. Then $M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Since P and Q lie on the ellipse,

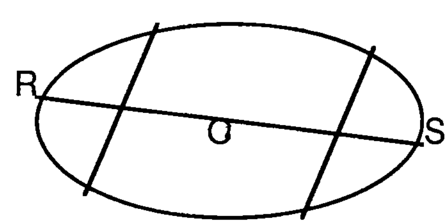
$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}=1$ and $\frac{x_2^2}{a^2}+\frac{y_2^2}{b^2}=1$. Subtracting and combining yields $\frac{y_2^2-y_1^2}{x_2^2-x_1^2}=-\frac{b^2}{a^2}$. But $\frac{y_2^2-y_1^2}{x_2^2-x_1^2}=$

$$\frac{y_2-y_1}{x_2-x_1}\cdot\frac{y_2+y_1}{x_2+x_1}=\text{slope } \overline{PQ}\cdot\text{slope } \overline{OM}.$$

If we take two such parallel chords, the center of the ellipse and the midpoints of the chords will be collinear. [Why?]

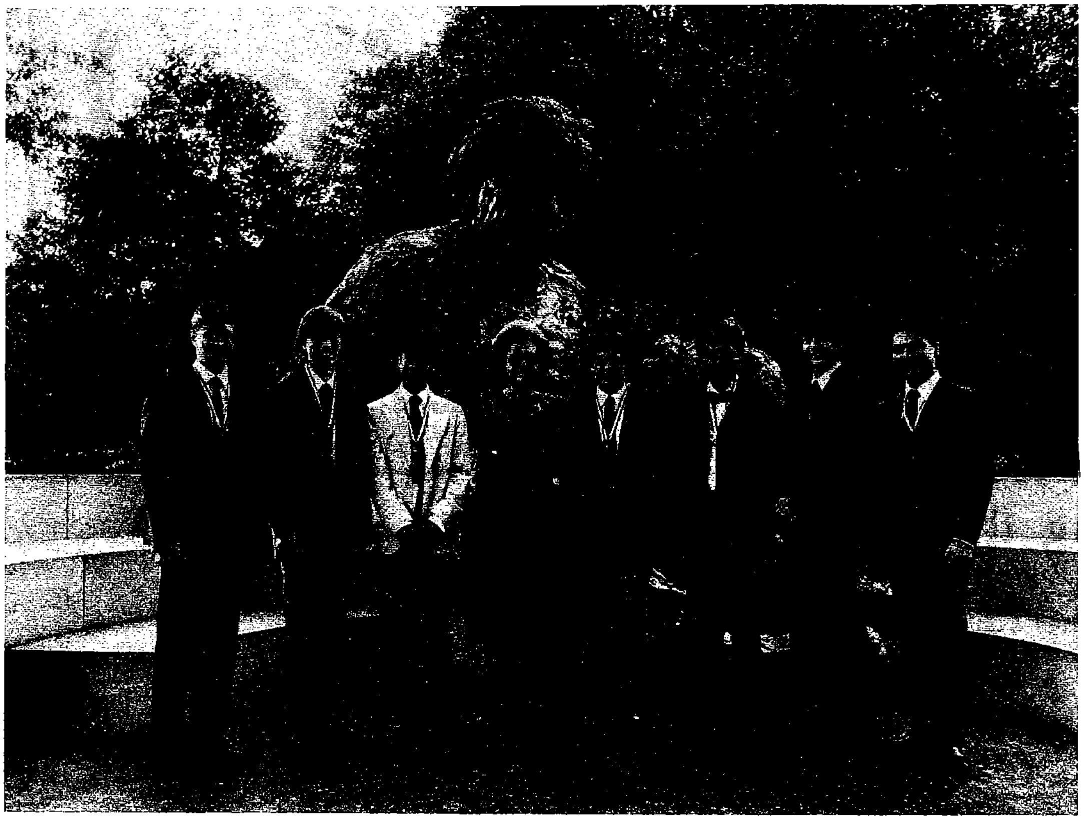
Thus to construct the center of an ellipse:

1. Construct two parallel chords [not containing the center or parallel to an axis]
2. Construct their midpoints.
3. The line containing these midpoints will intersect the ellipse in two points R and S.
4. The midpoint of $\overline{RS}$ is the center, O.



Problem Construct the axes and foci of the ellipse.

In a future issue, we will examine analogous constructions for the hyperbola and the parabola.



The USAMO [USA Mathematical Olympiad] medalists pose with Mu Alpha Theta President Carol McGill. They are [from the left] Michail Sunitsky, Joel Rosenberg, Lenhard Ng, Carol McGill, Robert Kleinberg, Kiran Kedlaya, J.P.Grossman, and Ruvim(Ruby) Breydo. What familiar face is ovelooking the proceedings?

OFFICERS 1992

The nominating committee for the 1992 convention welcomes your nominations for the following offices

- **President -elect**
- **Governor Region I**
- **Governor Region II**

Please send your suggestions to:

Dr. Betty Lichtenberg
Mathematics Education Dept.
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Tampa, FL 33620

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The Mathematical Log is the official publication of Mu Alpha Theta, national high school and junior college mathematics honor society and mathematics club federation. Founded in 1957 by Richard and Josephine Andree, Mu Alpha Theta is co-sponsored by the Mathematical Association ofAmerica (MAA) and the National Council of Teachers of Mathematics (NCTM) Correspondence may be directed to the Editor or to Mu AlphaTheta National Office, 601 Elm Ave., Room 423, Norman, OK 73019.

Mu Alpha Theta Slides

Jeanne Nelson, Governor of Region I, is preparing a slide show "Mu Alpha Theta Across the Country". If you have any slides of activities in your chapter that you would like to share, please send them to her at the address on page 2. Deadline is January 1, 1992.

1991 √At the Root of It All

Deborah Patonai Phillips, Activities Editor

As a mathematics teacher, I am always delighted to see college students planning careers in mathematics education. Since the numbers of capable, enthusiastic, and caring teachers are never enough, the importance of finding new ones is paramount. The Andree Mathematics Education Award, named in honor of Richard and Josephine Andree [the founders of MAΘ] salutes students interested in becoming mathematics teachers. Given to graduating seniors or to former MAΘ who have completed no more than two years of college, this award includes a $1,000 prize, a plaque, and a complimentary registration at the national convention. The runner-up receives $300. Selected by the Governing Council, √At the Root of It All honors the 1991 winners: Crystal Brandon and runner-upTanya Walter.

A 1991 graduate of Plant City, Florida, High School, Crystal Brandon is majoring in mathematics education at the University of South Florida. After graduation, she plans to return to teach mathematics at her high school alma mater. Her former mathematics teachers are " ... looking forward to the day when she can become a member of our faculty and continue to be a positive influence in Mu Alpha Theta."

Moving several times while she was in junior high school, Crystal encountered a rocky start in mathematics. At Plant City, she was initially placed in a low math class. While taking Algebra I in tenth grade, she was discovered to have high ability in mathematics. She was subsequently placed in geometry while still taking Algebra I. She continued by taking Algebra II, Trigonometry, Analytic Geometry,and Calculus over the next two years! Because of her achievements, she received the Outstanding Math Student Award for 1991.

Soon after she was placed in advanced courses, Crystal was asked to join MAΘ. Although unfamiliar with the organization, she was willing to try anything involving mathematics. Working at least two extra hours a day on mathematics, she competed on the Algebra I team that won several state awards. In her junior year, she was on the Algebra II team that placed second at the national convention. As a senior, she was a member of the calculus team that won several awards.

Crystal participated in many extra-curricular activities including Future Florida Educators, National Honor Society, Orchestra, Drama Club, and the Knowledge Master Open. Her MAΘ sponsor, Larry Baker, was proud of the way she volunteered to be a coach of the Algebra I team that included driving to the junior high school for after-school practice.

Wanting to be a teacher since kindergarten, Crystal had the opportunity to teach a few math classes on special occasions. She felt 'right at home'. "Teaching is something I really want to do. I want to give my future students what my teachers have given to me: confidence in myself, a desire to achieve, and an undying interest in mathematics."

Tanya Walter, the Andree runner-up, is a 1991 graduate from Macon, Missouri. Attending Northeast Missouri State University, she plans to major in mathematics education and minor in computer science. Relating mathematics to the real world, Tanya wants "to assist students in understanding the fascinating aspects of mathematics in addition to its use in everyday life."

Macon R-I School's 1991 valedictorian, Tanya maintained the top ranking while taking a rigorous academic program and participating in numerous other activities. In addition to the school activities such as SADD, Yearbook, Jets, and Choir, she also squeezed in time to be active in one of the local churches. She traveled to an out-of-state inner city area to present a Bible School for underpriviledged youngsters.

Contributing much to MAΘ , Tanya was selected by her peers as the Outstanding Senior Scholar. She participated in various mathematics contests, winning many awards and commendations. She also co-chaired Family Math Night where members helped the elementary students and their parents to better understand the nature of mathematics.

During the summer, Tanya was chosen as a student intern in an institute for students having difficulty in pre-algebra. According to her MAΘ sponsor/math teacher Myrna Main, Tanya "is very intelligent, and ... has a special ability to explain concepts to students with math anxiety and weak backgrounds. The way she relates to both mathematics and people, I know she will be an outstanding educator." Tanya hopes to pattern her teaching style after that Ms. Main whom she considers a role model.

A teacher must be a guide, a 'people person', and a role model as well as a conveyor of information. Both Crystal Brandon and Tanya Walter possess these attributes. Mu Alpha theta congratulates them on their career choice and wishes them well!

If you have any news for √At the Root of It All , please write to me at the address on page 2.

CONTEST CORNER

This month's column contains the Power Question from the 1991 ARML Contest. The first two parts will allow you to use your knowledge of some basic ideas from trigonometry in an interesting way. The question was written by Harry Ruderman, Gil Kessler, and Larry Zimmerman. The solution will be published in the next issue.

"NICE" ANGLES AND POLYGONS

Definition 1: We call angle A "nice" if both sin A and cos A are rational.
Definition 2: We call a convex polygon "nice" if all of its interior angles are "nice."

I. Prove each of the following:

(1a) If angle A is nice, its supplement will be nice.
(1b) If angle A is nice, $\frac{1}{2}A$ need not be nice, but 2A will be nice.
(1c) The set of nice angles is closed under addition.

(2a) [Note: A Pythagorean triangle is a right triangle with integer sides.] Every acute nice angle is the angle of some Pythagorean triangle.
(2b) If angle A is an acute nice angle and cos A = b/c, where b and c are relatively prime positive integers, then c will be odd.
(2c) There is no smallest positive nice angle.

II. (1) Prove that if the sides of a triangle are rational, and one angle is nice, then the other angles will be nice. [Be sure you satisfy both parts of Definition 1.]

(2a) Find the sides of an acute nice triangle whose sides are integers and whose perimeter is less than 20. [Be sure that this triangle is both acute and nice.]
(2b) Prove that if the sides of a nice triangle are integers, its area will be an even integer. [Hint: One approach would be to first show that its perimeter must be even.]

III. A convex quadrilateral has the following properties:
1. Its sides are integers whose product is the square of some integer.
2. It can be inscribed in a circle, and can be circumscribed about (another) circle.

Prove that this quadrilateral is nice.

REGION IV MINI-CONVENTION

Region IV, which includes all states east of Illinois and Wisconsin and north of Mississippi, Alabama, and Georgia is sponsoring a mini convention at Lakota High School in West Chester, Ohio on December 6 and 7. Make plans to attend. For more information, contact Lakota High School, 5050 Tylersville Road, West Chester , Ohio 45069, 513-874-8390

QUADRATIC QUERIES

Here are some ideas and problems involving quadratic equations. They are largely independent, so you can try them one at a time in any order.

• What is the relationship between the roots of $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$? Try some examples, make a conjecture, and justify it.

• Check the origin of the word 'quadratic' to see how it is related to the number 2 and not 4.

• For what values of n, $0 < n < 1000$ does the equation $x^2 - 2x - n = 0$ have two integer roots?

• Here is another way to justify the quadratic formula:

If $ax^2 + bx + c = 0$, then $4a(ax^2 + bx + c) = 0$ or $4a^2x^2 + 4abx + 4ac = 0$. Then $4a^2x^2 + 4abx + b^2 = b^2 - 4ac$, or $(2ax + b)^2 = b^2 - 4ac$. Hence $2ax + b = \pm\sqrt{b^2 - 4ac}$ or

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Can you justify each step?

• Here is an efficient way to compute the roots of a quadratic equation using the quadratic formula on a simple calculator with only one memory. Let D be the discriminant.

Compute $\frac{\sqrt{D}}{2a}$ and store it in the memory

Compute $\frac{-b}{2a}$

Subtract the stored number:

$$r_1 = \frac{-b}{2a} - \frac{\sqrt{D}}{2a}$$

Addthe stored number twice:

$$r_2 = r_1 + \frac{\sqrt{D}}{2a} + \frac{\sqrt{D}}{2a} = \frac{-b}{2a} + \frac{\sqrt{D}}{2a}$$

Try this method on a few examples. Can you explain why it works?

• Find all real values of x for which

$$(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1.$$

• Solve each of these equations by solving two quadratic equations:

a. $x^2 + x + \frac{12}{x^2 + x} = 8$

b. $x^2 - 5x + \sqrt{x^2 - 5x - 2} = 8$

•Ms. Matrix, the mathematics teacher, wrote a quadratic equation of the form $x^2 + bx + c = 0$ on the board and asked the students to find the two real roots. Tanya miscopied one of the coefficients and found 4 and 1 as roots. Crystal miscopied a different coefficient and found 3 and –2 as roots. What are the roots to the original equation?

• For what values of a does the equation $x^2 + |x| + a = 0$ have real roots? What are the roots [in terms of a]?

• For what values of a do the equations $x^2 + x + a = 0$ and $x^2 + ax + 1 = 0$ have at least one root in common?

• If p and q are odd integers, show that $x^2 + 2px + 2q = 0$ has no rational roots.

• Consider the computation $7 - 4\{7 - 4(7 - 4)\}$. If one mistakenly views this as $3\cdot3\cdot3$, one gets the correct answer. Why? For what pairs of numbers does this incorrect method yield the correct answer for computations of this type?

• If a, b, and c are whole numbers, what the smallest value of a so the equation $ax^2 + bx + c = 0$ has two real roots between 0 and 1?

• Suppose b and c are integers. What percent of quadratic equations of the form $x^2 + bx + c = 0$ have real roots? Write a computer program that checks values $-n < b < n$ and $-n < c < n$ for n = 10, 100, 1000, etc.

• If a, b, and c are integers and the discriminant of $ax^2 + bx + c = 0$ is D, for how many values of D, $78 \le D \le 83$ does the equation have two irrational and unequal roots? [Hint: The answer is not 5] .

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with Log Editor Tom Butts

This year's convention in Huntsville was a great success. Congratulations to the many award and contest winners. Special thanks to the Alabama host chapters who organized the convention, to the contest writers, and to all the others who made the convention such an enjoyable and worthwhile experience.

This column is intended as a forum for **your** questions and comments. So if you have a suggestion on what type of articles and features to include in the Log, a question about any area of mathematics - a person, a concept, a problem,... a comment about any of the articles , or anything else on your mind, write it up and send it to me.[Use the address on page 2.]

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If you're not sure of the significance of this title, look it up.

****

1

1 2

1 3 2

4 1 3 2

5 4 1 3 2

5 4 1 6 3 2

5 4 1 7 6 3 2

8 5 4 1 7 6 3 2

What are the next two rows in this triangle?

Richard Andree Memorial Crossword Puzzle

This puzzle was shown to students attending the 1976 convention in Philadelphia. You will get an insight into the type of person Dr. Andree was after you have solved this puzzle.

Paul Foerster

1	2	3	4
5			
6			
7			

<u>Across</u>
1. Insects
5. Organs of sight
6. To annoy
7. Comfort

<u>Down</u>
1. Dogs do it
2. Lions do it
3. Mosquitos do it
4. Snakes do it

CONTEST: ALGEBOGGLE

Remember the deadline for entires in the Algeboggle contest is December 1, 1991. Send your entries to the Editor [see p. 2 for address]
In this contest [see Oct. 1990 issue], design a 5-by-5 BOGGLE® board that contains a set of **algebra** words with the highest score.

You are to designate one letter for each of the 25 squares. The same letter may appear in more than one square. Words are formed from adjoining letters - either horizontally, vertically, or diagonally. Letters must be joined in sequence to spell a word that could be found in an algebra book. Proper nouns, abbreviations, contractions, and hyphenated words are not permitted. **Only one form of a word may be used.** No letter [from a given square] may be used more than once in a single word. Each word must have at least four letters. Scoring will be as follows:

No. of letters	4	5	6	7	8	9	10 or more
No. of points	1	2	3	5	8	13	21

Your entry should consist of:
1. Your 5-by-5 board with 25 letters.
2. An alphabetical list of the words that can be formed using your board.
3. The total score of the words formed in step (2).