The Mathematical Log

MU ALPHA THETA

Volume 34, Number 1 **February 1990**

FUN TO EXPLORE

Classic 'Measurement' Problems Feature Practical, Fanciful

Log 'Sampler' Offers 25 Early Questions

By Don Allen

I plac'd a bowl into the storm,
  To catch the drops of rain--
A half a globe was just its form,
  Two feet across the same;
The storm was o'er, the tempest past,
  I to the bowl repair'd--
Six inches deep the water stood,
  It being measur'd fair;
Suppose a cylinder, whose base
  Two feet across within,
Had stood exactly in that place,
  What would the depth have been?[1]

Mensuration, the study of measurement, had an important, if somewhat subordinate, place in school mathematics a century and more ago. Elaborate formulas for surface areas and volumes (capacities) were developed or presented (volume of an icosahedron, edge e!), but the topic of measurement, however useful, was considered more "practical" than "intellectual," and accordingly, by many influential authors, was consciously downplayed. Like most math text problems of the period, questions in mensuration could be highly imaginative, however, and there was an evident division between those intended to be "practical" (carpetings, plasterings, wallpaperings) and those which made no such pretense (globes perforated by cylinders, hollow cannonballs being submerged in ale [see inset, p. 4].

Early school math "story problems" we've systematically studied and collected over some decades, our chief sources having been old textbooks in libraries and bookstores and old examinations. A score or more of the more intriguing mensuration questions, here assembled for the first time, could provide interesting material for browsing, attempting, and discussing. No "prerequisite" knowledge is really necessary, beyond a few accessible formulas (which we provide) and simple insights and arithmetic and problem solving skills. Area relations for the square, rectangle, parallelogram, triangle, and circle (πr^2) are generally "committed to memory." Useful relations in "3D" include those for the surface area and volume of a sphere of radius r, given by $4\pi r^2$ and $\frac{4}{3}\pi r^3$, respectively; for volumes or capacities of prisms and cylinders of base B and vertical height h, given by Bh, and of corresponding pyramids and cones, given by $\frac{1}{3}$Bh. Surface areas of such figures are obtainable as sums of circular and polygonal areas, although specific formulas abound in older texts.

An initial grouping of mensuration "word problems" serves to apply and reinforce volume/capacity concepts and relations for the cube, sphere, cylinder, and cone--and in one instance the "oblong spheroid." The questions derive from fairly widely circulated nineteenth century sources, Tobias Ostrander of upstate New York (I, II are reference 10, page 200, #73, and 201, #74), Jeremiah Day of Yale (III, IV are 7, p. 57, #1, and p. 58, #9), James H. Porter (V derives from 11, p. 212, #58-60), and an Irish education commission (VI, from 15, p. 258, #26). One question (VII), by the prolific English textbook author, J. Hamblin Smith (12, p. 274, #18), considers a coin as a cylinder, and recalls the tradition of melting coined gold for its metallic content (the number of coins is improbable!)

I.
GLOBE IN A GLASS

If a conical glass, whose top diameter is 5 inches and altitude 6 inches, be filled 1/5 full of water, and a globe 4 inches diameter be be dropt in the glass, how much of the axis of the globe will be immersed in the water?

(See "'Measurement' Problems," page 4)

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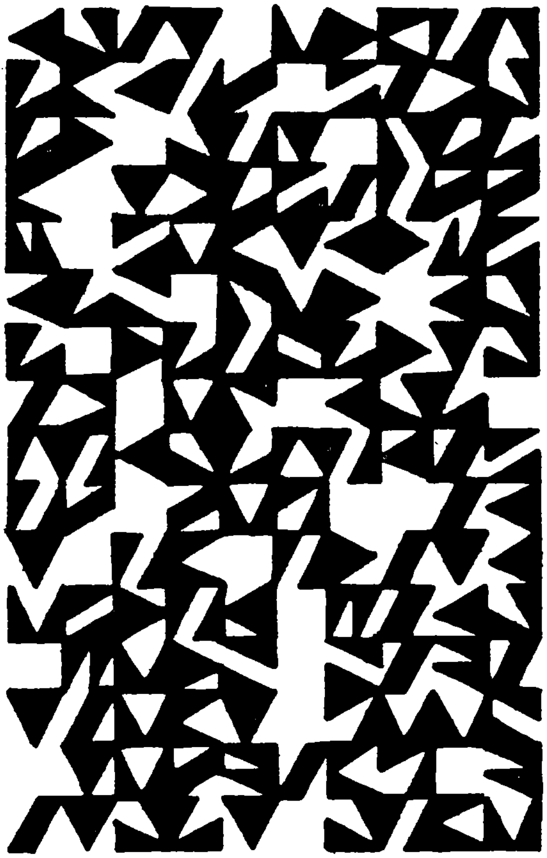
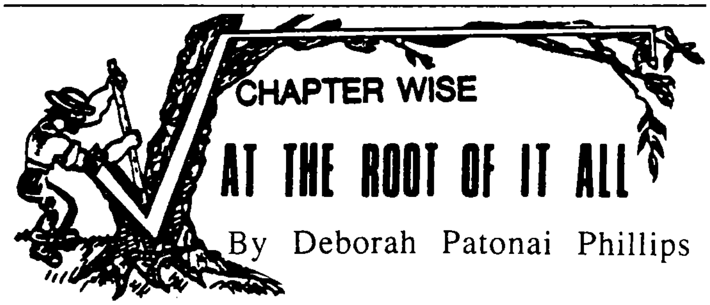
[1]The verse problem is from Tobias Ostrander ("Teacher of Mathematics"), The Elements of Numbers, or Easy Instructor (Canandaigua, N.Y., 1823), Miscellaneous Questions, p. 202, no. 76 (answer: 2.5 in.), reference 10 in the listing appended to this article. The concluding verse is from John Bonnycastle, An Introduction to Mensuration and Practical Geometry (Philadelphia, 1823), p. 233, no. 44 (answer: bottom diameter, 14.6401; top diameter, 24.4002 [inches]), reference 05. We have considered such verse problems (reference 03) and other early story problems (01, 02) at greater length. Annotations for the rest of this article will be to reference, page, problem number.



FRIENDLY CHALLENGE ... FROM THE GRAND MASTER. Architect, standards expert, and recreational cryptographer par excellence Henning Orlando of Stockholm, whose outstanding cryptarithms were featured in our February 1989 issue, offers his fans in Mu Alpha Theta a challenge in the form of an "ornamental" cipher, the cryptographic "type" for which he is best known. He tells us that it is an "aristocrat," which implies one-to-one correspondence between cipher characters (design elements) and "plaintext" letters; also that word divisions are retained. While HANO art frequently is featured on cryptographic journals, he tells us that this original was penned for Log readers. So, look for pattern elements, observe frequency counts and other word characteristics to identify individual letters, and discover what our world-class puzzlist is saying.



This past Summer at Mu Alpha Theta's nineteenth National Convention in Tampa, FL, a new addition to the already full schedule of contests was included, the club scrapbook competition. Each chapter in attendance could bring a scrapbook of club-related materials dating from the 1988, Knoxville convention to the time of the Florida convention. This innovative competition afforded all participating schools opportunity to "show off" (and share) nationally their year's Mu Alpha Theta activities. Scrapbooks in an incredible range of shapes, colors, and styles appeared.

Judges had a difficult time choosing winners. A distinguished list of Sponsors composed the judging panel: Doris Collins (Parkview High School), Ken Lloyd (Brookstone School), Ron Vavrinek (Illinois Mathematics and Science Academy), Lee Pedersen (Vinalhaven High School), and Don Allen (St. George's School of Montreal)—whose scrapbooks of Log submissions and correspondence have been passed around at many recent conventions.

Scrapbooks were judged according to a preset checklist and rating sheet available to all at Convention. Major areas considered were: (i) cover, (ii) content, (iii) organization, (iv) artistic quality, and (v) overall effect. Judges were to consider use of illustrations, volume and variety of content material; also, neatness, originality, lettering. Conveying something of the activities of each school's chapter, scrapbooks could include photographs, contest programs and results, travel forms, permission slips, and membership documents. They also could show items relating to national affiliation: receipts, membership lists, applications. To give the scrapbook its own personality, a chapter could highlight its school's mascot, colors, or emblem.

My favorite scrapbook didn't win an award, but I fell in love with the cover. Entitled "Cruisin' Back in Time ... on Highway '89," the scrapbook was the entry from North Little Rock High School. The cover was a Mu Alpha Theta dollhouse painted shades of aqua/turquoise. In front of the house, which was constructed like a fraternity house with a Mu Alpha Theta symbol on its gable, was a sports car of the same aqua color. The overall effect was gorgeous—it did not look like any scrapbook that I had ever seen. From its contents I learned that the chapter takes a canoe trip, and also adopts two families for Christmas.

The following is a rundown of Tampa's top ten scrapbook winners, with a brief description of each winning book. Finding words to describe these extraordinary scrapbooks was anything but easy!

First Place: Vero Beach. This scrapbook was loaded with photos with numerous captions and some hand-drawn comics interspersed throughout. Its large wooden cover was painted with a colorful Indian, the school's mascot. While looking through this scrapbook, I chanced upon this interesting quotation: "If the English language was truly graphic, logarithm would mean music played at the lumberjack ball."

Second Place: Grissom. This scrapbook was covered in cloth with a cross-stitched Mu Alpha Theta symbol, the school's tiger mascot, and the school emblem. The book contained plenty of good photos and captions with pages full of contest programs and results—most of which chronicled Grissom victories.

Third Place: Vestavia Hills. This scrapbook also had a cross-stitched cover. Using volumes of construction paper, this chapter opted for four-leaf clovers and collages to display programs, receipts, budgets, and newspaper clippings.

Fourth Place: King. With a grey painted wooden cover, this scrapbook was ... huge! Fortunately, the Convention was within driving distance of King High School because I doubt if a plane could have taken off carrying this scrapbook! The book began with a collage of pictures and a page with members' signatures. In addition to shhets of newspaper articles, contest materials, and programs, it featured colorful artwork throughout, which added to the overall effect. All pages were laminated. After seeing pages of various candy wrappers, I surmised that King's Mu Alpha Theta chapter sells chocolate bars as one of its fundraisers!

Fifth Place: Coral Springs. Even though this scrapbook was normal-sized, it was packed with pages upon pages of photos, newspaper clippings, receipts, and other documentation. This chapter worked in every piece of paper that related to Mu Alpha Theta activities.

Sixth Place: Miami Springs. This chapter held a scrapbook party to organize their book. During Spring Vacation, designing took place at the home of their sponsor. The chapter also had a carving party at which several students and their sponsor took six hours to convert their wooden cover into a golden hawk. Contained in the book were some distinctive items, including the chapter T-shirt and computer disks.

Seventh Place: A. Crawford Mosley. Having one of my favorite covers, this scrapbook was a four-foot dolphin

(See "At the Root of It All," page 4)

Familiar Integer Divisibility Rules Permit Instructive Extensions

By Ali R. Amir-Moéz
Mathematics Editor

Many of us have learned some simple rules for divisibility by 2, 3, 4, 5, 6, 8, 9, and 10 and, perhaps, we did not questions their validity. Even though one may test the divisibility of an integer by another integer using the calculator, one may find it interesting to obtain the rules logically. In this note we obtain the divisibility rules for 7 and 11. Then we suggest the rules for some other integers so that the reader may supply the reason for them.

1. Divisibility By Seven: In the decimal system, every integer is written as a linear combination of powers of 10 such as

$$a_0 + a_1(10) + a_2(10^2) + a_3(10^3) + \cdots + a_n(10^n),$$

where

$$0 \leq a_i \leq 9, i = 1, \cdots, n.$$

For example, 3907511 can be written as

$$1 + 1(10) + 5(10^2) + 7(10^3) + 0(10^4) + 9(10^5) + 3(10^6).$$

Now in order to obtain a divisibility rule for 7, we divide 10^n by seven as below

```
    1 4 2 8 5 7
7 | 1 0 0 0 0 0 0 . . . 0
    7
    3 0
    2 8
      2 0
      1 4
        6 0
        5 6
          4 0
          3 5
            5 0
            4 9
              1
```

note that we have divided until 1 has appeared which means the same set of integers will repeat in the remainders. From this division one may write

$1 = 7m + 1$
$10 = 7m + 3$
$10^2 = 7m + 2$
$10^3 = 7m + 6 = 7m + 7 - 1 = 7m - 1$
$10^4 = 7m + 4 = 7m - 3$
$10^5 = 7m + 5 = 7m - 2$
$10^6 = 7m + 1,$
$\cdots,$

where $7m$ means a multiple of 7. So an integer can be written as

$$N = a_0 + a_1(10) = a_2(10^2) + a_3(10^3) + a_4(10^4) + \cdots + a_n(10^n).$$

Instead we can write

$$\begin{aligned} N &= a_0[7m+1] + a_1[7m+3] + a_2[7m+2] \\ &- a_3[7m+1] - a_4[7m+3] - a_5[7m+2] \\ &+ a_6[7m+1] + \cdots. \end{aligned}$$

So

$$\begin{aligned} N = \text{multiple}(7) &+ a_0 + 3a_1 + 2a_2 \\ &- (a_3 + 3a_4 + 2a_5) \\ &+ \cdots \end{aligned}$$

This suggests that if

$$a_0 + 3a_1 + 2a_2 - (a_3 + 3a_4 + 2a_5) + \cdots$$

is divisible by seven, then N is divisibly by seven.

Example: We shall test 4'491'081. Here we look at

$$\begin{aligned} &1(1) + 3(8) + 2(0) \\ &-[1(1) + 3(9) + 2(4)] \\ &+1(4) \qquad = 21 - 36 = -7. \end{aligned}$$

So the number is divisibly by 7. Indeed testing with a calculator is much faster.

2. Divisibility By Eleven: Again we try the following division

```
        9
1 1 | 1 0 0 0 0 0 0 0
      9 9
        1
```

We did not have to go far. This division implies that

$1 = m(11) + 1$
$10 = m(11) + 10 = m(11) - 1$
$10^2 = m(11) + 1$
$10^3 = m(11) + 10 = m(11) - 1$
$\cdots$

So

$$N = a_0 + a_1(10) + a_2(10^2) + a_3(10^3) + \cdots + a_n(10^n)$$

can be written as

$$N = m(11) + a_0 - a_1 + a_2 - a_3 + \cdots + (-1)^n a_n.$$

If $a_0 - a_1 + a_2 - a_3 + \cdots + (-1)^n a_n$ is divisible by 11, then N is divisible by eleven.

Example: We shall test $N = 7684281$. Here we look at

$$\begin{aligned} &1 - 8 + 2 - 4 + 8 - 6 + 7 \\ &= 18 - 18 = 0. \end{aligned}$$

Therefore, N is divisible by eleven.

The reader may obtain the rules of divisibility by 13 and 17. Indeed, the rule for 17 is quite lengthy.

For divisibility by 13 one can consider

$$N = a_0 + a_1(10) + a_2(10^2) + a_3(10^3) + a_4(10^4) + a_5(10^5) + \cdots$$

and check

$$\begin{aligned} &1a_0 - 3a_1 - 4a_2 \\ &-[1a_3 - 3a_4 - 4a_5] + \cdots. \end{aligned}$$

For 17 the test period has eight terms which makes it tedious, but finding the rule is interesting by itself.

AT THE ROOT OF IT ALL

... FROM PAGE TWO

whose fins opened to a book of green and orange laminated pages. Of course, this school's mascot is the dolphin! Nicely printed letters and scores of photos of members, officers, and sponsors lent to a professional look. Along with competitions, this Mu Alpha Theta chapter's activities extend to a calendar girl contest and a computer dating service, the scrapbook documents.

Eighth Place: Central of Tuscaloosa. This Alabama chapter used the computer program, AUTO CAD, to make its scrapbook distinctive by creating numerous borders and layouts. Some of the club's reported activities have been a Christmas party, tournaments, and a picnic.

Ninth Place: Berkeley Prep. A wooden airplane with moving propellers adorned this chapter's scrapbook cover. Opening pages featured a plane pulling a huge Mu Alpha Theta banner through the clouds. As an engaging way to depict their officers, the chapter chose to use a deck of cards as backgrounds for excutive photos. The page title: "It's All in the Cards." Information in the scrapbook relates how chapter members really do benefit from working on fundraisers. After the Knoxville trip, those having earned a set amount of money were rewarded with a whitewater rafting excursion. For two days these Mu Alpha Theta members went canoeing on the Arkansas River, we're told!

Tenth Place: Miami Sunset. Appropriately, the cover of this scrapbook displayed the Mu Alpha Theta in a sea of water with an orange sun in the background. The book was a treasure trove of pictures and captions--many of them of Mu Alpha Theta sports.

One might conclude that Florida schools dominated this national competition; and, in fact, they did. One reason for this advantage might be that a scrapbook competition has been a major contest at Florida state conventions. These Florida schools have witnessed the quality and the creativity of the state competition, and have come to national prepared to "show their stuff."

Representing well each chapter's distinctive history, the scrapbook competition was a successful addition at Tampa convention. Not only did ten schools go home with trophies, but also all schools carried home with them a lasting book of memories. Scrapbooks can be a most attractive tradition!

'Measurement' Problems

... FROM PAGE ONE

II.
SPHERICAL SHELL

If a solid globe of glass at the furnace, whose diameter is 8 inches, be blown into a hollow globe till the shell is but 1/5 of an inch in thickness, what then will be its diameter, and how much wine will it hold?

CANNON BALLS IN THE ALE

Cannon balls in ale? Only in an early mensuration problem presented by Edward Brooks in his Normal Higher Arithmetic (Philadelphia: Christopher Sower, 1877), p. 502, #84. The old question reads as follows:

"How many cannon balls, 8 inches in diameter, are contained in a cubical vessel whose side measures 6 feet, and how many gallons of ale can be poured in after the vessel is filled with balls, each ball containing a hollow 5 inches in diameter, and the opening containing $1\frac{1}{2}$ cubic inches?"

III.
CUBIC VESSEL

How much water can be put into a cubical vessel three feet deep, which has been previously filled with cannon balls of the same size, 2, 4, 6, or 9 inches in diameter, regularly arranged in tiers, one directly above another?

IV.
PERFORATED GLOBE

If a globe 20 inches in diameter be perforated by a cylinder 16 inches in diameter, the axis of the latter passing through the centre of the former; what part of the solidity, and of the surface of the globe will be cut away by the cylinder?

V.
SPHERE IN A CONE

If a heavy sphere, whose diameter is 4 inches, be let fall into a conical glass full of water, whose diameter is 5 and altitude 6 inches. It is required to determine how much water will run over.

The dimensions of the sphere and cone being the same ..., and the cone only 1/5 full of water. Required what part of the axis of the sphere is immersed in the water.

The cone being still the same, and 1/5 full of water. Required the diameter of a sphere which shall be just covered by the water.

VI.
OBLONG SPHEROID

There is a bowl in form of the segment of an oblong spheroid, whose axes are to each other in the proportion of 3 to 4, the depth of the bowl one-fourth of the whole transverse axis, and the diameter of its top 20 inches; it is required to determine what number of glasses a company of 10 persons would have in the contents of it, when filled, using a conical glass, whose depth is 2 inches, and the diameter of its top an inch and a half?

VII.
CUBE OF GOLD

A sovereign is 7/8 in. in diameter and 1/16 in. in thickness. If 2964500 of them are melted and formed into a cube, find the length of the edge of the cube.

Word problems of a further grouping (VIII-XII) permit treatment in two dimensions--which sounds straightforward--but give rise to interesting considerations. Thomson (VIII) asks, in effect, the radius of a circle which encloses one acre (14, p. 397, #77). Ostrander (X) requires the circumference enclosing twice this area (10, p. 196, #46). Glashan (X) asks about concentric circular routings (08, p. 78, #32). Smith (XI) considers circular arc in an innovative way ... unwinding a string from around an equilateral triangle (12, p. 337, #67). In somewhat similar dynamics, Porter (XII) has a tree, having stood "upon an eminence," break off above the ground, its tip describing a circular arc in falling "down a declivity" (11, p. 219, #95). An elementary solution (to XII) should constitute a distinctive challenge.

VIII.
AN ACRE TO GRAZE

A man wished to tie his horse by a rope so that he could feed on just an acre of ground: how long must the rope be?

IX.
A CIRCULAR ENCLOSURE

A circular garden contains 2 acres; required the length of a stone wall which will enclose it?

X.
A CIRCULAR RACE-TRACK

A circular race-track 24 ft. wide encloses a circle of 50 yd. radius. How long would it take a man to run round the outer edge of the track at a speed which would take him round the inner edge in one minute?

(See "'Measurement' Problems," page 6)

Beyond the Curricular

Nationally-Acclaimed Sanctuary Offers Insights to Members

Scientists Tend to Disabled Birds Take Pride in Rehabilitation

Story and photos by Don Allen

Indian Shores, FL--Incongruous, as we leave downtown traffic for our beachfront parking area and afternoon of discovery, is the tall seabird, obviously fake or inexpertly stuffed, that someone has perched on one leg atop the low roof of the reception building that is our initial destination on our Gulf Coast adventure in reporting for students. A stuffed bird at a bird sanctuary! Crow calls and the screech of bluejays divert and beckon us, however. Anole lizards scurry amid sand and crushed shells underfoot. Sturdy sea grape, reaching twice our height with jungly foliage, provides a natural backdrop to the walkways past pens and cages that house such diversity of recovering or permanently crippled birds.

This is the nationally acclaimed Suncoast Seabird Sanctuary, and Ernest Simmons, outdoor supervisor, has agreed to meet, escort, and brief a Mathematical Log team from Mu Alpha Theta's Tampa convention. With the Editor are students Adrian Hackett and Michael Davide of South Pike High School, Magnolia, MS; their teacher, convention perennial Adolph Holbrook; other interested sponsors Jane Radtke, Milwaukee Technical High School, and Sister Scholastica, Blessed Sacrament Academy, San Antonio. The crow speaks up fearlessly, a gannet preens in the shade ... then "screech owl," "barn owl," "great horned owl."

We stroll, amid casual visitors of all ages, past the screened enclosures that protect injured birds, toward the clinic where highly competent care is routinely given.

We sense, rather than note, the slightest of motions. Atop the reception building a head has turned, ever so slightly. A sharp eye watches our progress, gauges our purpose. The incongruous bird is for real, a regular fixture though not a legitimate resident. He is a freeloader, hoping for leftovers at feeding time. His patience is likely to be rewarded, we are to be informed.

Ernest Simmons gives us all the time we need to ask our questions, explore the acre Sanctary and its beachfront, and meet some of its more celebrated winged residents. The Suncoast Seabird Sanctuary, Inc., was founded in 1971 by Ralph T. Heath, Jr., a zoologist, and is the largest wild bird hospital in the U.S., we learn. "Dedicated to the rescue, repair, recuperation and release of sick and injured wild birds," it receives no government funding but operates of tax-deductible donations from the general public.

We meet Gabrielle, a redtail hawk of fine plumage who had been brought in blinded by gunshots to her head. Pellets remain, unable to be safely removed, but she will live out her life at the aviary.

Injured birds are collected or brought in from all parts of the State, 12 to 15 a day, seven days a week. Some 10 000 birds of more than 100 species have been rehabilitated and released, we learn. On a given day up to 500 birds are receiving care. Sea birds predominate, but land birds and birds of prey also receive needed care.

We are struck by the vocal--but undersized--bluejays, so different from their big counterparts known to the Editor by Canadian mountain lakes. "A subspecies," Simmons points out, explaining that smaller varieties at lower latitudes is the rule for a number of species.

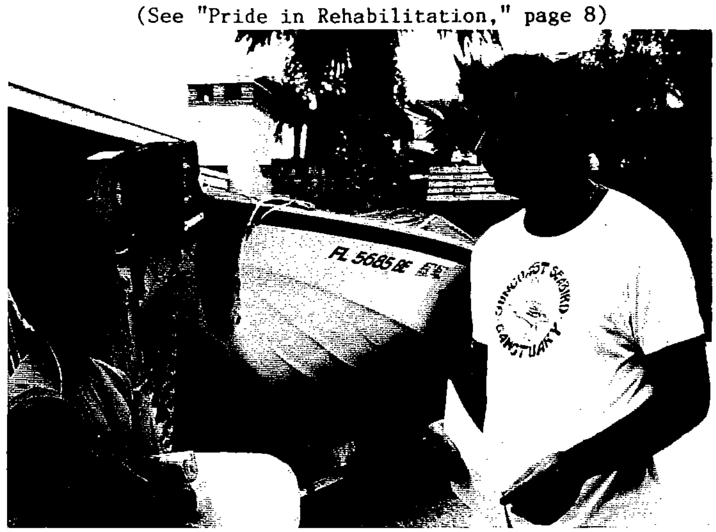
Approximately ninety per cent of birds brought to the Sanctuary have injuries directly or indirectly related to humans, we are told. Two basic problems are encroachment on what was the bird's own natural habitat, and environmental pollution. Protected birds may incur gunshot wounds, get caught in fish hooks and lines, be attacked by pets, fly into power lines and glass windows, or be poisoned by pesticides or environmental pollution.

The Sanctuary seeks to treat, rehabilitate, and release wild birds. Some, too injured ever to fend for themselves, live out their lives at the Sanctuary. A number of species which rarely breed in captivity have done so at the Sanctuary--including crippled brown pelicans--with the young flying away to join others of their species.

The Sanctuary is known throught the State, where 150 volunteers pick up and bring in injured birds, and far beyond, publicity helping to spread the message that the Sanctuary and its founder seek to share. National press and broadcast coverage has been extensive. Footage for an IMAX "giant screen" film on man's relationship with birds, first released in Japan, was produced with Sanctuary assistance, we learn.

Sponsors and Editor then stroll down to the shoreline to watch a ritual, the tossing of leftover food from the pens into the Gulf?

(See "Pride in Rehabilitation," page 8)



SEVENTEEN BIRDS A DAY has been the recent average for rescued sea and land birds brought to Florida's unique bird sanctuary, Ernest Simmons, outdoor supervisor (right) informs Mu Alpha Theta's Michael Davide, now in Grade X at South Pike High School, Magnolia, MS. A group from Tampa convention--editor, sponsors, and students--visited Suncoast Seabird Sanctuary to ask, learn, and inform concerning this major life science enterprise. The boat pictured has seen service in rescue operations.

CONGRATULATIONS, MAA ... NOW 75

'Measurement' Problems

from page four

XI.
WINDING A STRING

One extremity of a string is fastened to a corner of a board of the shape of an equilateral triangle, the side being 7 in. long, and the string is then wound around the triangle. It is then unwound, being kept stretched. Find the length of the distance moved over by the free end in one complete revolution.

XII.
HEIGHT OF A STUMP

A tree 120 feet in heighth, stood upon an eminence, but being partly broken off at a certain distance from the ground, and the top falling down a declivity considerably lower than the foot of the tree, but resting upon the stump, the distance from the foot to the top of the tree when down was 90 feet, and a line drawn from the foot, at a right angle to the perpendicular stump, to intersect the part broken off was 40 feet. Question, how high was the tree broken above the ground?

Application of length, area, and volume relations to such wholly "practical" pursuits as papering a room, plastering a ceiling, and carpeting a floor is considered by Glashan (reference 08: XIII is p. 79, #42, and XIV is p. 85, #92), who also gives attention to the complexity of laying a wooden sidewalk (XV; p. 80, #52). The Irish reference (15), as used in North America, looks at an interesting aspect of putting up a house (XVI; p. 259, #35).

XIII.
CARPETING AND WALL-PAPER

How many yards of carpet 27" wide will be required to carpet a room 25' 8" by 15' 8" allowing 9" per width for matching? How many rolls of wall-paper and how many yards of bordering will be required for the same room, allowing on the wall-paper a width of 42" each for 3 windows and 2 doors?

XIV.
PLASTERING WALLS AND CEILING

Find the cost of plastering the walls and ceiling of a room 27' 8" x 13' 4" x 9' 2" at 32 ct. per square yard, there being 3 windows 6' 9" x 4' 3" and 2 doors 7' 3" x 4' 3". How many cubic feet of plaster would be required to plaster the room, the average thickness of the plaster being half an inch?

XV.
COST OF A SIDE-WALK

What will be the cost of 1000 yards of side-walk 8 ft. wide, made of 3 in. plank laid on three lines of cedar stringers, if the planks cost $12.00 per M., the cedars $4\frac{1}{2}$ ct. per running-foot and preparing and laying the side-walk $3.50 per yard?

XVI.
HEIGHT AND BREADTH OF A HOUSE

A side wall of a house is 30 feet high, and the opposite one 40, the roof forms a right angle, at the top, the lengths of the rafters are 10 feet and 12; the end of the shorter is placed on the higher wall, and vice versa; required the length of the upright, which supports the ridge of the roof, and the breadth of the house?

A mensurational consequence of the world being round not flat, the distance of the "visible horizon" at stipulated heights, features in the occasional textbook problem, though it rather extends the measurement ideas being considered. Porter, writing in 1845 or somewhat earlier, tells of a hot air balloon ascent in New York state, producing an instructive problem (XVII, from 11, p. 216, #76). Exploring somewhat further, the Irish text asks (XVIII) the number of acres that can be seen from a tall steeple (from 15, p. 257, #19). Consideration of significant digits arises naturally in the Irish question, which suggests that taking the earth as a sphere is at best an approximation, as is the 25000 miles reported circumference. The textbook answer, nonetheless, is to an improbable 17 digits.

XVII.
HEIGHT OF A BALLOON

If a person with an air balloon ascends vertically from the city of Hudson, to that heighth that he may just see the top of a flag-staff hoisted on the top of the city hall, in New York, appear in the horizon. I demand his heighth above the surface of the earth, supposing the circumference to be 25000 miles, and the distance between Hudson and New York 150 miles, and the flag erected 100 feet from the ground.

XVIII.
ACRES OF SURFACE

How many acres of the earth's surface may be seen from the top of a steeple whose height is 400 feet, the earth being supposed to be a perfect sphere, whose circumference is 25000 miles?

One group of traditional problems, among the more challenging, calls for the partitioning, in equal parts or in stipulated proportionality, of something in geometric configuration. Greenleaf (reference 09) writes about a pyramidal haystack (XIX; p. 422, #23) and the more usual grindstone (XX; p. 443, #24), taken to be a cylinder. The Irish text (15) has a pleasant problem partitioning the consumption of, and hence payment for, a quantity of milk (XXI; p. 257, #20). A delightful old question (XXII) which tells of human frailty in a manner suggesting partitioning, but actually involving only proportion, is given by Greenleaf (09; p. 443, #25), and leads to interesting arithmetic investigation. (Greenleaf wrote 120 years ago, well before the advent of the pocket calculator, it should be noted!) A further question (XXIII), due to Porter, partitions a circular farm in an instructive way (11; p. 209, #40).

XIX.
PARTITIONING A HAYSTACK

Four men, A, B, C, and D, bought a stack of hay, containing 8 tons, for $100. A is to have 12 per cent more of the hay than B, B is to have 10 per cent more than C, and C is to have 8 per cent more than D. Each man is to pay in proportion to the quantity he receives. The stack is 20 feet high, and 12 feet square at its base, it being an exact pyramid; and it is agreed that A shall take his share first from the top of the stack, B is to take his share the next, and then C and D. How many feet of the perpendicular height of the stack shall each take, and what sum shall each pay?

XX.
PARTITIONING A GRINDSTONE

A, B, and C bought a grindstone, for which they paid $10.60. B paid 20 per cent more than A, and 10 per cent less than C. The diameter of the stone was 65 inches, and the diameter of the place for the shaft 3 inches. What sum did each pay, and how much must each grind off from the semidiameter to obtain his proper share of the stone?

XXI.
PROPORTIONS OF THE MILK

Two boys meeting at a farm-house, had a tankard of milk set down to them; the one being very thirsty drank till he could see the centre of the bottom of the tankard; the other drank the rest. Now, if we suppose that the milk cost $4\frac{1}{2}$ d., and the tankard measured 4 inches diame-

(See "'Measurement' Problems," page 7)