

Mathematical Log

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International Perspective

Range, Diversity of Questions Highlight Two-Round High School Competition

A school mathematics competition with an international flavor again offers Mathematical Log readers an individual or small-group challenge and good material to "work up" for a math club meeting. The contest this issue is the 24th Old Mutual Mathematics Olympiad (1989), sponsored by a major life assurance society and written in secondary schools of South Africa. First-round problems in this competition are multiple choice, while second round problems call for formal solutions. Questions are shared with Mathematical Log readers with permission of the Olympiad committee.

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FIRST ROUND PROBLEMS

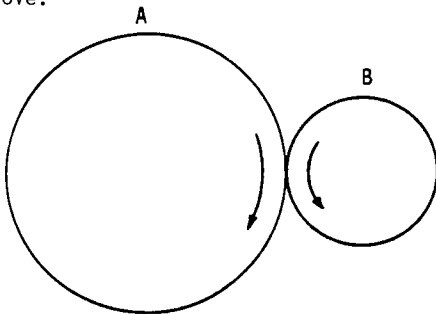
- The polynomial $f(x) = 2x^5 + ax^4 - bx^2 - 30x - 6$ is divisible by $x - 2$. What is the remainder if $f(x)$ is divided by $x + 2$?
 (A) $4a - b$ (B) -9 (C) 8 (D) $8a - 2b - 5$
 (E) -8 .
- Instead of walking along the two sides of a rectangle, one can also proceed along a diagonal, and thereby decrease the total distance travelled by one third of the length of the longer side of the rectangle. What is the ratio of the length to the breadth of the rectangle?
 (A) $\frac{3}{2}$ (B) $2:1$ (C) $11:5$ (D) $2\frac{2}{5}$
 (E) None of the above.
- How many pairs of real numbers have the property that their sum and their product are both equal to 1989?
 (A) 0 (B) 1 (C) 2 (D) 3 (E) Infinitely many.
- Which of the following numbers is the smallest?
 (A) 19.89 (B) $\frac{198}{9}$ (C) $1 + \sqrt{98} + 9$
 (D) $\sqrt{198} + 9$ (E) $19 + \frac{8}{9}$.
- A number of teams entered a competition, and each team played once against every other team. The total number of matches played was 66. How many teams entered the competition?
 (A) 11 (B) 12 (C) 13 (D) 66 (E) 67.
- The positive integers p and q are such that, when divided by q
 p leaves a remainder of 2
 p^2 leaves a remainder of 4
 p^3 leaves a remainder of 8
 p^4 leaves a remainder of 5.
 What is q ?
 (A) 9 (B) 10 (C) 11 (D) 13
 (E) None of the above.
- How many isosceles triangles can be formed by joining points of the accompanying lattice? (The points form a square and are equidistant.)
 (A) 48 (B) 36 (C) 32 (D) 24
 (E) None of the above.
- The graph of the function $y = x^2 - 2|x| + 1$ is:

(See "High School Competition," page 2)

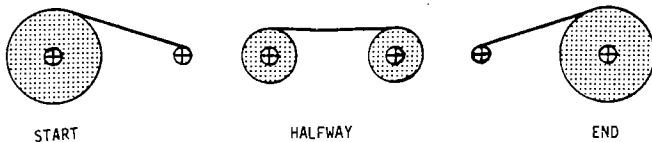
High School Competition

...FROM PAGE ONE

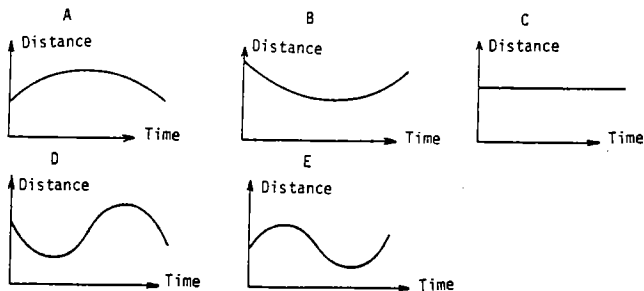
9. A boy receives \$x pocket money per month on condition that he saves x% of the amount. What amount of pocket money should he receive so that he retains a maximum amount after the part to be saved has been subtracted?
 (A) \$50 (B) \$40 (C) \$20 (D) \$5
 (E) There is no answer since the more the boy receives, the more he retains.
10. What is the minimum value of $x^3 + 4y^3$ if $x + y = 6$?
 (A) 27 (B) 96 (C) 128 (D) 135 (E) none of the above.



11. A metal wheel A and a rubber wheel B rotate together so that A makes 100 revolutions per minute. Originally the radius of wheel B was half of that of A, but after a time, due to wear, the radius of B has decreased by 10%. At how many revolutions per minute does B rotate now?
 (A) $222\frac{2}{9}$ (B) 220 (C) 200 (D) 180 (E) $177\frac{7}{9}$.
12. A cassette tape is wound from one reel to another as in the figure. Note that the spindles of the reels are fixed.



The shortest distance between the reels is measured at regular intervals, and the results are represented graphically. Which of the following graphs best represents the results?

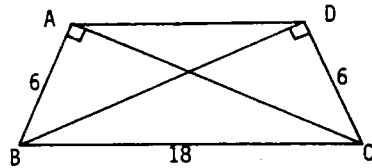


13. Two equal and parallel chords of a circle are 4 cm apart. If the radius of the circle is 4 cm, what is the area of that part of the circle that lies between the chords?
 (A) $8\sqrt{3} + \frac{16}{3}\pi$ (B) $\frac{16}{3}\pi - 8\sqrt{3}$
 (C) $8\sqrt{3} - \frac{32}{3}\pi$ (D) $4\sqrt{3} + \frac{16}{3}\pi$ (E) $\frac{42}{3}\pi$.

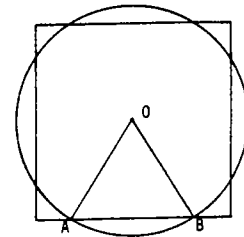
14. How many numbers, which do not start with 0 and which

can be formed by permuting the digits 0, 2, 3, 4, and 5, are divisible by 11?

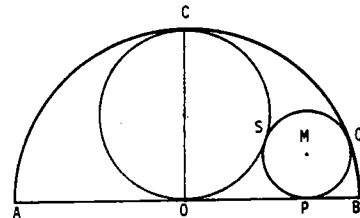
- (A) 4 (B) 8 (C) 12 (D) 16 (E) 20.



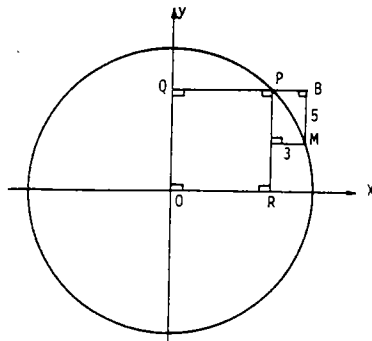
15. In the quadrilateral ABCD it is given that $\hat{BAC} = \hat{BDC} = 90^\circ$. If $AB = DC = 6$ and $BC = 18$, what is AD ?
 (A) 12 (B) 14 (C) 15 (D) 16 (E) $16\frac{1}{2}$.



16. In the figure the circle and square have a common center O. One side of the square intersects the circle at A and B. If the circle and the square have equal areas, what is the value of $\cos \frac{1}{2} \hat{AOB}$?
 (A) 0.5 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{2}\sqrt{\pi}$ (E) $\sqrt{\frac{\pi}{4}}$

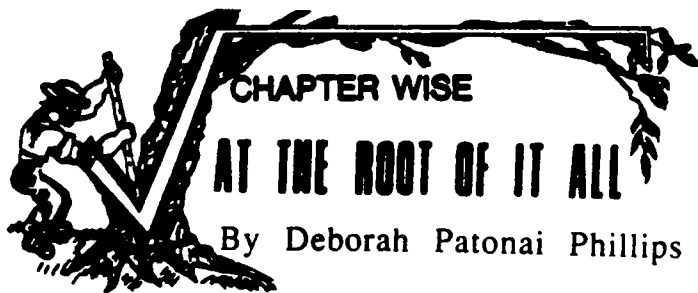


17. A semicircle with center O has diameter AB. The radius OC is perpendicular to AB. A circle with center M touches AB at P, the semicircle at Q, and the circle with OC as diameter at S. If the radius of the semicircle is R, what is the radius of circle M?
 (A) $\frac{2}{9}R$ (B) $(2 - \sqrt{3})R$ (C) $\frac{1}{4}R$
 (D) $\frac{1}{2}(3 - \sqrt{5})R$ (E) $\frac{3}{10}R$.



18. P is a point on the circumference of a circle with center at the origin of a set of axes. PQOR is a square. Another point M on the circumference is 3 units distant from PR and 5 units distant from PQ. What is the equation of the circle?
 (A) $4x^2 + 4y^2 = 289$ (B) $x^2 + y^2 = 34$
 (C) $x^2 + y^2 = 289$ (D) $2x^2 + 2y^2 = 289$
 (E) $x^2 + y^2 = 130$.

(Concluded on page 3)



Since 1977 Mu Alpha Theta has added both student delegates and officers to its governing body. Each summer, every school attending the national convention chooses one of its own Mu Alpha Theta members to represent it at the Delegate Assembly. Meeting together several times during the conference, often very early in the morning, these delegates discuss club activities and concerns involving all Mu Alpha Theta chapters.

After a few meetings together, the delegates divide themselves into Mu Alpha Theta's four "regions" in order to meet others from the same geographical area and to talk over regional topics. As an outgrowth of these divisions, one officer is selected from each region. In a general election by the Delegate Assembly, these four officers are voted in as student president, vice-president, secretary, and sergeant-at-arms. Afterwards, these student officers have the responsibility of working with the delegate Assembly and of relaying their thoughts and concerns to the Governing Council. In fact, the student president is invited to speak to the Mu Alpha Theta Governing Council during one of its meetings. The student president is the direct link between Mu Alpha Theta members and sponsors.

This past summer in Tampa, the Delegate Assembly chose the following students as elected officers for 1989-90: President, Jane Huang, Hanford High School (WA); Vice-President, James Gomez, Berkeley Prep High School (FL); Secretary, Kerry Whiteside, Riverside High School (LA); and Sergeant-at-Arms, Keith Berry, Lebanon High School (TN). In order to acquaint Mu Alpha Theta national membership with these fine officers, this column is honored to introduce each one:

President Jane Huang is a gregarious 160 cm (5'3") senior. At Hanford she is her Mu Alpha Theta chapter's vice-president. In addition to activities such as National Honor Society, Key Club, ASB Executive Council, and the City of Richland Ecology Commission, Jane enjoys playing soccer, tennis and piano.

Loving mathematics, Jane considers Mu Alpha Theta a way to view the subject as exciting, challenging, and fun. For example, her Mu Alpha Theta chapter organizes a problem of the Week designed to stimulate student minds.

Each week a different member picks three fun problems for grades 7-8, 9-10, and 11-12. Distributing the problems to the teachers, the Mu alpha Theta members reward the students who have correct answers. Each winner receives Spudnuts...which are a famous variety of doughnuts in Richland. Mu Alpha Theta provided Jane the opportunity to attend the national convention in Tampa. Experiencing Florida to its fullest, she declares, "I had the opportunity to challenge my mind and bring all the math skills I have ever acquired into use, including speed and accuracy. The competition was intense!"

As National President, Jane has a lot of new ideas for the upcoming year. She intends to correspond with delegates who plan to attend nationals, to start more chapters, and "to give everyone in Mu Alpha Theta a more general awareness of what happens in other chapters." Jane would also love to hear from any member.

Vice-President James Gomez is a Berkeley Prep senior, full of ambition and leadership potential. He leads his school as president of the Math Club as well as of Mu Alpha Theta. He is also actively involved in other clubs, including the Environment Club, the Service Club, and the French Club. In his second year of calculus in preparation for engineering in college, he plans to attend Harvard University or the United States Naval Academy.

Henry David Thoreau said, "The mass of men lead lives

of quiet desperation." James' goal is "to prove Thoreau incorrect" and to give back a little of what his family, friends, teachers, and church have given him.

Secretary Kerry Whiteside is an aspiring, zealous senior at Riverside. A member of the varsity dance team and the junior varsity softball team, she has held the office of class president for her freshman, sophomore, and junior year. Presently, she is serving as the Student Council historian, National Honor Society vice-president, and her Mu Alpha Theta chapter's secretary. Participating in the school's Gifted and Talented Program, Kerry has been selected as a National Merit Semi-Finalist.

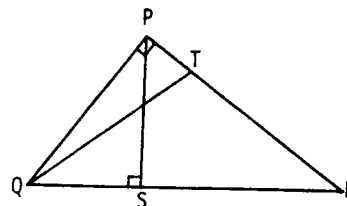
Sergeant-at-Arms Keith Berry is a vivacious, energetic Lebanon senior. Most of his days contain a full schedule of activities. A member of the Beta Club, Junior Classical League, and senior play cast, Keith is also vice-president of the Fellowship of Christian Athletes, Student Council treasurer, and captain of the wrestling team. Not only is Keith serving Mu Alpha Theta as a national officer, he is also chapter president and state vice-president. In his spare time he is a swimming instructor and works with Lads to Leaders, a program designed to develop leadership skills. In this program Keith has won self-starter and spiritual progress awards. Recently, Keith has been a Tandy Scholarship nominee and an applicant for the Century III Leaders Scholarship.

With these impressive lists of credentials, the student officers of Mu Alpha Theta possess the knowledge, the experience, and the skill necessary to make a positive contribution to the MAΘ organization!

High School Competition

...FROM PAGE TWO

19. The minimum value of $\frac{y}{x}$ subject to the condition $(x - 3)^2 + (y - 4)^2 = 5$ is equal to
- (A) $-\infty$ (B) $\frac{8 - \sqrt{5}}{6 + \sqrt{5}}$ (C) $\frac{4 - \sqrt{5}}{3}$ (D) $\frac{1}{2}$
 (E) $\frac{3}{4}$.



20. In triangle PQR it is given that $QP \perp PR$, $PS \perp QR$, and $SR = QT = TR = a$. What is the length of PR in terms of a?
- (A) $\sqrt[3]{2}a$ (B) $\frac{\sqrt{2}}{a}$ (C) $\sqrt{2}a$ (D) $\frac{3\sqrt{2}}{a}$
 (E) $\sqrt{3a}$.

* * *
SECOND ROUND PROBLEMS

- The diagonals of a quadrilateral ABCD intersect at a point O inside the quadrilateral. Show that $(\text{area } AOB)^{\frac{1}{2}} + (\text{area } COD)^{\frac{1}{2}} \leq (\text{area } ABCD)^{\frac{1}{2}}$ and that equality holds if and only if AB CD.
- Assume that p, q, and r are three integers such that the numbers $p^{1/3}$, $q^{1/3}$, and $r^{1/3}$ are three terms (not necessarily consecutive) of an arithmetic sequence. Prove that $(pqr)^{1/3}$ is a rational number.
- It is given that $\sin 18^\circ = a + b\sqrt{5}$, where a and b are rational numbers. Determine a and b.
- Determine all five-digit numbers x which are such that the last five digits of the number x^2 is again x.

FOR CHAPTER CONSIDERATION

Some Theorems, Conjectures, and Open Questions
Lending Themselves to Number Exploration

By Don Allen

A recent public lecture on experimentation and exploration in the mathematical "number" strand caused us to consolidate and share a thirty-year collection of elementary theorems, conjectures, and open questions lending themselves to this kind of student investigation. Student response to the questions leads us to offer the list to Mu Alpha Theta readers desiring to investigate one or more of the questions and to share findings at a chapter-level student presentation.

Take "number" to refer to a counting number, 1, 2, 3, ..., and "prime number" to be a counting number having exactly two divisors, 1 and itself.

1. Products and powers of odd numbers are odd. Products and powers of even numbers are even.

2. Square numbers never end in the digits 2, 3, 7, or 8, and triangular numbers never end in 2, 4, 7, or 9.

3. The "digital root" of a number is found by summing the digits of the number, then summing the digits of the resulting sum, etc., repeating the process until a single digit, 1 to 9, is obtained. Thus, the digital root of 3794 is found as $3 + 7 + 9 + 4$, summing to 23; then $2 + 3$, summing to the single digit (digital root) 5. The digital root of a square number must be 1, 4, 7, or 9; of a triangular number must be 1, 3, 6, or 9; and of a cubic number must be 1, 8, or 9.

4. Three consecutive cubes must add to a multiple of 9.

5. Between any number (greater than 1) and its double is at least one prime (Tchebychef, 1850)

6. Sums $1 + 2 + 4$, $1 + 2 + 4 + 8$, $1 + 2 + 4 + 8 + 16$, ..., are alternately prime and composite. (Tartaglia, 1556)

7. Even numbers greater than 4 can be written as the sum of two odd primes. (Goldbach's conjecture, 1742)

8. Odd numbers greater than 3 can be written as a prime plus a power of 2. (de Polignac, 1848)

9. Odd numbers can be written as a prime (or 1) plus twice a square. (Goldbach, 1752)

10. Odd numbers greater than 5 can be written as a prime plus twice a prime. (Lagrange, 1775)

11. Numbers greater than 9 can be written as the sum of distinct odd primes.

12. Numbers greater than 11 can be written as the sum of two composites.

13. Consider two consecutive odd numbers, one of them one less than, the other one more than, a multiple of 6. One or both of the numbers will be prime. (DeBouvelles, 1509)

14. Multiply all the primes up to a stipulated prime, then add 1. The result will be a new prime. (Considered by Euclid)

15. Multiply all numbers up to a stipulated number, n , then add 1. If the result is divisible by $n + 1$, then $n + 1$ is a prime ... and conversely. (Wilson, 1770)

16. "Prime-rich" polynomial expressions include $n^2 + n + 17$ (Barlow), $n^2 + n + 41$ (Euler), $2n^2 + 29$, and $n^2 - 7n + 1601$.

17. Units digits 1, 3, 7, and 9 are equally likely in the sequence of primes. All nine digital roots are equally likely.

18. "Numbers of the even type" are numbers having an even number of prime factors, as $360 = 5 \times 3 \times 3 \times 2 \times 2 \times 2$, which has six. "Numbers of the odd type" have an odd number of prime factors, as $540 = 7 \times 3 \times 3 \times 2 \times 2$, which has five. Counting from 2 up, at any given stage the number of odd-type numbers will exceed the number of even-type numbers. (Polya, 1919)

In addition to "theorems, conjectures, and open questions," students at our Montréal lecture were left with

three further "invitations to number exploration":

1. Hailstone Numbers, the $3N + 1$ Problem, or the Collatz Conjecture. Choose as starting value any positive integer, N . Necessarily, N is even or N is odd. If N is even, then divide N by 2. If N is odd, multiply N by 3, then add 1. Repeat the procedure, applying it to the new integer so obtained. Continue repetition, and observe the resulting integer sequence. For most starting values, integers strikingly rise and fall, hence the aptness of the allusion to hailstone formation. Any starting value from which the derived sequence ultimately tumbles to 1 (entering a "1 4 2" loop) is termed a hailstone number. Informal investigation will suggest possible lines of inquiry. Which positive integers, N , are indeed hailstone numbers? Which, if any, are not? For a given starting value, say $N = 27$: (i) how many steps are required to reach 1? (ii) how high does the sequence rise (to what maximum value) before its fall? Can we, in general (for a stipulated N) predict path length, maximum value, or hailstone status? The problem is attributed to one Lothar Collatz, when a student in the 1930's: it has attracted considerable attention, notably in computer circles, in recent years. Basic questions remain unanswered.

2. Palindromes from Number Reversals. Choose as starting value a positive integer of four--or more--digits. Reverse the digits, retaining any zeros, and add the original number and the new number so obtained. (For starting value 3549, reverse to 9453, then $3549 + 9453 = 13002$.) Repeat the process until a palindromic number (one whose digit sequence reads the same in both directions) is reached. For different starting values give thought to (i) the inevitability of the conclusion and (ii) the number of steps required.

3. Kaprekar's Constant. Choose as starting value a four-digit positive integer, N , having not all digits the same. Denote by n_1 the integer obtained by writing the digits of N in descending order, and by n_2 the integer obtained by writing the digits of N in ascending order. Form from N a new integer, N' , given by $n_1 - n_2$, retaining any initial zeros at each step. (For starting value $N = 7280$, $n_1 = 8720$, $n_2 = 0278$, and $N' = n_1 - n_2 = 8442$.) Repeat the process with N' . Continue until you reach an unchanging value. This value will be 6174, known as Kaprekar's constant, for D. R. Kaprekar, the mathematician who investigated the phenomenon. Query: Are there numbers analogous to Kaprekar's constant for number bases other than ten or for integers of other than four digits?

New Student Journal

If: $a + b + c = -3$

$$a \cdot b \cdot c = 6$$

and $(1 - a)(1 - b)(1 - c) = -4$,

then what is: $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$?

The question (H19, November-December 1989), addressed to secondary students, appears in a new periodical which has found its way to the Mathematical Log editorial desk. Subtitled "A Mathematical Journal for High School and University Students, Mathematical Mayhem is being published five times a year with financial support from the Samuel Beatty Foundation at Canada's University of Toronto.

Subscriptions to U.S. addresses are available for US \$10, payable to Mathematical Mayhem and sent to co-editor Patrick Surry, 87 Wakefield Crescent, London, Ontario, Canada N5X 1Z6.

Cultural Heritage

Realms of Mathematics and of Real People Come Together in Remarkable Interface

(In talks at 11 Mu Alpha Theta national conventions, the retiring editor has underlined the fascination of the "interface" between mathematics and "real people." In his final article for *Log* readers, he develops some related ideas.)

By Don Allen

Some of the oldest ideas of the Theory of Numbers--primes, divisor sums, amicable numbers--are surely the "purest" of mathematics, concepts elaborated for their innate interest or attractiveness, with no "justification" in terms of utility. The Greek philosopher-mathematician could savor the logical coherence of definition and extension of such concepts, reflect upon the elegance of reasoning and demonstration.

Pure mathematics! Amicable numbers are pairs of [counting] numbers, the sums of the proper divisors of each of which is equal to the other number. (Proper divisors are divisors less than the number itself: 6 has divisors 1, 2, 3, and 6; and proper divisors 1, 2, and 3. Numbers such as 6, 28, and 496 which are the sum of their respective proper divisors were considered "perfect," and rather special; but that's another story.) The number 220 has proper divisors 1, 2, 4, 5, 10, 11, 10, 20, 22, 44, 55, and 110, which sum to 284. Now, 284 has proper divisors 1, 2, 4, 71, and 142, which sum to 220, making 284 and 220 an "amicable" pair, a fact which was known to Pythagoras.

Hundreds of amicable pairs have been found, including 17 296 and 18 416, attributed to Fermat, and 9 363 584 and 9 437 056, credited to Descartes. (Burton, p. 234)

Public response? In the Middle Ages it was common to split a talisman inscribed with such an amicable pair as 220 and 284, each partner carrying one part--and one amicable number--as a token, and promoter, of mutual affinity!

The incident is not isolated. After all, "7" is a "lucky" number in many societies, "13" unlucky, and even and odd numbers were declared to be masculine and feminine by both the Pythagorean brotherhood (essentially a number cult) and the ancient Chinese--who adopted opposite conventions as to which was which.

Mathematics is logical, rational (in the "reasoned" sense); human nature can be anything but! The interface between "numbers" and real people, according, can be a most fascinating realm to explore.

Number mumbo-jumbo persists, living and well in a post-industrial information age. A numerology textbook chanced upon in a used book store underscores as Pythagorean philosophy that "man could grasp the nature of the universe only through number and form." (Goodman, p. 13) The four elements, Fire, Water, Air, and Earth, "comprised the holy fourfoldness, or tetraktys, to which Pythagoreans [prayed]." The same work assures us that we personify "all that is sought of in a collaborator," because we were born on the 2nd day of a month, but that hourly, daily, four-month, and yearly "cycles" demanded consideration, as did vowel and consonant "vibrations."

Numerology, like astrology, thrived in an age of ignorance, superstition, and reasonable fear of the less-than-understood. Today, both could be laughed off, though few take such an approach. Martin Gardner does, chronicling the utterances and adventures of his all-too-believable "Dr. Matrix" (*Scientific American* columns, and 1985 reference). Elsewhere, in more serious vein, Gardner takes on the whole "paranormal" establishment (1981 and 1988 references). Having recently watched a computer printer churn out highly overpriced sine waves as "biorhythms," we urge Gardner's fine writings as material for ingestion and reflection.

Currently being marketed locally: four octahedral dice (of the kind favored for imaginative board games), and a text to aid in seeing one's "destiny" in the random numbers so generated ... \$29.95.

Meanwhile, clovers with four leaflets are "lucky," and (as we've noted) our recent departure at Philadelphia airport had no gate between 12 and 14.

Superstition and allied silliness set aside, the reaction of "the man in the street" on coming face to face with mathematics or a mathematician can be revealing. "I never could do it," is a common, unsolicited, initial response. But listen carefully, as we have, to such observations, and build your insights. The first reader of each new *Log* is a high-IQ pediatrician, husband of a colleague, and the importance of his math-major background to his present perspective is evident. Our dentist, when we can't talk back, boasts how he, a Phys Ed student (and top athlete) led his Calculus class, beating student Engineers and such ... perhaps thirty years ago. He remembers. Our lawyer, in a recent property acquisition, grateful accepts a stack of *Logs* for evening reading. He was a math major at university, he has chosen to point out. Surprising, isn't it, how the ex-math students you meet are the successes ... often in fields not directly related to computation.



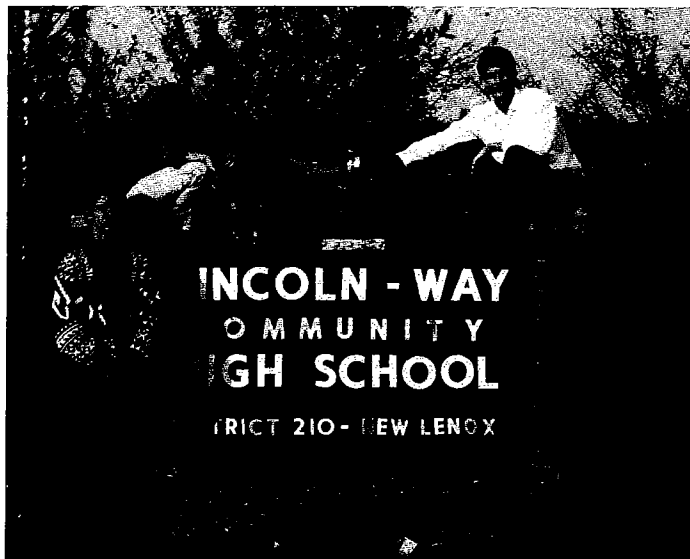
AUSTRALIAN "DICE GAME" POSES QUESTIONS

For others, for far too many in North America, mathematical feeling and appreciation are poor, the ability to "ball park" an estimate or size up a quantitative or logical situation is underdeveloped. Two areas were intuition commonly is inadequate is randomness and, relatedly, the mathematics of chance. Talk to people about lotteries or weather records or sports scores, and listen to what is not said in their responses.

The interface between real math and real people can be one of fear--"math phobia" and all that that implies--or one of challenge and fun. Whole areas of recreational mathematics await exploration--check the section in your community or university library. You'll be glad that you did.

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RECEPTION COMMITTEE. Mu Alpha Theta members at Lincoln-Way soon will be scouting Chicago O'Hare, familiarizing themselves with jet-age and down-to-earth gateways to Illinois and to Mu Alpha Theta's forthcoming 20th National Convention. On hand to meet planes and other arrivals will be chapter president John M. Drozdek, secretary Eric A. Twietmeyer; also (front) vice-president David Mundo and treasurer James P. Bergstrom. Convention co-chairs are Ron Vavriner and Patti Bowler-Johnson, and meetings will be at Northern Illinois University, DeKalb, August 4-9. Don Allen photo.

Quebec Contest Questions Seek Elegance of Solution

Chapters and individual members who find challenge and good discussion in nonroutine questions from competitions and similar sources may wish to look to a recent Canadian competition, the 1990 Junior Mathematics Competition of the Quebec Association of Mathematics Teachers. Questions, while somewhat tied to a government syllabus, are of interest in being non-multiple choice and placing a premium on "succinctness and/or elegance in the solutions."

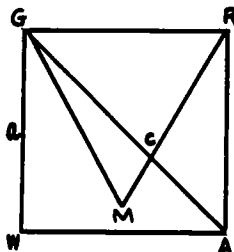
The three-hour examination (no calculators permitted) was presented in two parts, with a total of 11 questions.

PART A

- If $f(x) = 2^{4x-2}$, find $f(x)f(1-x)$.
- If $A = 4^x + 4^{-x}$ and $B = 4^x - 4^{-x}$, evaluate $A^2 - B^2$.
- If your mathematics textbook is open and the product of the page numbers is 2256, on what pages are you?
- For all positive real numbers, $a*b$ is defined by the equation $a*b = \frac{ab}{a+b}$. If $2*x = 3*4$, find x .
- Find x if $\log_k x \cdot \log_5 k = 3$.
- Solve for x : $8^{1/6} + x^{1/3} = \frac{7}{3 - \sqrt{2}}$.
- The length of the median of triangle XYZ is 28 and the length of the side to which it is drawn is 42. If the perimeters of each of the two smaller triangles are equal, find the perimeter of triangle XYZ.

PART B

1. If GRAW is a square of side length a , and triangle GMR is equilateral, what is the exact area of triangle RCA?



2. If the sum of n terms of $2+7+12+\dots$ equals the sum of n terms of $50+47+44+\dots$, find the value of n .

3. A root of a third degree equation is one. When the cubed term of the equation is eliminated, the resulting quadratic equation has a root of two. When the squared term is also eliminated, the resulting linear equation has a root of three. Reconstruct the original third-degree equation, expressing it in the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c , and d represent relatively prime integers.

4. The number 695 is to be written with a factorial form of numeration $[695 = a_1 + (2!)a_2 + (3!)a_3 + \dots + (n!)a_n]$; a_1, a_2, \dots, a_n are integers such that $0 \leq a_k \leq k$ and $n!$ means $n(n-1)(n-2)(n-3)\dots(3)(2)(1)$. Find a_4 .

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